Brute force quantum-assisted computation

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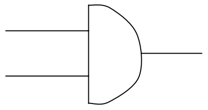
Standard approach to computation

- Set up problem as that of computing F(x,y,z,…)
- Define a series of gate operations which will achieve computation
- Present input values x,y,z…
- Display output F

CLASSICAL INFORMATION PROCESSING:

Information represented as binary digits (0 *or* 1)

Processing of information performed by logic gates (e.g. AND gates)



	0	1
0	0	0
1	0	1

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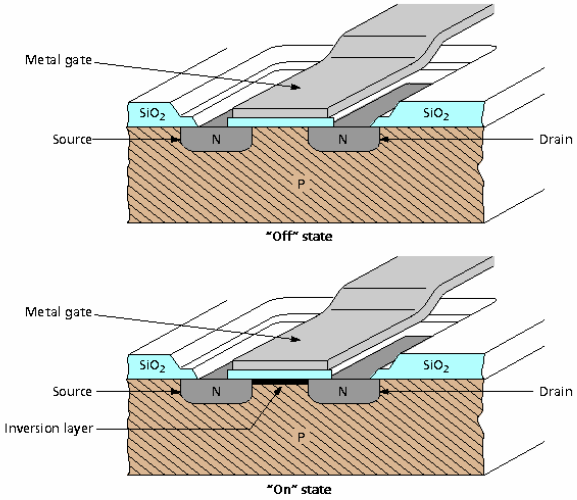
realisation: the silicon MOSFET



Simulated annealing approach to computation (Kirkpatrick et al)

- Cast computation as a minimization problem, i.e. want to find global minimum for F(x,y,z,…) subject to some constraints
- Instead of developing gate-based algorithm for searching phase space(x,y,z…), introduce ‘temperature’ to weight state probabilities, ‘warm’ the system up & cool slowly to find optimum, exactly as happens for natural systems settling into their ground states

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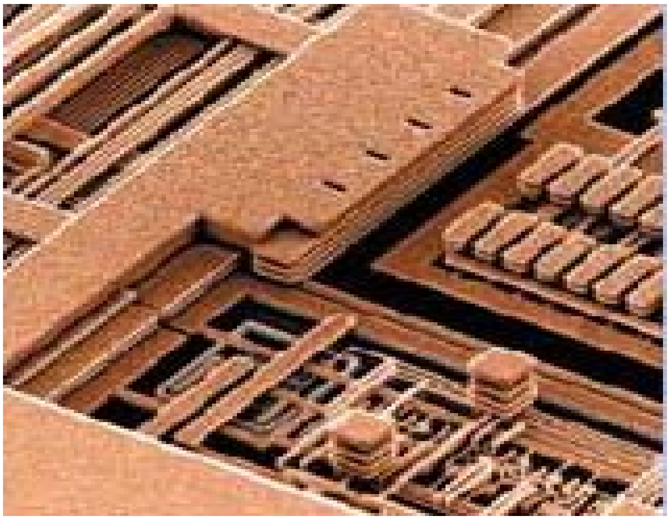
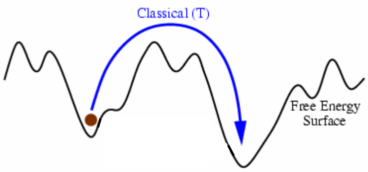
Traveling salesman

Circuit optimization

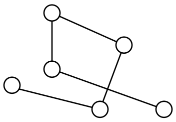


Materials physics:
Solve Complex
problem
Via
Thermal Annealing



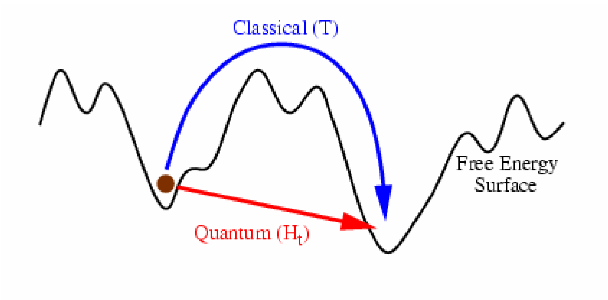
Computation:
Solve Complex problems
Via
Simulated Annealing

- Traveling Salesman



- Circuit Optimization

What happens when we add quantum mechanics?



Is there added efficiency?

Experimental test

- Find a material with a complex free energy surface where quantum and thermal fluctuations can be tuned independently
- Establish that tunneling picture is appropriate
- Try thermal and quantum annealing protocols to see if anything different

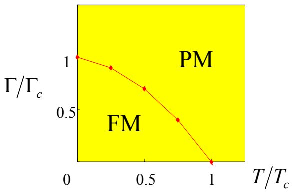
Simplest quantum magnet

Ising model in a transverse field:

$$H = -\sum_{i,j}^{N} J_{i,j}\sigma_i^z\sigma_j^z - \Gamma\sum_{i}^{N}\sigma_i^x$$

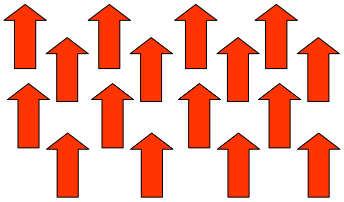
Quantum fluctuations matter for $\Gamma \neq 0$:

$$-\frac{\hbar}{i}\frac{\partial S_z}{\partial t} = [H, S_z] \neq 0$$



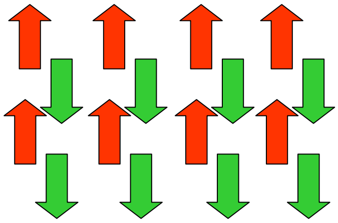
$\Gamma_c \sim kT_c \sim J$

Interactions J_{ij} can be simple …



Ferromagnet

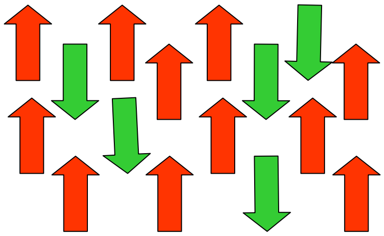
J<0



Anti-ferromagnet

J>0

Or, they can introduce complexity..



Disorder
FM with
some
AFM
bonds in
addition
to FM

Why is the disordered magnet problem hard?

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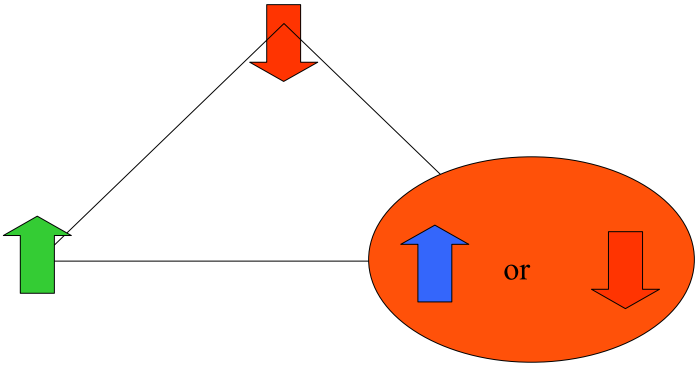
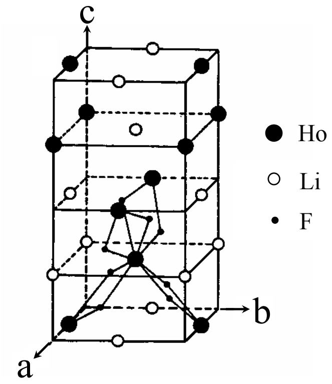
frustration



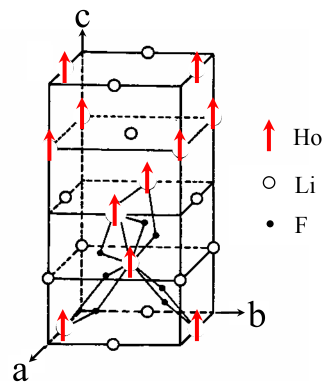
Need a magnet with independently adjustable J_{ij} and Γ...

Realizing the transverse field Ising model, where can vary Γ – $LiHoF_4$



- g=14 doublet
- 9K gap to next state
- dipolar coupled

Realizing the transverse field Ising model, where can vary Γ – LiHoF$_4$



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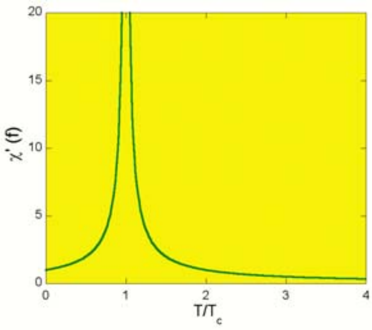
Susceptibility

$$\chi \equiv \frac{dm}{dh}$$

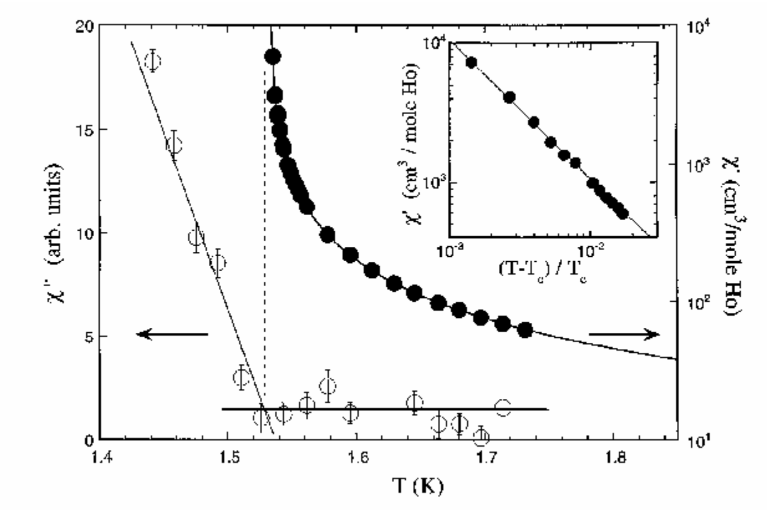
$$\chi(f) = \chi'(f) + i\chi''(f)$$

- Real component diverges at FM ordering
- Imaginary component shows dissipation

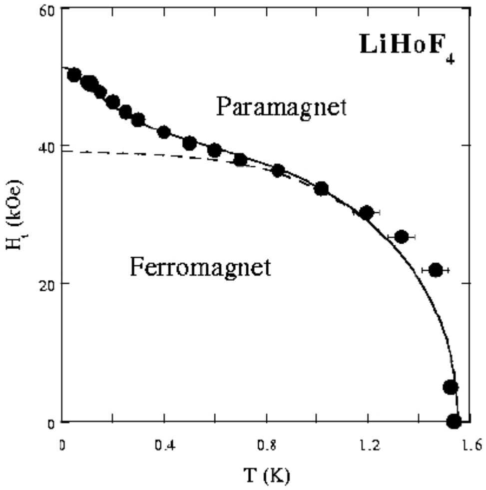


•D. Bitko, T. F. Rosenbaum, G. Aeppli, *Phys. Rev. Lett.*77(5), pp. 940-943, (1996)

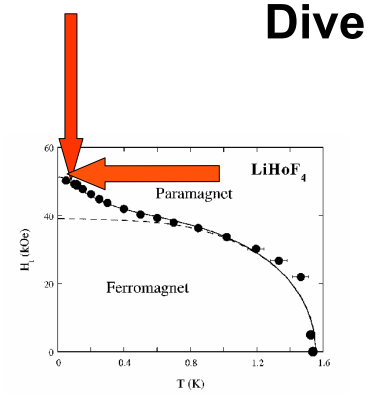
χ vs T for H_t=0



Mean-Field Ferromagnet



Diverging χ



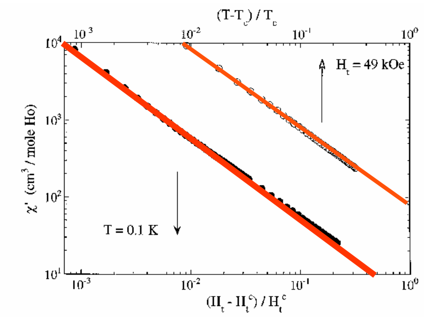


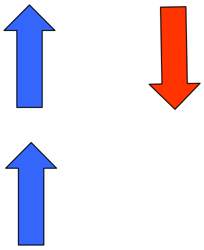
FIG. 2. Mean-field critical behavior of the magnetic susceptibility in the $T \to 0$ limit as functions of reduced temperature (open circles, $T_c = 0.114$ K, $H_t = 49.0$ kOe) and reduced transverse field (filled circles, $H_t^c = 49.3$ kOe, $T = 0.100$ K).

Introducing complexity via randomness & dipolar interaction …

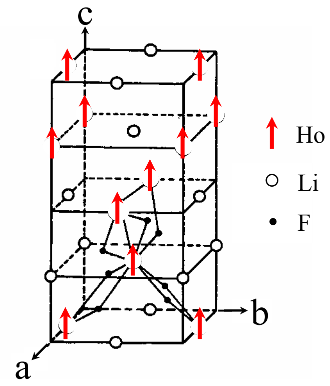
dipolar interaction between randomly placed spins leads to frustration

$$E=S_1S_2g^2M_B^2[1-3(r_z/r)^2]/r^3$$

ferro for $(r_z/r)^2 > 1/3$
antiferro for $(r_z/r)^2 < 1/3$



Experimental realization of Ising model in transverse field $LiHoF_4$



- •g=14 doublet
- •9K gap to next state
- •dipolar coupled

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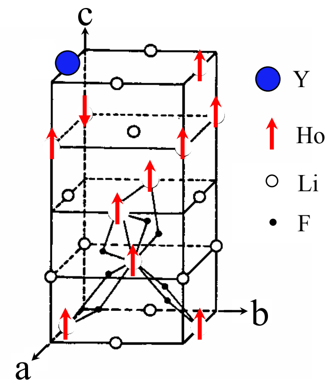

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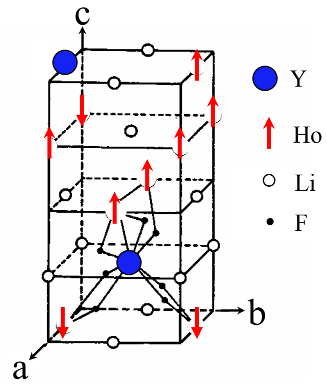

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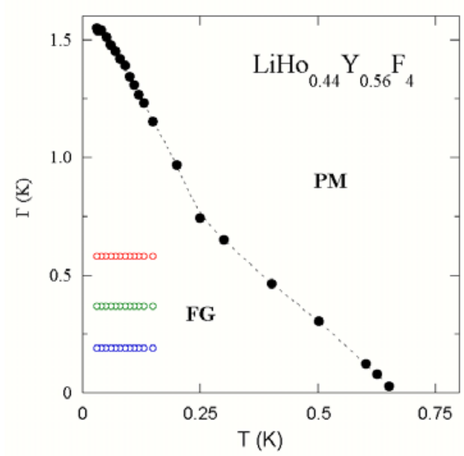
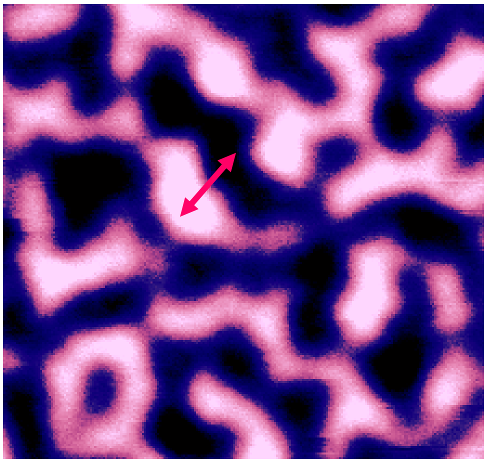


Y-A. Soh and G.A.,unpublished(2002)

A domain wall state pinned by random configurations of Y not much different from that at 300K in PdCo-

What about domain wall dynamics?



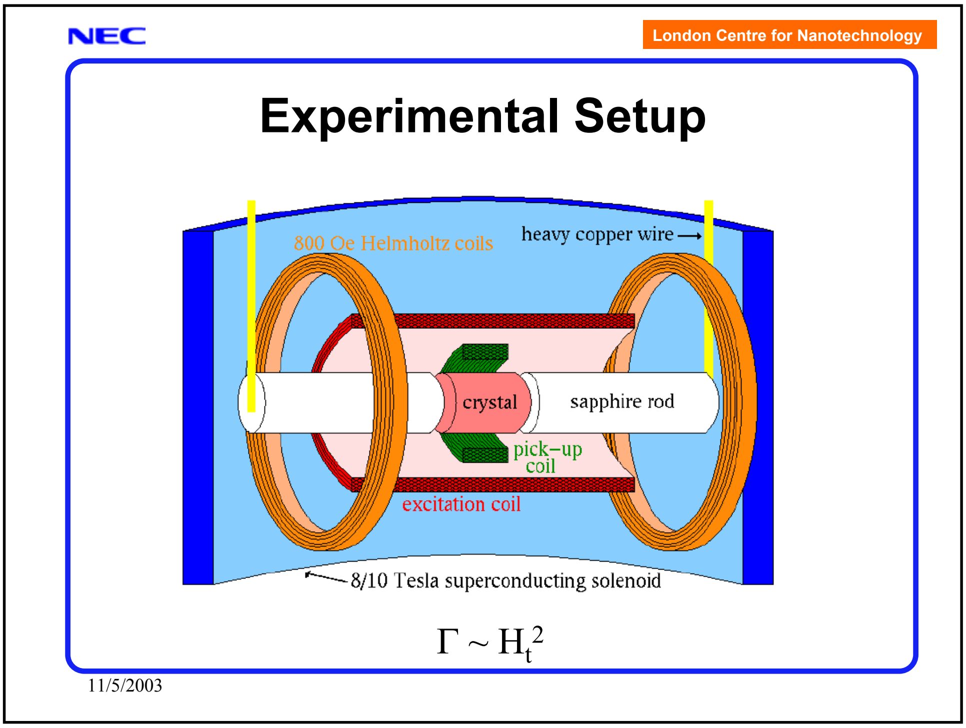
How to see?

- **Measure small signal response**

$M(t)=\chi'(\omega)h\cos(\omega t)+\chi(\omega)''h\sin(\omega t)$

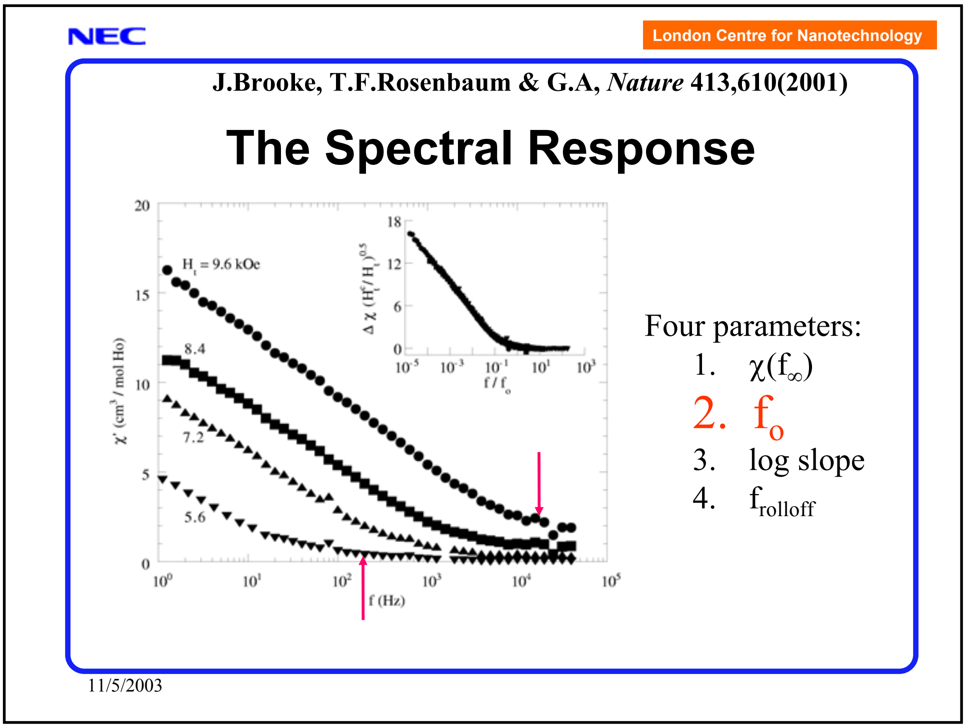
where

- $\chi=\chi'+i\chi''$ **is complex susceptibility**
- $h\cos(\omega t)$ **is excitation**



Experimental Setup

$$\Gamma \sim H_t^2$$

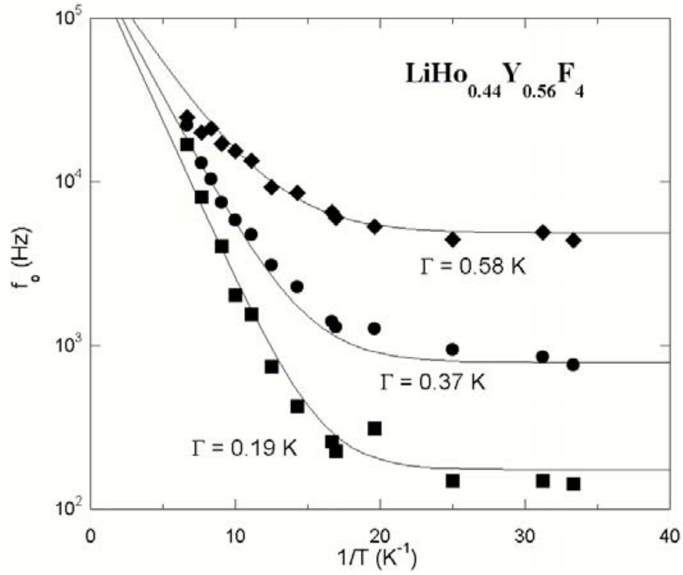


J.Brooke, T.F.Rosenbaum & G.A, *Nature* 413,610(2001)

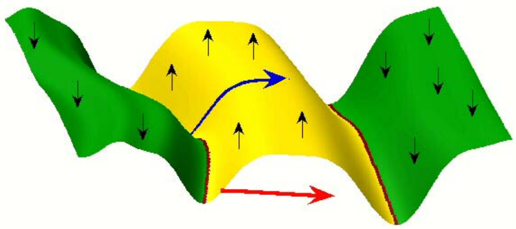
The Spectral Response

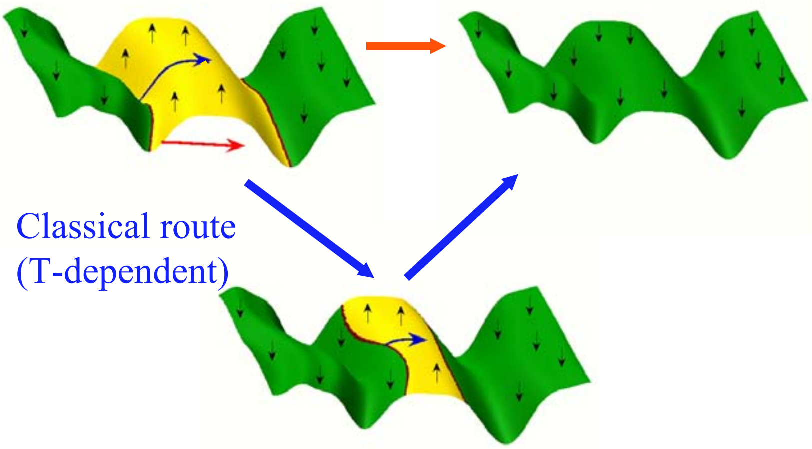
Four parameters:

1. $\chi(f_\infty)$
2. f_o
3. log slope
4. $f_{rolloff}$



$LiHo_{0.44}Y_{0.56}F_4$ Domain Wall Dynamics

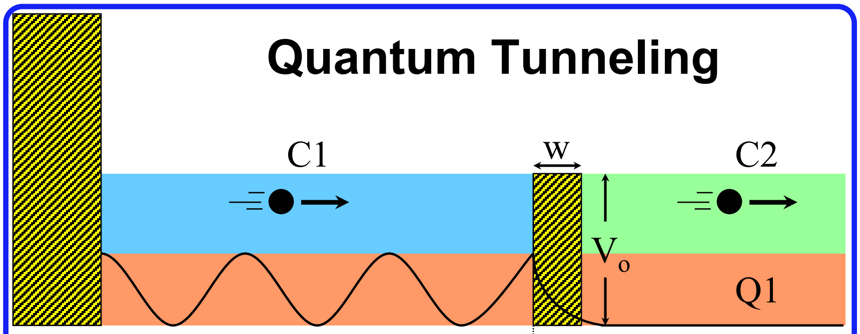




Quantum route(T-independent)

Classical route
(T-dependent)

Quantum Tunneling



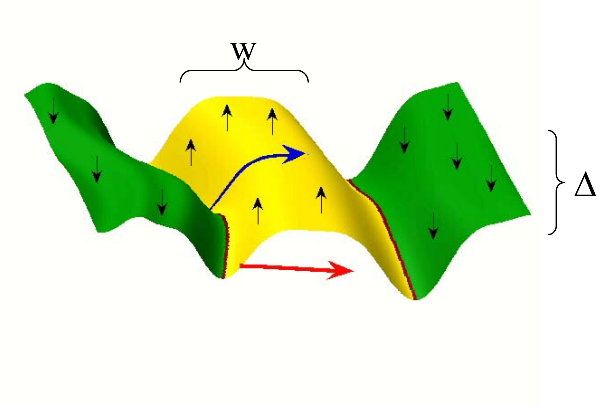
$$\sim \exp(-ikx) = \exp\left(-i\sqrt{2ME/\hbar^2}\,x\right) \qquad \sim \exp\left(-w\sqrt{\frac{2M}{\hbar^2}(V_o - E)}\right)$$

Map onto a magnetic system…

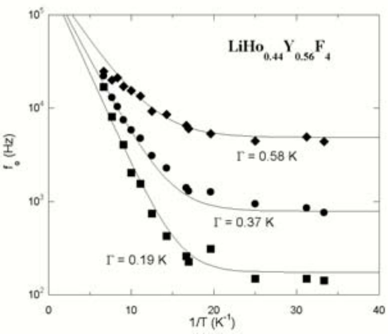
- Particle (E, M) → Domain Wall (E, M=Nm)
- Barrier (V_o, w) → Pinning&Exchange Potentials(V_o, w)

Domain Wall Tunneling

$$\sim \exp\left(-w\sqrt{\frac{2M}{\hbar^2}E_B}\right)$$



Evolution of the most mobile Domain Walls

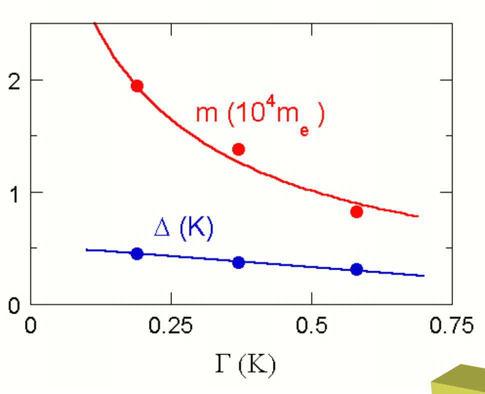


quantum tunneling

thermal hopping

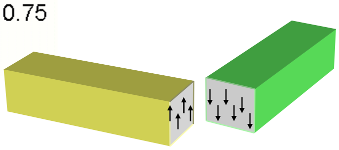
$$f = F_o\left\{\exp(-\Delta_\Gamma/T)+\exp\left(-2w_o\sqrt{\frac{2m_\Gamma}{\hbar^2}\Delta_\Gamma}\right)\right\}$$

Domain Wall Parameters



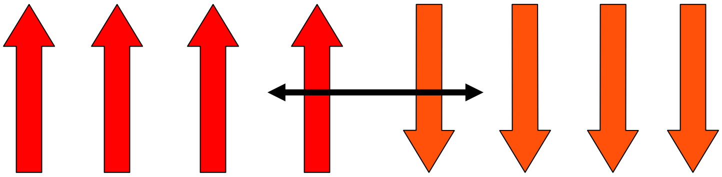
$$m_{DW} = N\cdot m_{spin} = N\cdot \frac{\hbar^2}{2a^2(\Gamma+\Gamma_i)}$$

N≈ 10



Tunable quantum tunneling of ferromagnetic domain walls

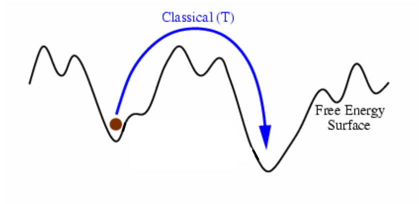
- **Simplest WKB approach where domain walls are particles works**
- **collective tunneling of wall segments with area=10 spins**
- **Mass of particles varied by external field**
- **potential energy landscape fixed by combination of field and random Y,Ho configuration**



Back to optimization problems-

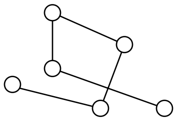
- Can be cast as energy minimization problems involving spin variables
- $H=\Sigma h_i S_i+J_{ij}S_i S_j + \dots$
- in real world problems, couplings are ‘random’ , leading to complicated energy landscape as function of configuration coordinates $\{S_i\}$

Real-World:
Solve Complex
Problems via
Thermal Annealing



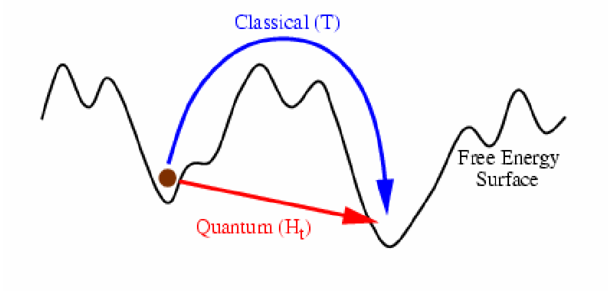
Computation:
Solve Complex
Problems via
Simulated Annealing

- Traveling Salesman



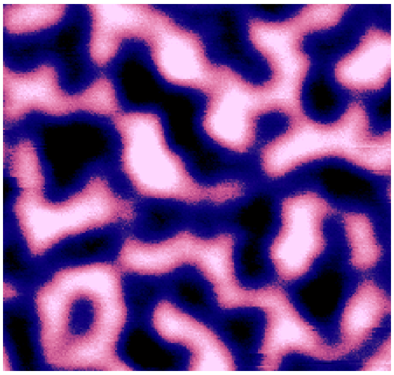
- Circuit Optimization

Quantum Tunneling adds a new path



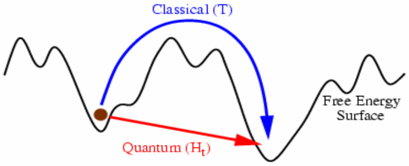
Which channel is more efficient?

Complex problem to solve: positioning ferromagnetic domain walls in a random ferromagnet

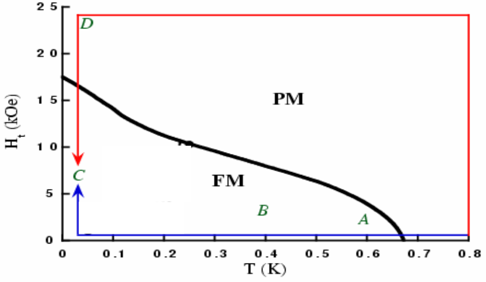


- J. Brooke, D. Bitko, T. F. Rosenbaum, G. Aeppli, *Science* 284, pp. 779-781, (1999)

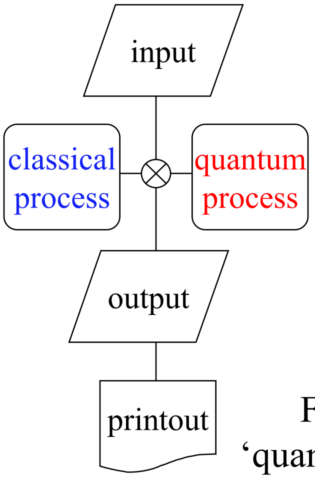
Quantum and Classical "Algorithms"



$LiHo_{0.44}Y_{0.56}F_4$
Disordered Ferromagnet



Process Flowchart



input

classical process

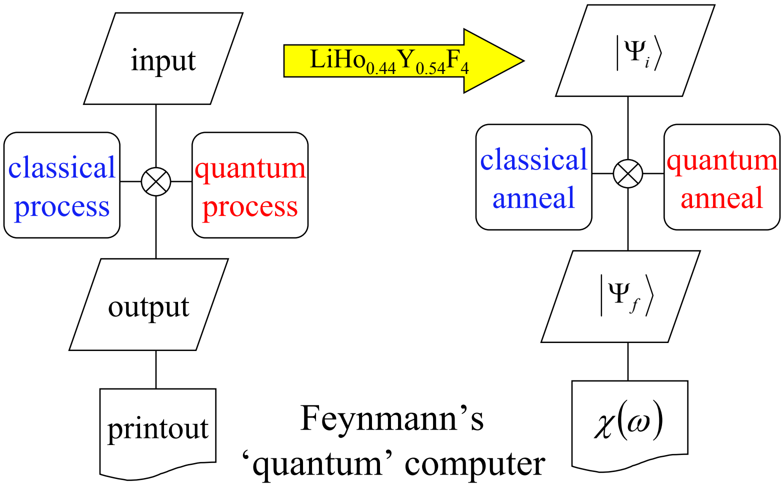
quantum process

output

printout

Feynmann's
'quantum' computer

Process Flowchart



input

classical process

quantum process

output

printout

$\mathrm{LiHo}_{0.44}\mathrm{Y}_{0.54}\mathrm{F}_4$

$|\Psi_i\rangle$

classical anneal

quantum anneal

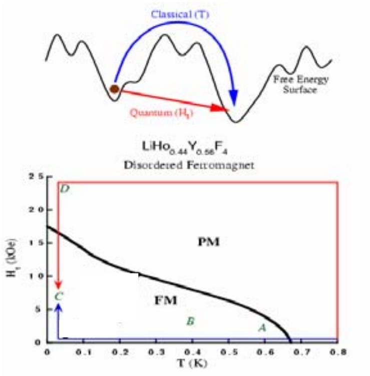
$|\Psi_f\rangle$

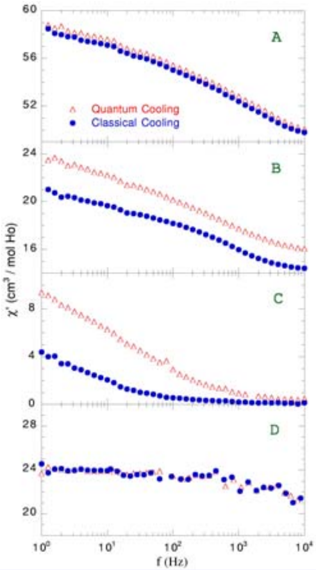
$\chi(\omega)$

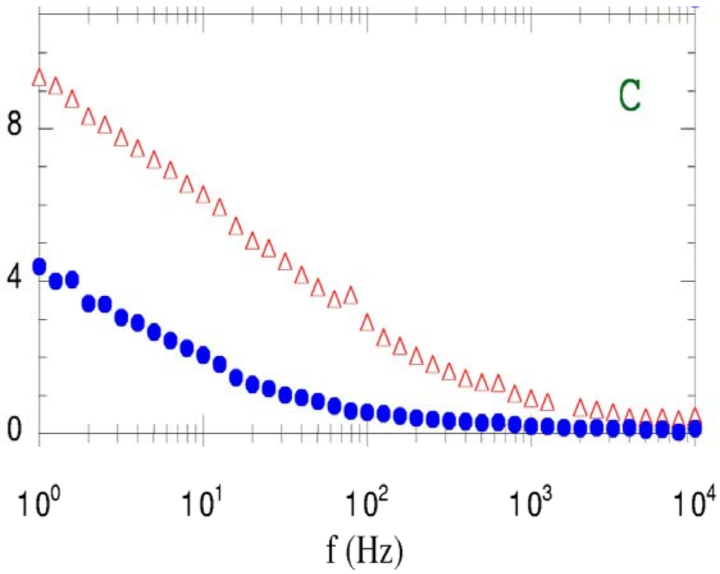
Feynmann's
'quantum' computer

- **J. Brooke, D. Bitko, T. F. Rosenbaum, G. Aeppli, *Science* 284, pp. 779-781, (1999)**

Fluctuation Response

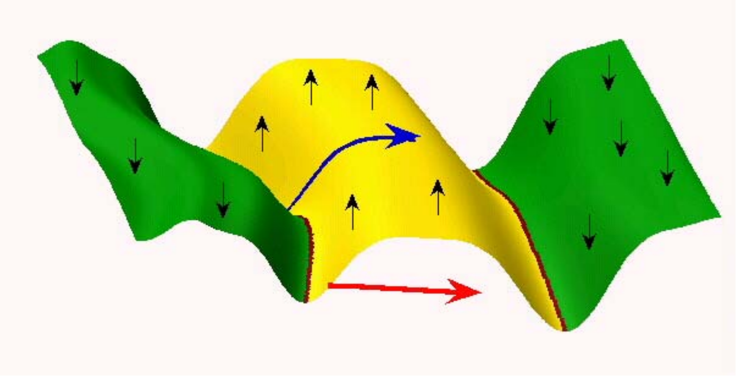


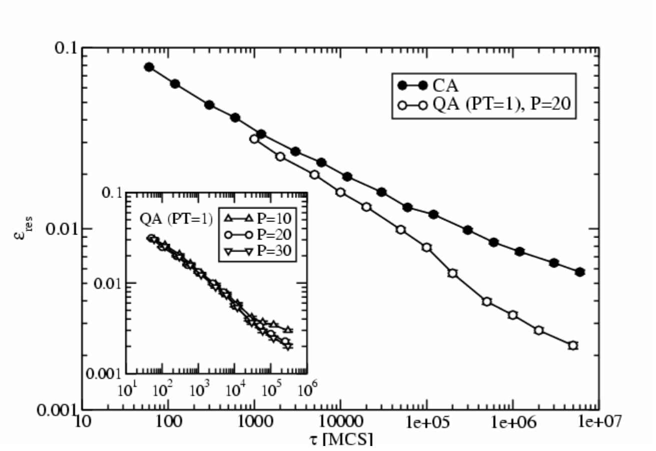




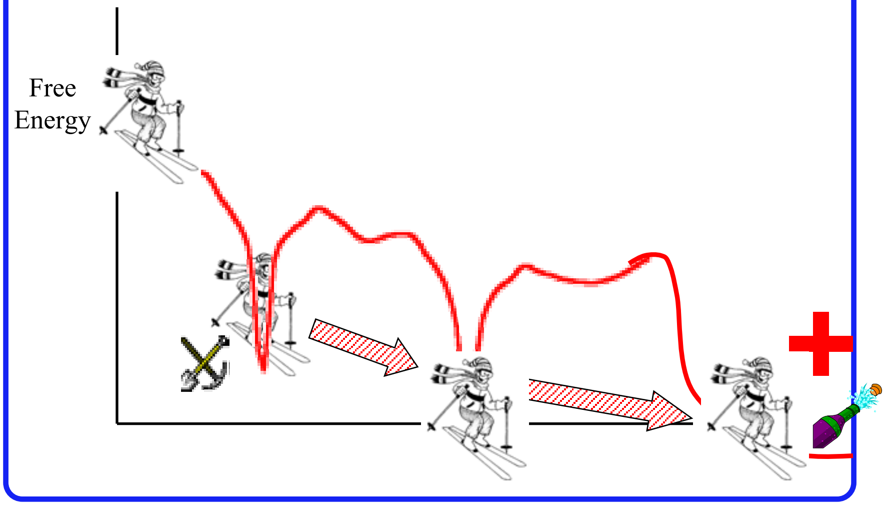
Results

- Quantum annealing is more efficient than thermal annealing at “uncovering” $\chi \sim \log(f)$.
- It pays off to look for quantum channels.





Santoro,Martonak,Tosatti & Car(2002)



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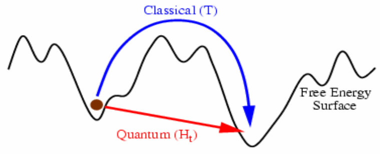
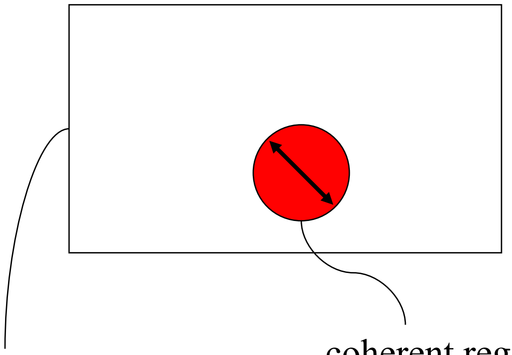
summary

- A different, brute-force approach to quantum computing – quantum annealing
- Identified a model solid state system to test quantum annealing ideas – Li(Ho,Y)F_4 where quantum fluctuations and ground state complexity can be regulated independently
- Tunneling picture of domain wall dynamics applies – introduces multiple spin moves into 'computation'
- Quantum annealing allows search of different minima than thermal annealing



Relation to ‘conventional’ quantum computation



total system-evolves incoherently

coherent region – computes as if gate-based

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references from 1999 and before

- “Quantum Annealing of a Disordered Magnet”, J. Brooke, D. Bitko, T. F. Rosenbaum, G. Aeppli, *Science* **284**, pp. 779-781, (1999).
- “Quantum Critical Points – Experiments”, G. Aeppli, T. F. Rosenbaum, *Dynamical Properties of Unconventional Magnetic Systems”*, A. T. Skjeltorp and D. Sherrington (eds.), Kluwer Academic Publishers, pp. 107-122 (1998)
- “Quantum Critical Behavior for a Model Magnet”, D. Bitko, T. F. Rosenbaum, G. Aeppli, *Phys. Rev. Lett.***77(5),** pp. 940-943, (1996).
- “High-Frequency Dynamics and the Spin-Glass Transition”, D. Bitko, N. Menon, S R. Nagel, T. F. Rosenbaum, G. Aeppli, *Europhysics Letters*, **33(6)**, pp. 489-494 (1996).
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