

# Cheap Vortices and New Collective Modes in High $T_c$ Superconductors

Patrick Lee

Collaborators:

Xiao-Gang Wen

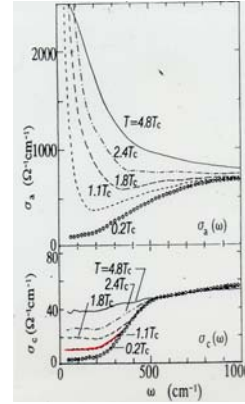
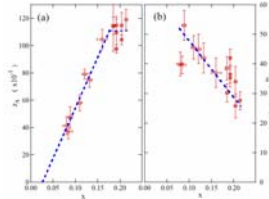
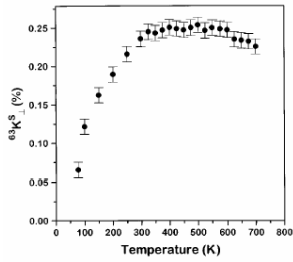
Naoto Nagaosa

Dima Ivanov

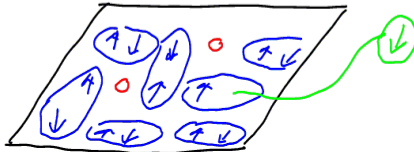
Carsten Honerkamp .....

- High  $T_c$  superconductivity is the problem of doping a Mott insulator.
- Focus on underdoped region: pseudo-gap.
- Competing state needed to form **cheap** vortex core.
- Candidate: staggered flux phase with **fluctuating** orbital currents.
- Evidence from projected wavefunctions.
- Proposed experimental tests.
- New collective modes in superconductor.

Pseudogap is the formation of spin singlet.



Photoemission: gap near  $(0, \pi)$



This is the resonating valence bond (RVB) idea of Anderson: Spins form singlets. Superconduct when holes are phase coherent.

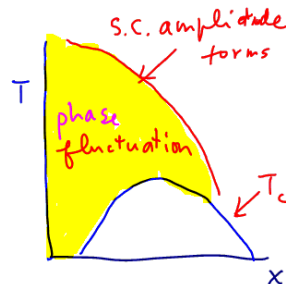
Not so fast!

Is there a more conventional explanation?

Small superfluid density ( $\kappa$ ) means small phase stiffness.

Then we can have strong phase fluctuation above  $T_c$  but amplitude and energy gap remains.

(Emery and Kivelson)



Is it RVB or is it superconductor with robust amplitude and strong Phase fluctuations?

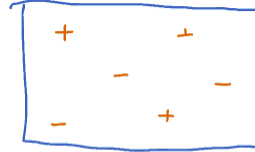
The phase fluctuation picture is incomplete:

1. Where does large SC amplitude come from in the first place?
2. BKT theory of phase fluctuation: vortex unbinding.

$$\text{vortex density} \sim e^{-E_c/kT}$$

$$E_c = \zeta^2 \frac{\Delta^2}{\xi_F}, \quad \zeta = \frac{\xi_F}{\Delta_0} a$$

$$\Rightarrow E_c \sim E_F \gg kT_c \text{ in BCS theory}$$



Need **cheap** vortex core:  $E_c \sim T_c \ll \Delta$ , ie,  
 Need **competing state** which is nearly degenerate in energy.  
 Candidate: **staggered flux state**.

3. If we assume  $E_c \sim T_c$ , then vortices proliferate and overlap around  $2T_c$ .  
 Above this  $T$ , pseudogap is determined by physics of the vortex core.

Summary so far concerning underdoped cuprates:

- Large energy gap (energy scale of  $J$ ), low  $T_c$ .
- Small superfluid density (=doping density  $x$ ):  $T_c$  controlled by phase coherence.
- Superconductor does not evolve out of quasi-particles pairing. Pairing mechanism in the traditional sense of q.p. exchanging Boson is not the right question.
- Wanted: **competing state**.

t-J model. Constraint of no double occupation is enforced by introducing fermion  $f$  which carries spin and boson  $b$  which represent vacancy.

$$c_{\sigma}^{\dagger} = f_{\sigma}^{\dagger} b$$

$\uparrow$   
 $\circ$

$f_{\uparrow}^{\dagger}$   
 $b^{\dagger}$

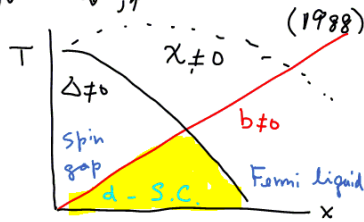
$$J \vec{S}_i \cdot \vec{S}_j \rightarrow -J f_{i\alpha}^{\dagger} f_{j\alpha} f_{j\beta}^{\dagger} f_{i\beta}$$

$$= -J (f_{i\uparrow}^{\dagger} f_{j\downarrow} - f_{i\downarrow}^{\dagger} f_{j\uparrow}) (c.c.)$$

RVB decoupling:

$$\chi_{ij} = \langle f_{i\alpha}^{\dagger} f_{j\alpha} \rangle$$

$$\Delta_{ij} = \langle f_{i\uparrow}^{\dagger} f_{j\downarrow} - f_{i\downarrow}^{\dagger} f_{j\uparrow} \rangle$$

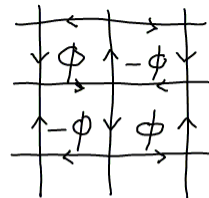
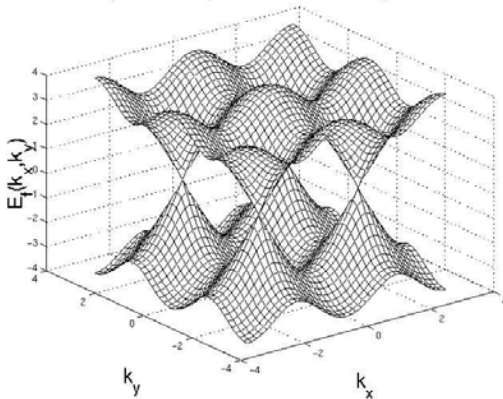


A problem with U(1) theory: cannot construct cheap  $hc/2e$  vortex

SU(2) symmetry at half-filling. Aflleck et al (88)

d wave SC and staggered flux state has the same q.p. dispersion.

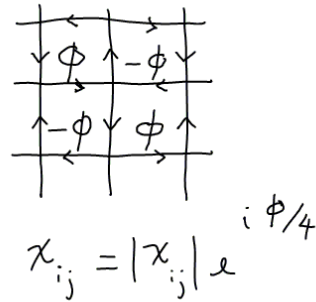
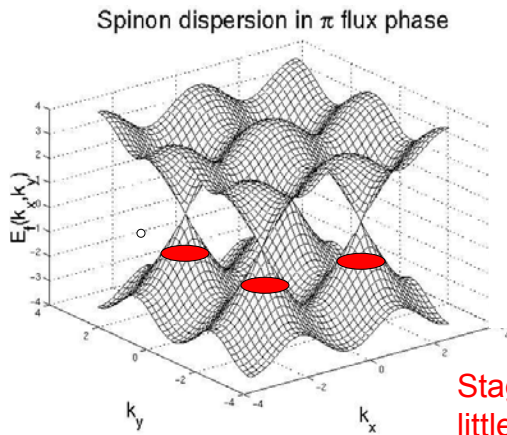
Spinon dispersion in  $\pi$  flux phase



$$\chi_{ij} = |\chi_{ij}| e^{i\phi/4}$$

SU(2) symmetry at half-filling. Affleck et al (88)

d wave SC and staggered flux state has the same q.p. dispersion.



Staggered flux state is a little more costly, by  $x^{3/2}$ .

Half-filling:

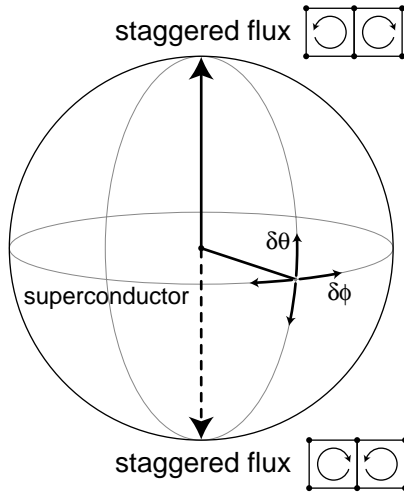
Flux phase + gauge fluctuation

= Dirac fermion + compact gauge fields

Familiar problem in QCD: confinement leads to “chiral symmetry breaking” which translate to Neel order in our case.

Flux state is our route to AF order.

X.-G. Wen and P. Lee (96) extend SU(2) symmetry to doped case to include fluctuation between d-SC and staggered flux.

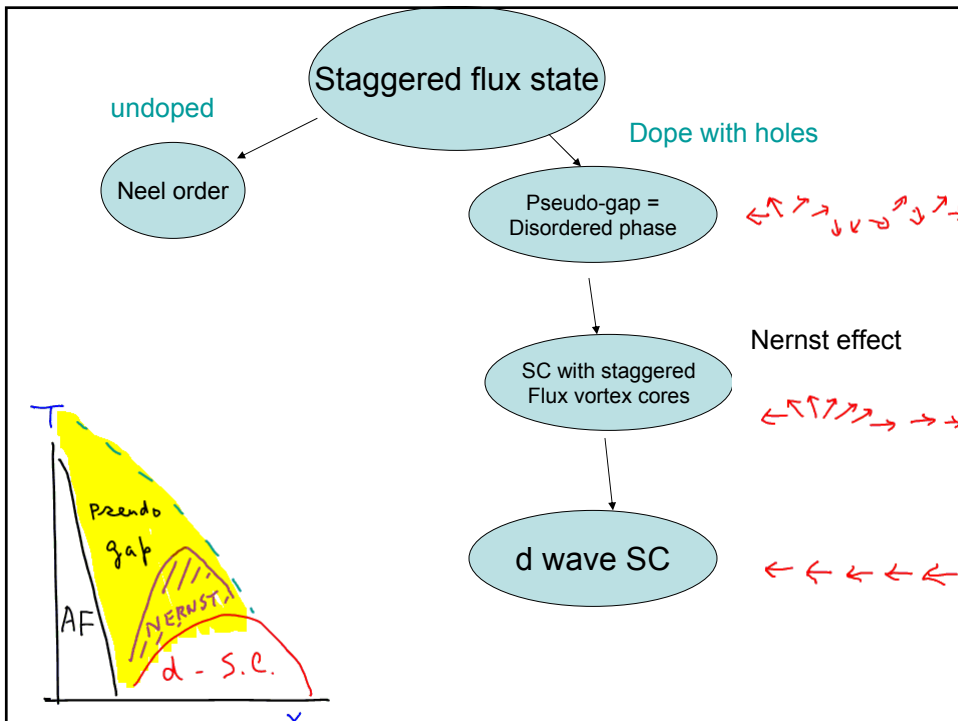


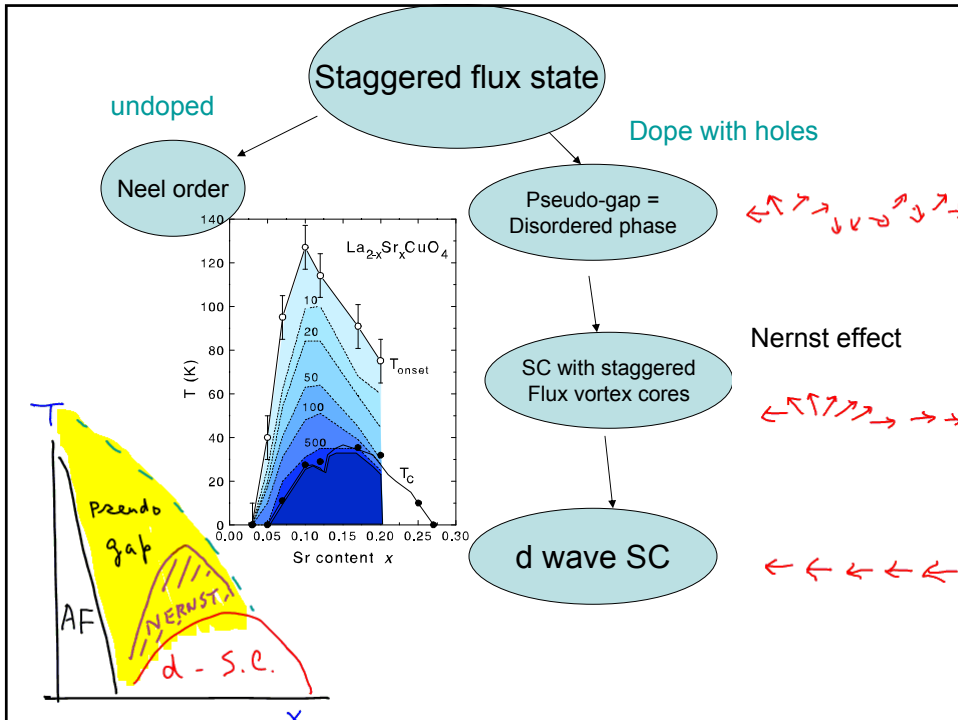
3 phase angles:

$\alpha$  (not shown) is the superconducting phase.

$\theta$  and  $\phi$  are internal phases which are gauged.

Arrow is the quantization axis





Trial wavefunctions:

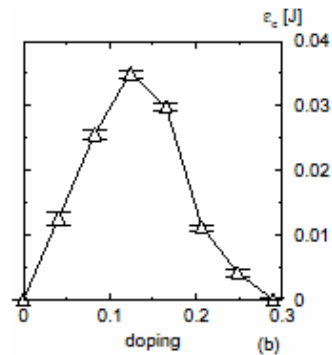
Gutzwiller projection:  
Remove by hand on a computer  
all amplitudes for double occupation.

SC=projected d wave BCS

SF=projected staggered flux

$\epsilon_c$  = condensation energy per site  
=energy difference between SC  
and SF.

(Ivanov and Lee, unpublished)

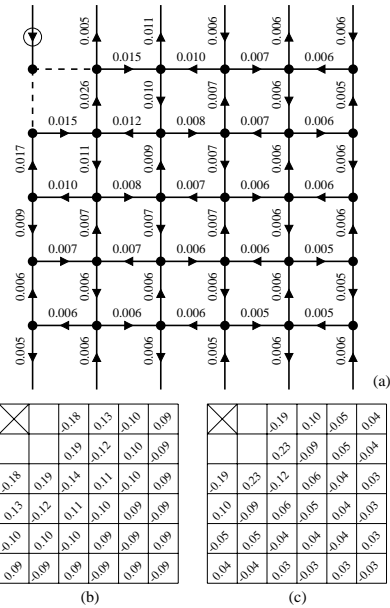


Ivanov, Lee and Wen (PRL 2000) studied projected BCS d-wave wavefunction, ie remove doubly occupied component and fix particle number.

$\langle j(\text{bond } m) \rangle = 0$ ,  
 but current-current correlator  
 $\langle j(\text{bond } m) j(\text{bond } n) \rangle$  shows a staggered pattern.

This pattern is absent before projection and is a consequence of enforcing no double occupation constraint.

This pattern was later found in exact ground state of 2 holes in 32 site t-J model by Leung.

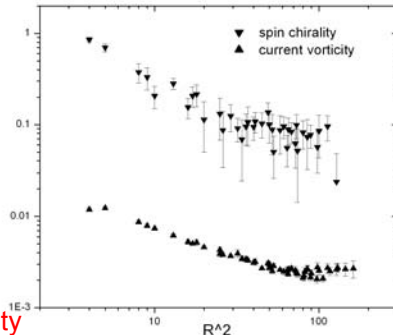
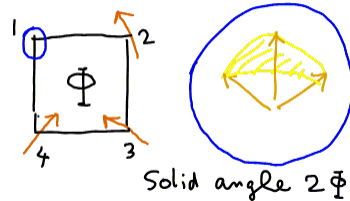


### What is the microscopic origin of orbital current?

Hole hopping around a square sees changing spin quantization axis and picks up Berry's phase  $\Phi$ . This phase is represented by the flux  $\Phi$ , which drives the hole clockwise or anti-clockwise depending on the sign of  $\Phi$ . To test this idea we computed hole-chirality correlator.

$$\chi_H = \vec{s}_1 \cdot \vec{s}_2 \cdot \vec{s}_3 \times \vec{s}_4$$

$$\langle \chi_H(n) \chi_H(m) \rangle$$



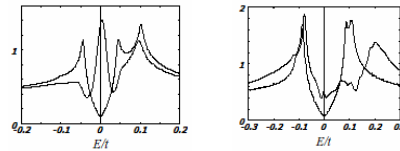
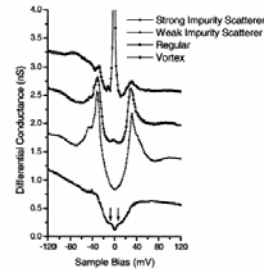
Orbital current is the symptom of spin chirality



Lee and Wen constructed vortex with staggered Flux core.

Numerical solution of mean field equation by Han and D.-H. Lee and by Ogata et al confirm the existence of Orbital currents in the core.

- Vortex is cheap
- LDOS in vortex core is similar to d SC (explains STM)
- Expect AF fluctuation inside the core since staggered flux is precursor to Neel state (NMR result)
- Mean field solution finds small moment SDW for small doping. (Staggered flux is our route to AF)



Ogata et al , right, with staggered flux  
left, without staggered flux

Low density of states inside core: breakdown of Bardeen-Stephen dissipation

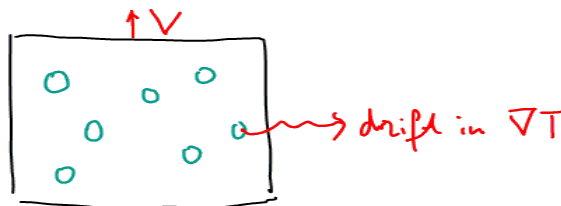
Low dissipation : small friction coefficient  $\eta$  for vortex motion.

**Cheap and fast vortices !**

Flux-flow resistivity =  $B/\eta$

small  $\eta$  means small flux flow contribution to conductivity and large Nernst effect.

Nernst coefficient =  $S_v/\eta$



## Experimental Search for orbital currents:

1. Pseudogap phase: currents are fluctuating. No phase transition and no time reversal breaking.
2. Orbital currents may be quasi-static inside vortex core and generate a magnetic field of order 20 gauss.
  - $\mu$ SR, Miller et al reports 18 gauss field in underdoped YBCO below 30K. Cannot distinguish between spin or orbital origin. If it is due to spin, it corresponds to 1/20 of insulator moment.
  - NMR Y site uniquely sensitive to orbital moment, but there is bi-layer cancellation. Need 2-4-7.
  - Neutron: quasi-elastic but short range peak at  $(\pi, \pi)$  which grows with H.
  - New collective modes in SC.

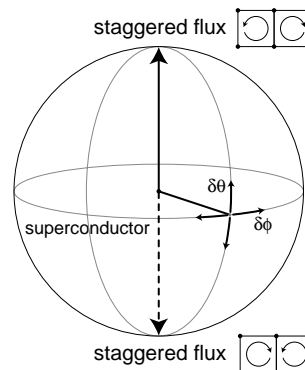
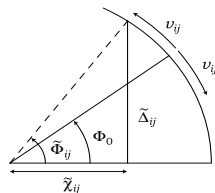
New Collective modes. (Lee and Nagaosa)

Ordinary SC : order parameter  $\Delta$ . amplitude and phase modes.

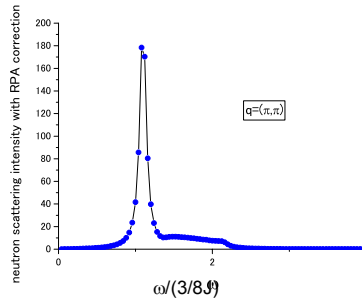
Here hopping matrix element  $\chi$  is also a dynamical variable, because hopping is strongly coupled to spin configuration. Therefore expect additional collective mode. SU(2) theory allows us to classify them and compare with numerical calculation of fluctuation about mean field. (RPA)

$\theta$  mode : fluctuation of staggered flux

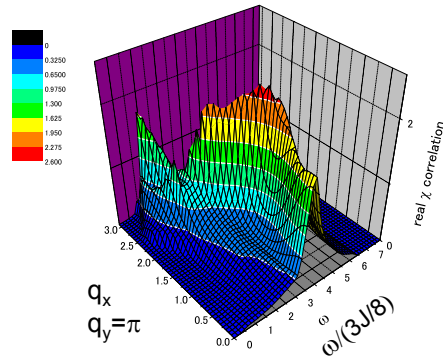
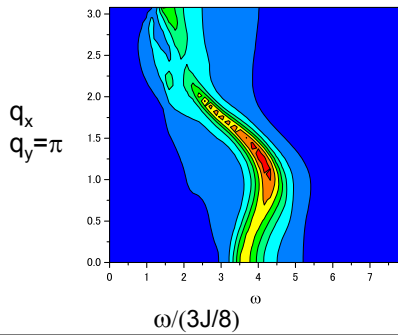
$\phi$  gauge mode: staggered modulation of the amplitude of  $\chi$  and  $\Delta$ .



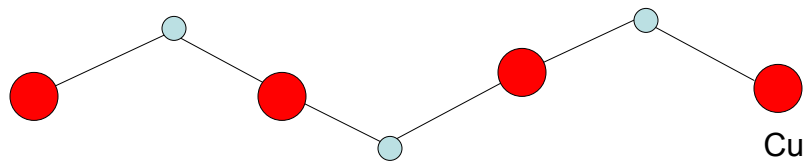
$\theta$  mode creates local orbital moment:  
in principle detectable by neutron  
scattering as inelastic peak at  $(\pi, \pi)$ .



Inelastic x-ray scattering couple to  
fluctuation in  $\chi$ . (**bond charge density**)

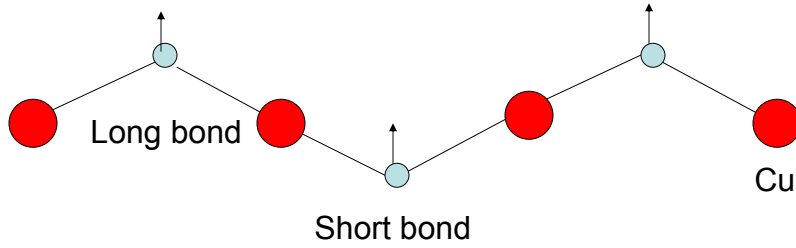


How to couple to  $(\pi, \pi)$  using optics?



LTO phase in LSCO

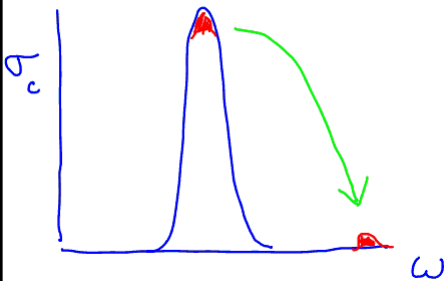
How to couple to  $(\pi, \pi)$  using optics?



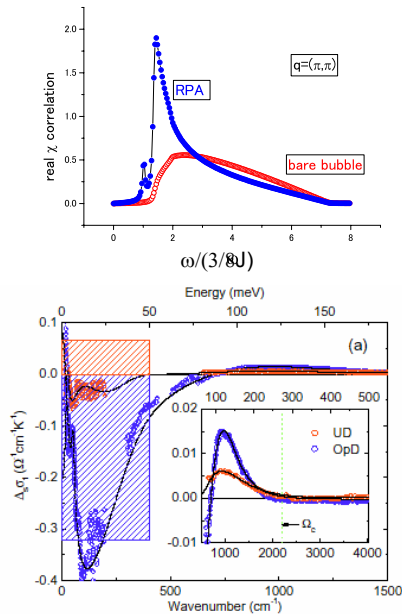
LTO phase in LSCO

Bonds are modulated by  $(\pi, \pi)$  for  $q=0$  displacement  
Of oxygen.

Predict transfer of spectral weight from buckling  
Phonon mode to collective mode

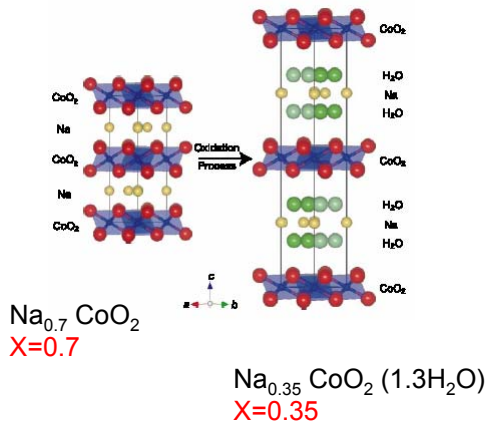


Note signal is small only  
Because the coupling via  
The phonon is weak.

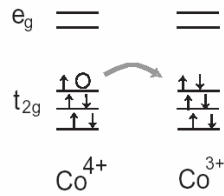


Are the Cuprates unique?  
Is SC out of doped Mott insulator a  
general phenomenon?

- Low dimension
- S=1/2
- Large J

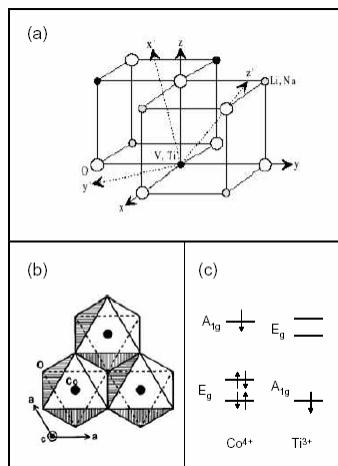


Ti  $3+ d^1$   
Co  $4+ d^5$



Takada et al, Nature,  
March, 2003.

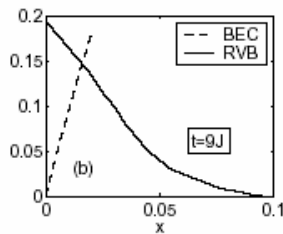
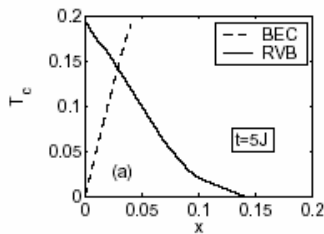
$\text{Na}_{0.35}\text{CoO}_2 (1.3\text{H}_2\text{O})$   
is a 5K superconductor



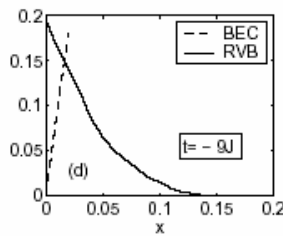
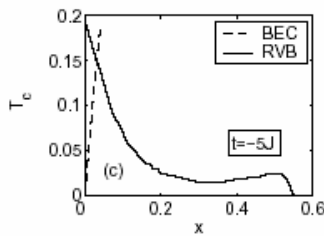
Electron doping  
 $\text{Na}_x\text{CoO}_2$

Hole doping  
 $\text{Na}_{1-x}\text{TiO}_2$

$\Delta = \delta_0 \exp(i2\theta)$  d+id **time reversal breaking** superconductor



Electron doping  
 $\text{Na}_x\text{CoO}_2$



Hole doping  
 $\text{Na}_{1-x}\text{TiO}_2$

Qiang-Hua Wang, Dung-Hai Lee and Patrick Lee, cond-mat 0304377

Critique of slave boson / orbital current:

- uncontrolled approximation ( $1/N$  but  $N=1$ ), strong gauge fluctuations.
- no adequate treatment of confinement in the low doping region. It is not clear whether predicted orbital currents will survive, or it is just a formal route to AF.
- Current carried by quasi-particles are of order  $x$ , so that  $\rho = x - x^2 T$ , whereas expt gives  $x - T$ .

**Need experiments!**

## Summary

Staggered Flux as a competing state explains many of the important features of the underdoped cuprates and allows us to make concrete predictions.

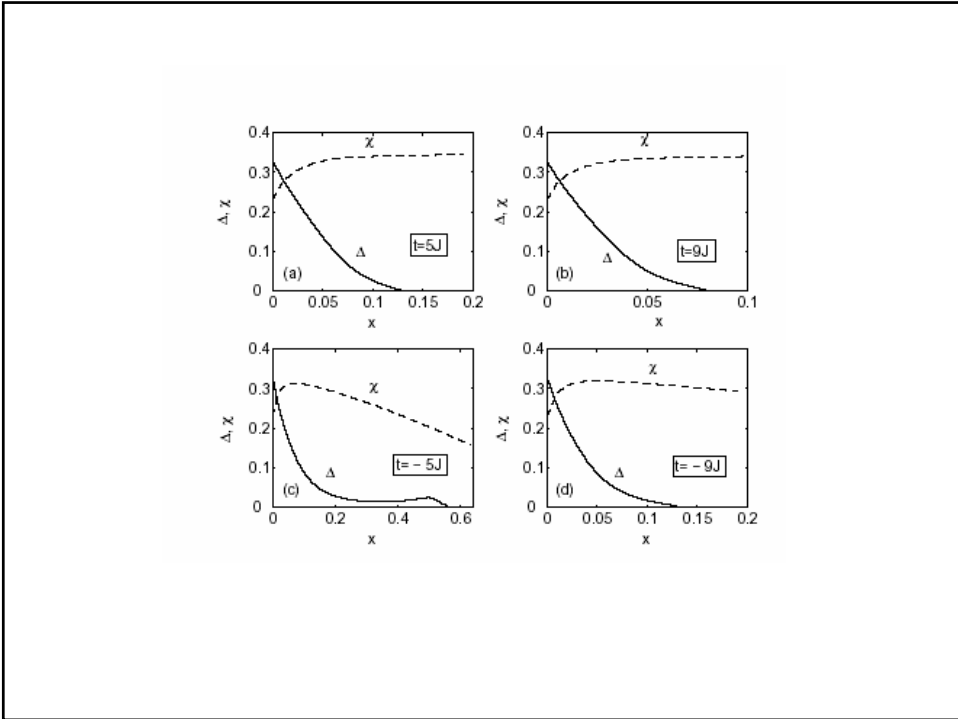
The orbital current (fluctuating in the bulk and possibly static in the vortex core) is a good diagnostic tool.

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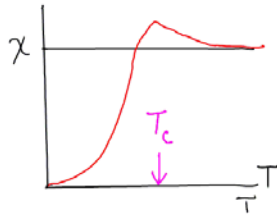
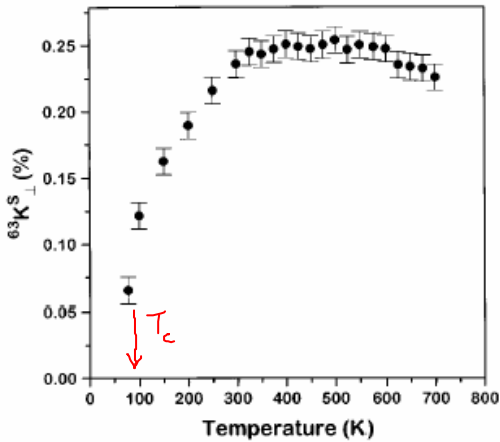
The orbital current (fluctuating in the bulk and possibly static in the vortex core) is a good diagnostic tool.

**Experimentalists, please HELP!**



**Pseudo-gap** phenomenon: energy gap appears in some quantities and not others:

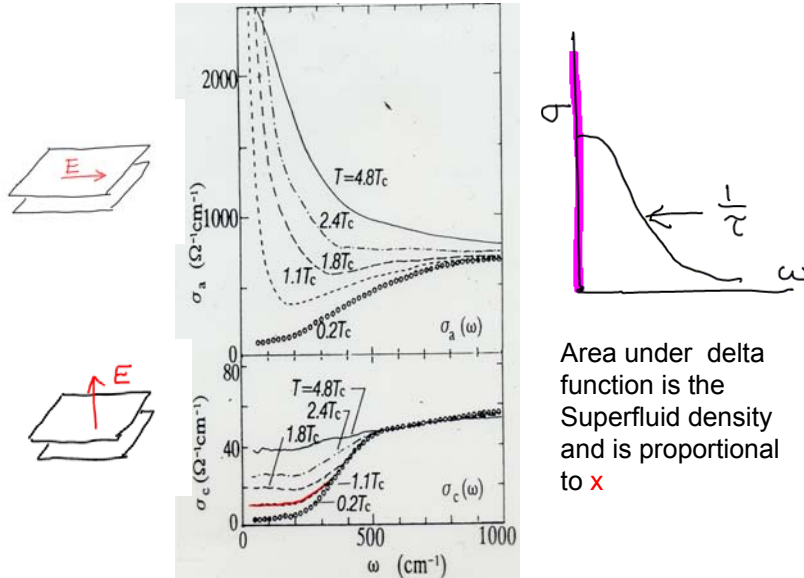
1. Spin Susceptibility is reduced below 300K, while  $T_c$  is 79K. Indicative of singlet formation much above  $T_c$ .



Curro et al, PRB (94)



2. No gap for conductivity in the plane,  
Gap for conductivity perpendicular to plane.



Area under delta function is the Superfluid density and is proportional to  $x$

3. Angle resolved photoemission:  
Above  $T_c$  spectrum is incoherent.  
Below  $T_c$  coherent peak appears with weight  $x$ .  
Pseudo-gap visible above  $T_c$ .  
Energy gap  $\Delta$  grows with decreasing doping and  $T_c$ .

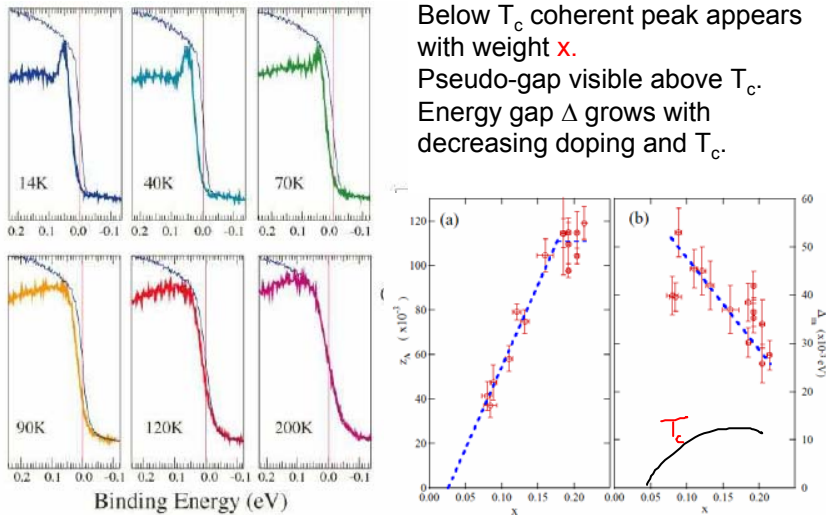


Fig. 26. ARPES spectra along the  $(\pi, 0) \rightarrow (\pi, \pi)$  direction for an 83 K underdoped sample at various temperatures (solid curves). The thin curves in each panel are reference spectra from polycrystalline Pt used to accurately determine the zero of binding energy at each temperature.