Quantum Computers and Decoherence: Exorcising the Demon from the Machine

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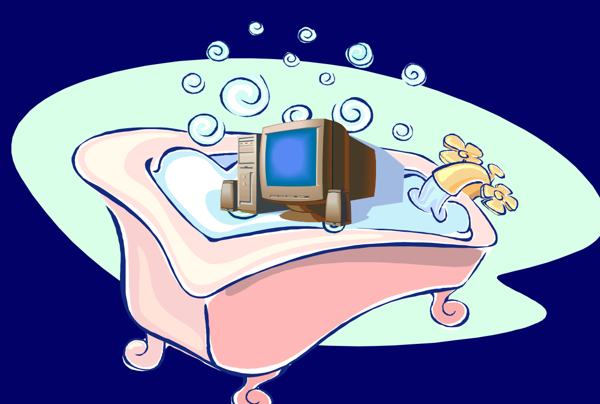
The Problem

Decoherence



The Arsenal

- Active Quantum Error Correction: Error correcting codes
- Passive Error Prevention: Decoherence-free subspaces and (noiseless) subsystems
- Dynamical Decoupling: Strong and fast "bang-bang" pulses
- Topological & Holonomic Methods: Nonabelian anyons, Toric codes, Adiabatic elimination, …
- Continuous Quantum Control: Closed-loop feedback

Underlying Paradigm

Adapt decoherence-resistance method to a model of decoherence

E.g.:

•Quantum error correction: assumes local, uncorrelated errors

•Decoherence-free subspaces: assumes a symmetry in system-bath interaction

•Dynamical decoupling: assumes bath with long correlation time

Focus on different Primary Object: Set of "Naturally" Available Interactions and Measurements

For given proposed realization of a QC:

- What are the controllable terms in the *internal* Hamiltonian?
- What are the possible *external* unitary control options?
- What are the possible measurements?

Determines options for *both* **decoherence control** and **quantum computation** (universality of logic gates), typically via an encoding

Interaction capable of doing both will be called "Super-Universal"

Examples of "Naturally Available" Interactions

- Electrons spin in quantum dots, nuclear spin in doped atom arrays: Heisenberg exchange interaction easily controllable, single-spin operations are hard
- Linear optics: single-photon gates easy, photon-photon interaction is hard
- Trapped ions: relative phase between lasers easily controllable, absolute phase is hard
- Superconducting flux qubits: application of local bias magnetic field hard, controllable Josephson coupling easy
- …

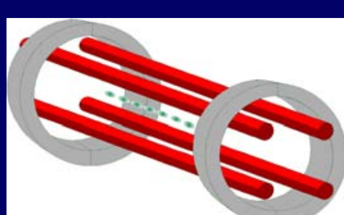
Plan

Show how options for

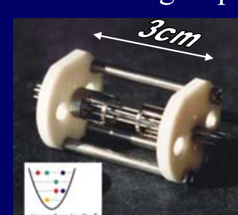
1. Universal QC
2. Decoherence reduction

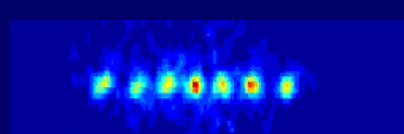
   are determined naturally by set of available and controllable interactions.

Trapped Ions



Innsbruck group





few μm

Qubit: two hyperfine states of trapped ion

Natural control options

- Efficient single-qubit measurements (cycling transition)
- Sorensen-Molmer gates (insensitive to heating of ions center of mass motion)

$\propto$ Rabi freq. Laser phase on ions 1,2

$$U_{12}(\theta,\phi_1,\phi_2)=\exp[i\theta(\sigma_x\cos\phi_1+\sigma_y\sin\phi_1)\otimes(\sigma_x\cos\phi_2+\sigma_y\sin\phi_2)]$$

How to avoid control of absolute phase??

Two-Qubit DFS Encoding

$$|0\rangle_L=|\downarrow\rangle_1\otimes|\uparrow\rangle_2$$
$$|1\rangle_L=|\uparrow\rangle_1\otimes|\downarrow\rangle_2$$

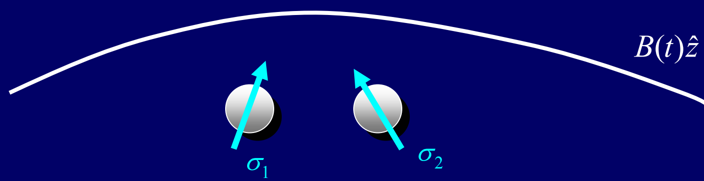
$$U_{12}(\theta,\phi_1,\phi_2)=\exp[i\theta(\sigma_x\cos\phi_1+\sigma_y\sin\phi_1)\otimes(\sigma_x\cos\phi_2+\sigma_y\sin\phi_2)]$$

$$\overset{DFS}{\mapsto}\exp[i\theta(\bar{X}\cos(\phi_1-\phi_2)+\bar{Y}\sin(\phi_1-\phi_2))]$$

$\therefore$ Can generate all single DFS-qubit operations by controlling **relative** laser phase.
Same true for controlled-phase gate between two DFS qubits

Same encoding protects against *collective dephasing*: the chief source of decoherence in trapped ions

Collective Dephasing



Long-wavelength magnetic field B (bath) couples to spins:

$$H_{\text{int}} = -gB\left(\sigma_1^z + \sigma_2^z\right) = \begin{pmatrix} -2gB & & & \\ & 0 & & \\ & & 0 & \\ & & & 2gB \end{pmatrix} \quad \begin{matrix} |\downarrow\rangle_1|\downarrow\rangle_2 \\ |\downarrow\rangle_1|\uparrow\rangle_2 \\ |\uparrow\rangle_1|\downarrow\rangle_2 \\ |\uparrow\rangle_1|\uparrow\rangle_2 \end{matrix}$$

Encode qubit into states with M_z=0:

$|0\rangle_L = |\downarrow\rangle_1 \otimes |\uparrow\rangle_2$

$|1\rangle_L = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2$

$|\psi\rangle_L = a|0\rangle_L + b|1\rangle_L$ is decoherence-free

"A Decoherence-Free Quantum Memory Using Trapped Ions"
D. Kielpinski et al., Science **291**, 1013 (2001)

Bare qubit:
two hyperfine states of trapped $^9Be^+$ ion

Chief decoherence sources:
(i) fluctuating long-wavelength ambient magnetic fields;
(ii) heating of ion CM motion during computation

DFS encoding: $|0\rangle_L = |\downarrow\rangle_1 \otimes |\uparrow\rangle_2$
into pair of ions $|1\rangle_L = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2$

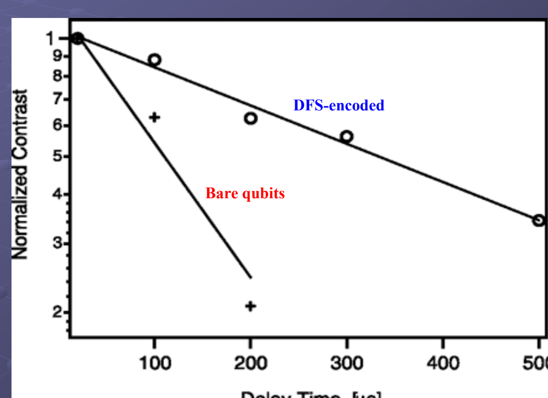


Figure 2. Decay of the DFS-encoded state (circles) and the test state (crosses) under ambient decoherence. We vary the delay time between encoding and decoding to give the ambient noise a variable time to act. Coherence data are normalized to their values for zero applied noise. The fit lines are exponential decay curves for purposes of comparison and are not theoretical predictions. The decay rate of the test state is $(7.9 \pm 1.5) \times 10^3/\mu s$, whereas the decay rate of the DFS state is $(2.2 \pm 0.3) \times 10^3/\mu s$. **Because the coherence time of the DFS-encoded state is much longer than that of the test state, we see that the chief source of ambient decoherence is collective dephasing**.

Gate Control Option Motivates Classification of all Decoherence Processes on Two Qubits (Ions)

$$H_{SB} = H_{DFS} + H_{Leak} + H_{Logical}$$

$|0\rangle_L = |\downarrow\rangle_1 \otimes |\uparrow\rangle_2$

$|1\rangle_L = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2$

immune

storage:

collective dephasing (σ_z):

$$H_{DFS} = \{\frac{ZI+IZ}{2}, \frac{XY+YX}{2}, \frac{XX-YY}{2}, ZZ, II\} \otimes B$$

differential dephasing:

$$H_{Logical} = \{\bar{X} = \frac{XX+YY}{2}, \bar{Y} = \frac{YX-XY}{2}, \bar{Z} = \frac{ZI-IZ}{2}\} \otimes B$$

computation:

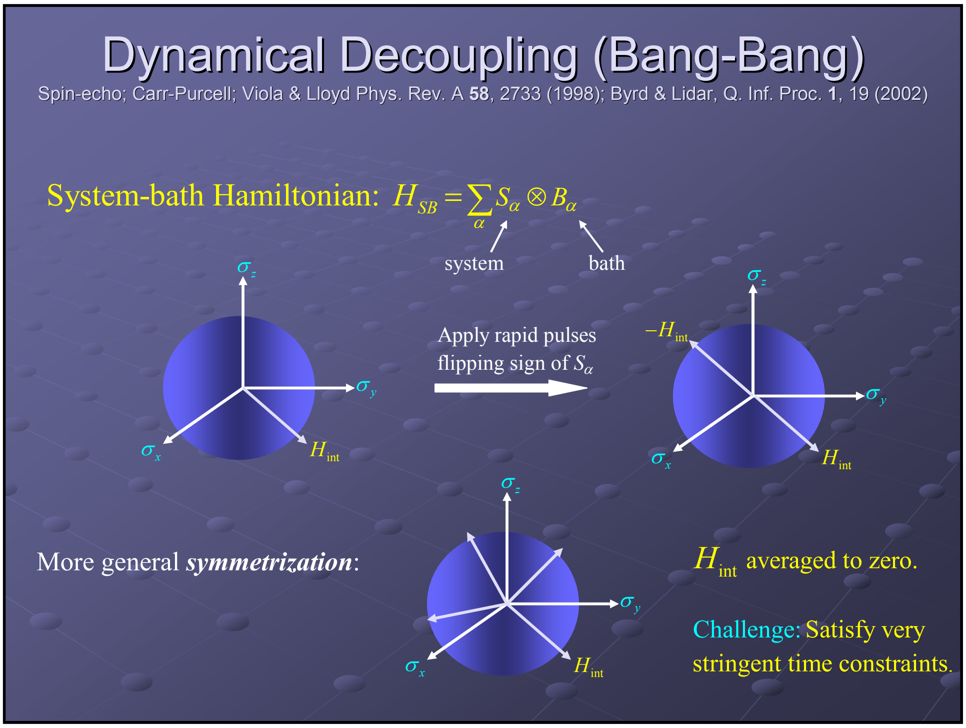
motional decoherence:

$$H_{Leak} = \{XI, IX, YI, IY, XZ, ZX, YZ, ZY\} \otimes B$$

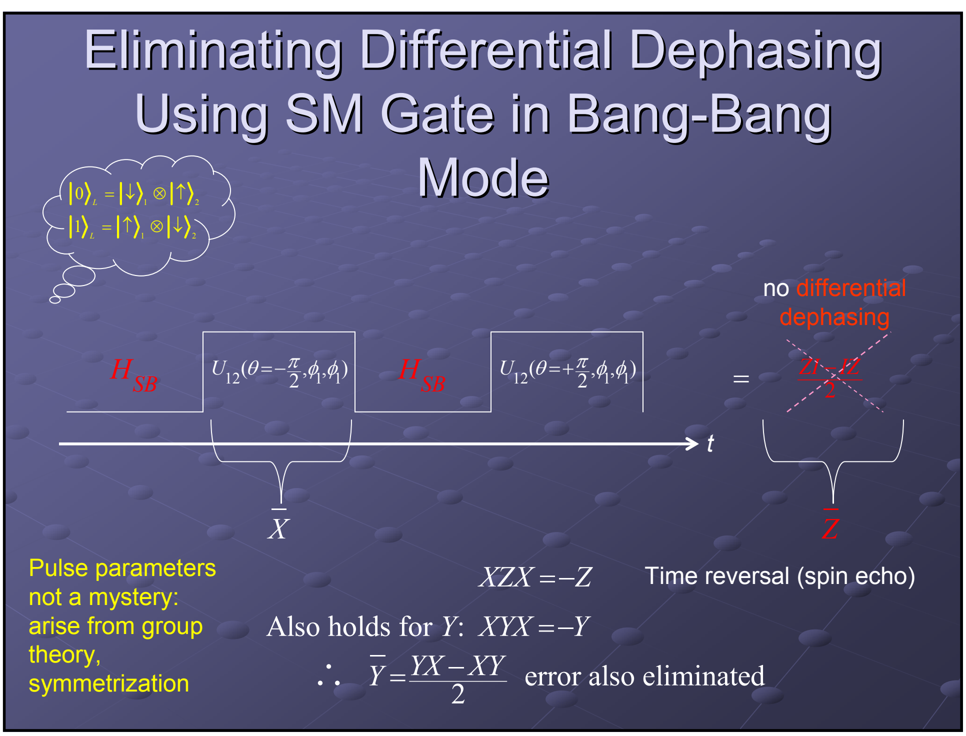
Can *all* decoherence be eliminated using just DFS encoding & Sorensen Molmer gates?

Options:

- Apply active quantum error correction. Problem: not known how to do using only Sorensen Molmer gates.
- Topological, Holonomic: ??
- Dynamical decoupling. ✓

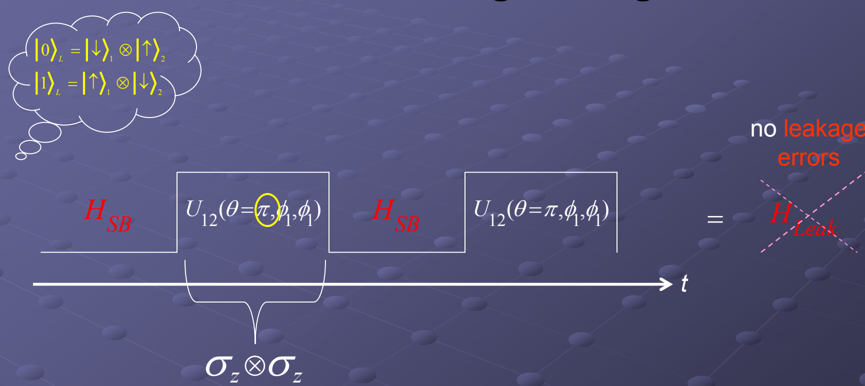


Dynamical Decoupling (Bang-Bang)
Spin-echo; Carr-Purcell; Viola & Lloyd Phys. Rev. A **58**, 2733 (1998); Byrd & Lidar, Q. Inf. Proc. **1**, 19 (2002)
System-bath Hamiltonian: $H_{SB} = \sum_\alpha S_\alpha \otimes B_\alpha$
system
bath
σ_z
σ_y
σ_x
H_{int}
Apply rapid pulses flipping sign of S_α
$-H_{\text{int}}$
More general ***symmetrization***:
H_{int} averaged to zero.
Challenge: Satisfy very stringent time constraints.



Eliminating Differential Dephasing Using SM Gate in Bang-Bang Mode
$|0\rangle_L = |\downarrow\rangle_1 \otimes |\uparrow\rangle_2$
$|1\rangle_L = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2$
no differential dephasing
H_{SB}
$U_{12}(\theta = -\frac{\pi}{2}, \phi_1, \phi_1)$
H_{SB}
$U_{12}(\theta = +\frac{\pi}{2}, \phi_1, \phi_1)$
$= \frac{ZI - IZ}{2}$
t
$\bar{X}$
$\bar{Z}$
Pulse parameters not a mystery: arise from group theory, symmetrization
$XZX = -Z$
Time reversal (spin echo)
Also holds for Y: $XYX = -Y$
$\therefore \quad \bar{Y} = \frac{YX - XY}{2}$ error also eliminated

Elimination of all Leakage Using SM Gate in Bang-Bang Mode



$$\sigma_z \otimes \sigma_z H_{Leak} \sigma_z \otimes \sigma_z = -H_{Leak}$$

$$H_{Leak} = \{XI, IX, YI, IY, XZ, ZX, YZ, ZY\} \otimes B$$

SM Pulses are Super-Universal

- Methods above can be used to eliminate all dominant errors (differential dephasing + leakage) in a 4-pulse sequence
- To eliminate ALL two-qubit errors (leaving DFS encoding intact) need a 10-pulse sequence.
- Scheme entirely compatible with SM-gates to perform universal QC inside DFS.

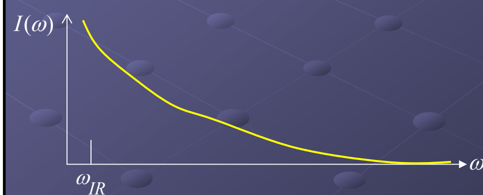
L.-A. Wu, M.S. Byrd, D.A.L., *Phys. Rev. Lett.* **89**, 127901 (2002).
L.-A. Wu, D.A.L., *Phys. Rev. Lett.* **88**, 207902 (2002).
D.A.L. and L.-A. Wu, *Phys. Rev. A* **67**, 032313 (2003).

Are the time-scales feasible?

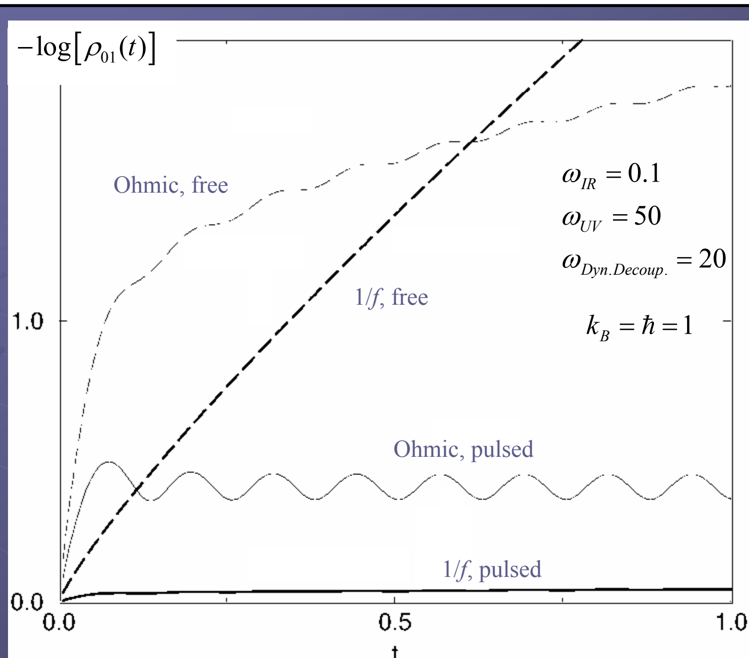
Standard BB time-scale assumption: *pulses need to be faster than fastest bath time-scale* (inverse of bath high-freq. cutoff): ~10ns for fluctuating patch potentials. Not feasible with SM pulses: 1μs.

However, this relies on bath with Debye-like spectral density:

Measurements for trapped ions indicate **1/*f*-type spectrum:**

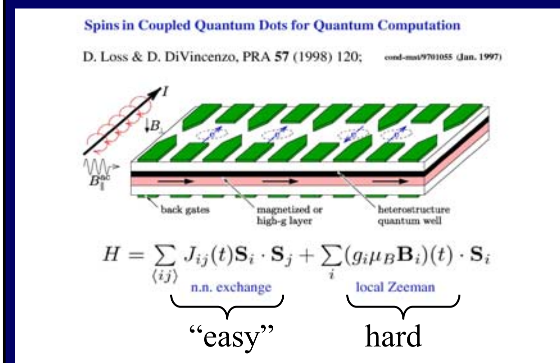


Implies much relaxed time-constraints (K. Shiokawa & D.A.L., quant-ph/0211081): time-scale set by bath **low**-freq. cutoff. Our scheme then appears *feasible. Experimental verification welcome.*

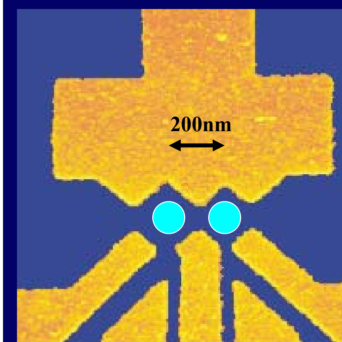


K. Shiokawa, D.A.L., quant-ph/0211081

Nanofabricated Quantum Dots



Delft qubits



Natural control options

- Two-spin measurements distinguishing singlet from triplet
- Heisenberg exchange gates generated from

$$H_{\text{Heis}} = \sum_{i<j} J_{ij}(t)\vec{\sigma}_i \bullet \vec{\sigma}_j$$

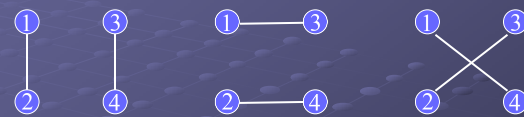
$J_{ij}(t)$ controllable via applied gate voltages + global magnetic fields

Challenge:

Implement **everything** (universal QC, decoherence elimination) using only Heisenberg exchange interactions.

Four-Qubit DFS Encoding

$|s\rangle_{ij} = \frac{1}{\sqrt{2}}\left(|0\rangle_i |1\rangle_j - |1\rangle_i |0\rangle_j\right)$



$|0\rangle_L = |s\rangle_{12} \otimes |s\rangle_{34}$ $\quad |a\rangle = |s\rangle_{13} \otimes |s\rangle_{24}$ $\quad |b\rangle = |s\rangle_{14} \otimes |s\rangle_{23}$

$|1\rangle_L = \frac{1}{\sqrt{3}}\left(|a\rangle + |b\rangle\right)$

∴ Can generate all single encoded-qubit operations by controlling Heisenberg exchange interactions:

$$\overline{Z} = -\sigma_1 \cdot \sigma_2 \qquad \overline{X} = -\frac{2}{\sqrt{3}}\left(\sigma_1 \cdot \sigma_3 + \frac{1}{2}\sigma_1 \cdot \sigma_2\right)$$

Same is true for controlled-phase gate between two DFS qubits

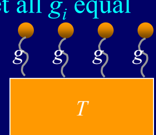
D. Bacon , J. Kempe, D.A.L. and K.B. Whaley, *Phys. Rev. Lett*. **85**, 1758 (2000).

This encoding protects against *collective decoherence.*

Collective Decoherence

$$H_{\text{int}} = \sum_{i=1}^{n} g_i^x \sigma_i^x \otimes B_i^x + g_i^y \sigma_i^y \otimes B_i^y + g_i^z \sigma_i^z \otimes B_i^z$$

Collective Decoherence:
set all g_i equal



Collective interaction:

$S_z = \sum_{i=1}^{n} \sigma_i^z$ etc., total (pseudo-)angular momentum operators

$H_{\text{int}} = S_x \otimes B_x + S_y \otimes B_y + S_z \otimes B_z$

S_z only: Collective dephasing (abelian)

$\{S_x, S_y, S_z\}$: Collective decoherence ($su(2)$)

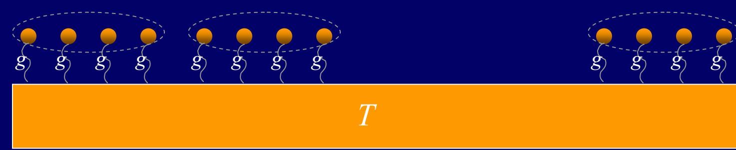
Decoherence-free states:

Singlets: states with zero total angular momentum J,

$\vec{J} = (S_x, S_y, S_z)$

Scaling Up

$\{|0_L\rangle, |1_L\rangle\} \otimes \{|0_L\rangle, |1_L\rangle\} \otimes \cdots \otimes \{|0_L\rangle, |1_L\rangle\}$



Assumption of collective decoherence less accurate the larger the number of physical qubits.

Other sources of decoherence necessarily appear.

Just as in two-qubit (trapped-ion) case, all other sources can be classified as

- Leave DFS invariant
- Leakage
- Logical errors

Can be eliminated using dynamical decoupling with Heisenberg

Dynamical Generation of Collective Decoherence

By rapid pulsing of $H_{\text{Heis}} = \frac{J}{2}\left(\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y + \sigma_1^z \sigma_2^z\right)$
collective decoherence conditions can be created
for arbitrary linear system-bath interaction:

$$H_{\text{int}} = \sum_{i=1}^{n} g_i^x \sigma_i^x \otimes B_i^x + g_i^y \sigma_i^y \otimes B_i^y + g_i^z \sigma_i^z \otimes B_i^z$$
$$\longrightarrow S_x \otimes B_x + S_y \otimes B_y + S_z \otimes B_z$$

Requires sequence of 6 $\pi/2$ pulses to create collective decoherence conditions over blocks of 4 qubits.

Details: L.-A. Wu, D.A.L., *Phys. Rev. Lett.* **88**, 207902 (2002).

Bilinear system-bath interaction (e.g., $\sigma_i^x \sigma_j^y \otimes B$) causes
logical errors ($\sigma_i \sigma_j$) and leakage.
Leakage part can be eliminated using Heisenberg, with two π pulses.

Details: L.-A. Wu, M.S. Byrd and D.A.L, *Phys. Rev. Lett*. **89**, 127901 (2002).

Heisenberg is Super-Universal

- Heisenberg exchange is naturally available interaction for spin-coupled Q. dots, doped atom arrays.
- It alone suffices for
  - Universal QC
  - Dynamical generation of collective decoherence
  - Leakage elimination
- This works in conjunction with DFS encoding

Generalization and Summary

- The available/controllable interactions $\{H_i\}$ are the primary object in Q. information processing
- They define an associative algebra
- The commutant of this algebra are the system-bath interactions that leave the system invariant
- This endows Hilbert space with a preferred encoding: the DFS
- In some cases the $\{H_i\}$ suffice to dynamically generate the commutant from an arbitrary system-bath interaction. In this case the $\{H_i\}$ are "super-universal".

- Heisenberg exchange
- Group algebra of the permutation group
- Collective decoherence processes
- The 4-qubit code (for example)
- Generation of collective decoherence from arbitrary linear system-bath interaction; leakage elimination

Similar conclusions seen for Heisenberg hold for anisotropic exchange models (e.g., XY, XXZ).

The role of the controllable interactions is primary in universality and combatting decoherence

Open question:

Can the duality

controllable ←→uncontrollable interactions be used in quantum error correction, topological codes, etc.?

Thanks

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Further Reading

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