Weak Lensing and CMB: Parameter forecasts including a running spectral index

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Scale invariant scalar power spectrum

- Primordial scalar power spectrum

$$P(k) = P(k_o)\left(\frac{k}{k_o}\right)^{n_s(k_o)+\frac{1}{2}\alpha_s \ln\left(\frac{k}{k_o}\right)}$$

- Most theories of the very early universe predict almost scale invariance with

$$n_s \sim 1 \qquad \alpha_s = \frac{dn_s}{d\ln k} \sim (1-n_s)^2$$

Measurement of a significant running of n_s

- WMAP team reported that from CMB data alone a scale invariant power spectrum is a good fit (e.g. Spergel et al. 2003), however
- From CMB+ 2dGRS+Lyman-α forest, their analysis favors a slowly varying spectral index (Spergel et al. 2003)

$$\alpha_s = -0.031^{+0.016}_{-0.017}$$

- Addition of Lyman-α forest to the analysis made the difference but was seriously questioned (Seljak et al. 2003)

We will know more soon

- WMAP and other CMB missions continue to operate, so results will improve
- Also, more to come soon from the Lyman-α forest analysis (e.g. from the SDSS)
- But …

The need for another probe

- There are uncertainties associated with the baryonic physics of the Lyman-α forest
- It is important to be able to constrain the running using another technique
- Weak Lensing (WL) + CMB is a good candidate.

Why Weak Lensing?

- Weak Lensing systematics are unrelated to the systematics of Lyman-α forest
- WL probes directly the dark matter distribution on various scales rather then the light distribution (e.g. galaxy surveys)
- WL breaks degeneracies present in other methods (e.g. CMB)

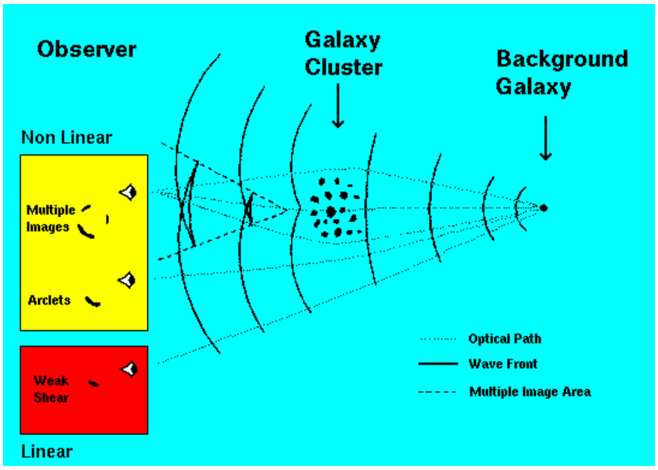
Gravitational lensing overview in 5 minutes

- A review of Weak Lensing as a tool of precision cosmology will be presented on Sunday by Henk Hoekstra
- I will mention the minimum necessary for my talk

Observations of Gravitational Lensing

- The resulting observations depend on the alignment of the source, the lens and the observer

Gravitational Lensing by clusters of galaxies (picture from MPA lensing group)

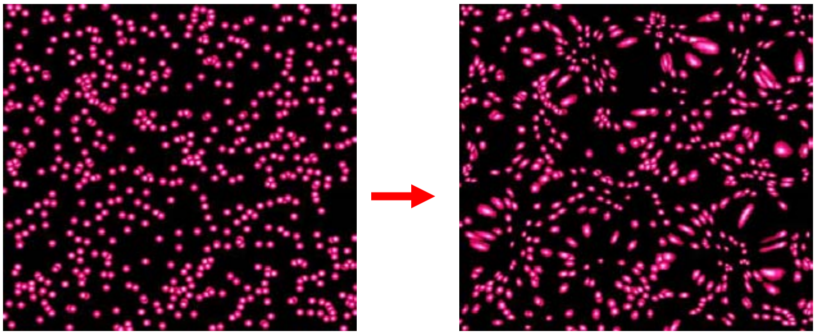


Gravitational arcs. Here distorted images of a distant galaxy population by a cluster of galaxies

Abell 2218: A Galaxy Cluster Lens, Andrew Fruchter et al. (HST)

Weak Gravitational Lensing

Distortion of background images by foreground matter



Unlensed **Lensed**

Credit: SNAP WL group

Distortion matrix, shear and convergence

The distortion matrix

$$A_{ij} = \frac{\partial \beta_i^{source}}{\partial \theta^j} = \begin{pmatrix} 1-\kappa-\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

γ_1 and γ_2 are the shear components.

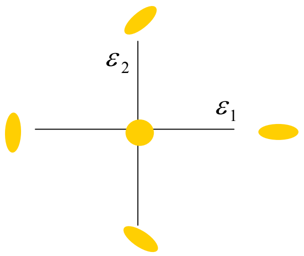
κ Is the convergence

Cosmic shear

Image quadrupoles: $$Q_{ij} = \int d^2x x_i x_j w(x) I(x)$$

Ellipticity “vector” ε:

$$\varepsilon_1 = \frac{Q_{11} - Q_{22}}{Q_{11} + Q_{22}}, \varepsilon_2 = \frac{2Q_{12}}{Q_{11} + Q_{22}}$$



Shear : γ is proportional to <ε>

The convergence κ and the shear components are related

Convergence Power Spectrum

$$P_\kappa(l) = \frac{9}{4} H_o^4 \Omega_m^2 \int_0^{\chi_H} \frac{g^2(\chi)}{a^2(\chi)} P_{3D}\left(\frac{l}{\sin_K(\chi)}, \chi\right) d\chi$$

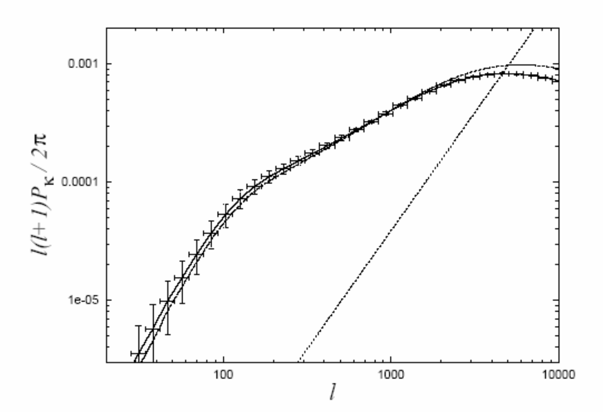
$$g(\chi) = \int_\chi^{\chi_H} n(\chi') \frac{\sin_K(\chi' - \chi)}{\sin_K(\chi')} d\chi'$$

Source redshift distribution: $$n(z) = \frac{z^2}{2z_o^3} e^{-\frac{z}{z_o}}$$

Work done

- We generated the convergence power spectra using a weak lensing code that includes the running, redshift distributions and tomography bins.
- We used the recent HALOFIT (Smith et al. 2003) mapping procedure to calculate the non-linear power spectra
- We also checked the Fisher matrix constraints using simulations with a running index (agreement to better than 10%)

Convergence power spectra for the reference survey (f_{SKY}=0.01 and n_g=56 gal/arcmin^2) with α_s= -0.04 (solid curve), 0.04 (dashed) (ΛCDM cosmology)



An aside: We investigated the effect of shear calibration biases

Power calibration parameter for the no-tomography case

$$\hat{P}_\kappa(l) = (1+\zeta_s)P_\kappa(l)$$

Relative calibration parameter for the tomography case

$$\tilde{P}_\kappa^{AA}(l) = (1+f_B\zeta_r)\hat{P}_\kappa^{AA}(l)$$

$$\tilde{P}_\kappa^{AA}(l) = (1+\frac{f_B-f_A}{2}\zeta_r)\hat{P}_\kappa^{AB}(l)$$

$$\tilde{P}_\kappa^{BB}(l) = (1-f_B\zeta_r)\hat{P}_\kappa^{BB}(l)$$

Also, we included the characteristic redshift z_p in the parameter uncertainty analysis

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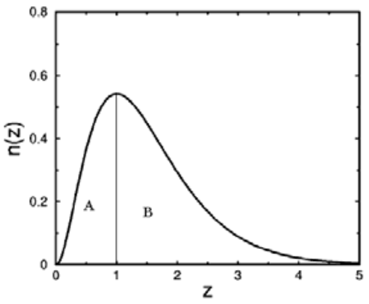
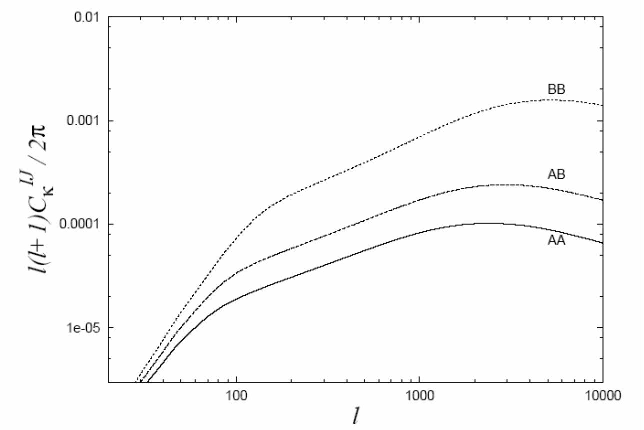
We implemented tomography in the analysis



$$n_A(z) = \frac{z^2}{2z_o^3(1-5/e^2)}e^{-\frac{z}{z_o}}$$

$$n_B(z) = \frac{e^2z^2}{10z_o^3}e^{-\frac{z}{z_o}}$$

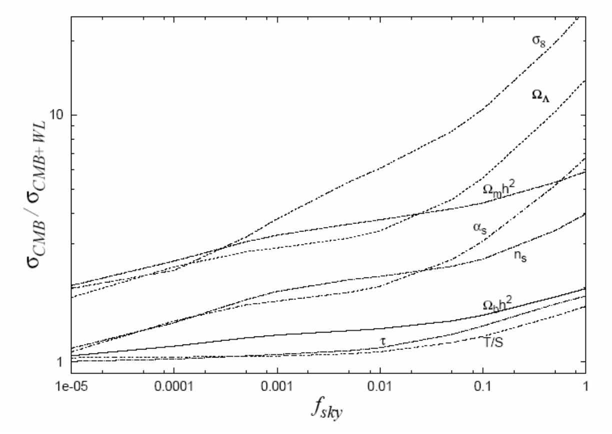
Tomography power spectra



Combining WL tomography with CMB

- Fisher matrix formalism was used to:
  - Derive parameter uncertainties for WL
  - Combine tomography bins
  - Combine WL and current CMB from (WMAP+CBI+ACBAR)

Improvement of the CMB parameter estimation with Weak Lensing. The effect of increasing f_{SKY} is displayed



Results

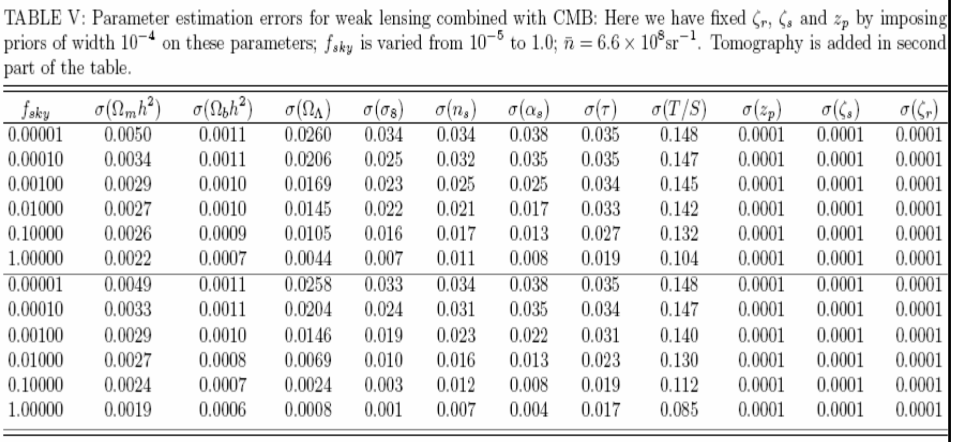


TABLE V: Parameter estimation errors for weak lensing combined with CMB: Here we have fixed ζ_r, ζ_s and z_p by imposing priors of width 10^{-4} on these parameters; f_{sky} is varied from 10^{-5} to 1.0; $\bar{n} = 6.6 \times 10^8 \mathrm{sr}^{-1}$. Tomography is added in second part of the table.

f_{sky}	$\sigma(\Omega_m h^2)$	$\sigma(\Omega_b h^2)$	$\sigma(\Omega_\Lambda)$	$\sigma(\sigma_8)$	$\sigma(n_s)$	$\sigma(\alpha_s)$	$\sigma(\tau)$	$\sigma(T/S)$	$\sigma(z_p)$	$\sigma(\zeta_s)$	$\sigma(\zeta_r)$
0.00001	0.0050	0.0011	0.0260	0.034	0.034	0.038	0.035	0.148	0.0001	0.0001	0.0001
0.00010	0.0034	0.0011	0.0206	0.025	0.032	0.035	0.035	0.147	0.0001	0.0001	0.0001
0.00100	0.0029	0.0010	0.0169	0.023	0.025	0.025	0.034	0.145	0.0001	0.0001	0.0001
0.01000	0.0027	0.0010	0.0145	0.022	0.021	0.017	0.033	0.142	0.0001	0.0001	0.0001
0.10000	0.0026	0.0009	0.0105	0.016	0.017	0.013	0.027	0.132	0.0001	0.0001	0.0001
1.00000	0.0022	0.0007	0.0044	0.007	0.011	0.008	0.019	0.104	0.0001	0.0001	0.0001
0.00001	0.0049	0.0011	0.0258	0.033	0.034	0.038	0.035	0.148	0.0001	0.0001	0.0001
0.00010	0.0033	0.0011	0.0204	0.024	0.031	0.035	0.034	0.147	0.0001	0.0001	0.0001
0.00100	0.0029	0.0010	0.0146	0.019	0.023	0.022	0.031	0.140	0.0001	0.0001	0.0001
0.01000	0.0027	0.0008	0.0069	0.010	0.016	0.013	0.023	0.130	0.0001	0.0001	0.0001
0.10000	0.0024	0.0007	0.0024	0.003	0.012	0.008	0.019	0.112	0.0001	0.0001	0.0001
1.00000	0.0019	0.0006	0.0008	0.001	0.007	0.004	0.017	0.085	0.0001	0.0001	0.0001

Results for the reference survey: f_{SKY}=0.01 and n_g=56 gal/arcmin^2 (flat LCDM cosmology)

C=CMB W=WL T:Tomography S: simulations

a, b, c: priors on z_p, ζ_s and ζ_r

Survey type	$\sigma(\Omega_m h^2)$	$\sigma(\Omega_b h^2)$	$\sigma(\Omega_\Lambda)$	$\sigma(\sigma_8)$	$\sigma(n_s)$	$\sigma(\alpha_s)$	$\sigma(\tau)$	$\sigma(T/S)$	$\sigma(z_p)$	$\sigma(\zeta_s)$	$\sigma(\zeta_r)$
C	0.013	0.0012	0.054	0.083	0.036	0.039	0.035	0.154			
CWa	0.0036	0.001	0.019	0.024	0.021	0.017	0.033	0.143	0.042	0.092	0.099
CWb	0.003	0.001	0.017	0.023	0.021	0.017	0.033	0.143	0.041	0.02	0.02
CWc	0.0027	0.001	0.015	0.022	0.021	0.017	0.033	0.142	0.0001	0.0001	0.0001
CTa	0.0034	0.001	0.017	0.021	0.02	0.016	0.031	0.14	0.029	0.075	0.024
CTb	0.0029	0.0009	0.016	0.017	0.019	0.015	0.03	0.138	0.027	0.02	0.015
CTc	0.0027	0.0008	0.007	0.01	0.016	0.013	0.023	0.13	0.0001	0.0001	0.0001
CWb[S]	0.0032	0.001	0.017	0.024	0.022	0.019	0.034	0.142	0.045	0.02	0.02
CWc[S]	0.0027	0.001	0.015	0.023	0.022	0.019	0.034	0.142	0.0001	0.0001	0.0001
CTb[S]	0.003	0.0009	0.015	0.017	0.015	0.014	0.03	0.125	0.024	0.02	0.014
CTc[S]	0.0025	0.0008	0.007	0.01	0.014	0.013	0.022	0.123	0.0001	0.0001	0.0001

Varying fsky and putting strong priors on z_p, ζ_s and ζ_r

fsky	$\sigma(\Omega_m h^2)$	$\sigma(\Omega_b h^2)$	$\sigma(\Omega_\Lambda)$	$\sigma(\sigma_8)$	$\sigma(n_s)$	$\sigma(\alpha_s)$	$\sigma(\tau)$	$\sigma(T/S)$	$\sigma(z_p)$	$\sigma(\zeta_s)$	$\sigma(\zeta_r)$
0.00001	0.005	0.0011	0.026	0.034	0.034	0.038	0.035	0.148	0.0001	0.0001	0.0001
0.0001	0.0034	0.0011	0.021	0.025	0.032	0.035	0.035	0.147	0.0001	0.0001	0.0001
0.001	0.0029	0.001	0.017	0.023	0.025	0.025	0.034	0.145	0.0001	0.0001	0.0001
0.01	0.0027	0.001	0.015	0.022	0.021	0.017	0.033	0.142	0.0001	0.0001	0.0001
0.1	0.0026	0.0009	0.011	0.016	0.017	0.013	0.027	0.132	0.0001	0.0001	0.0001
1	0.0022	0.0007	0.004	0.007	0.011	0.008	0.019	0.104	0.0001	0.0001	0.0001
With tomography											
0.00001	0.0049	0.0011	0.026	0.033	0.034	0.038	0.035	0.148	0.0001	0.0001	0.0001
0.0001	0.0033	0.0011	0.02	0.024	0.031	0.035	0.034	0.147	0.0001	0.0001	0.0001
0.001	0.0029	0.001	0.015	0.019	0.023	0.022	0.031	0.14	0.0001	0.0001	0.0001
0.01	0.0027	0.0008	0.007	0.01	0.016	0.013	0.023	0.13	0.0001	0.0001	0.0001
0.1	0.0024	0.0007	0.002	0.003	0.012	0.008	0.019	0.112	0.0001	0.0001	0.0001
1	0.0019	0.0006	8E-04	0.001	0.007	0.004	0.017	0.085	0.0001	0.0001	0.0001

Results

- A “reference” cosmic shear survey with f_{SKY}=0.01 and n_g=56 gal/arcmin^2 can reduce the uncertainties on n_s and α_s by roughly a factor of 2 relative to the current CMB data alone.
- Almost an order of magnitude on α_s for more ambitious future WL surveys (optimistic scenario)
- Even with 10% uncertainty on the calibration parameters, the precision on the cosmological parameters is only slightly degraded
- The uncertainties on cosmological parameters are more sensitive to the uncertainty in zp

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Conclusions

- Weak Lensing constraints reduce significantly the uncertainties on sigma_8, Omega_l and Omega_m h^2 (known)
- Weak Lensing surveys can supplement the CMB to constrain the running of the spectral index in the near future (new)
- This will be an important and necessary check that is independent from the constraints from CMB+Lyman alpha forest
- If a significant running (~0.03) is confirmed, this will have significant implications for theories of the early universe