Microwave sky confronts Topology of the Universe

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With:

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And, recently:

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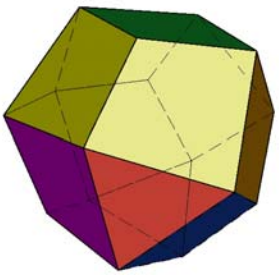
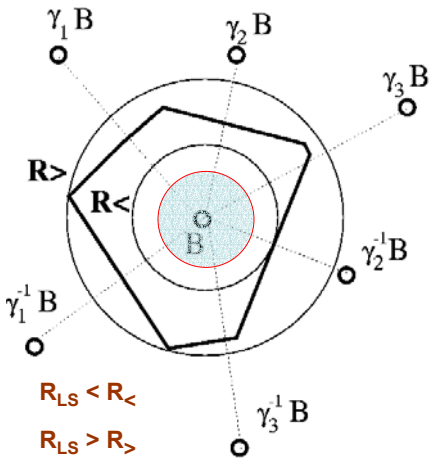
Adam Hincks

Dirichlet domain and dimensions of the compact space

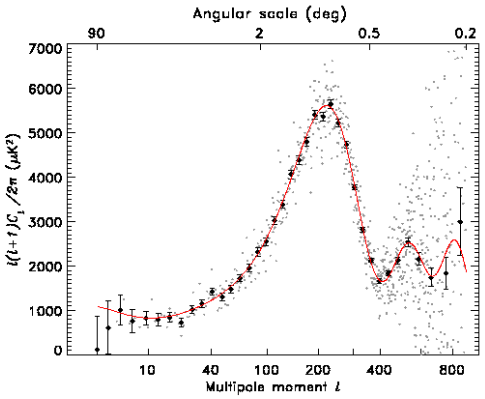




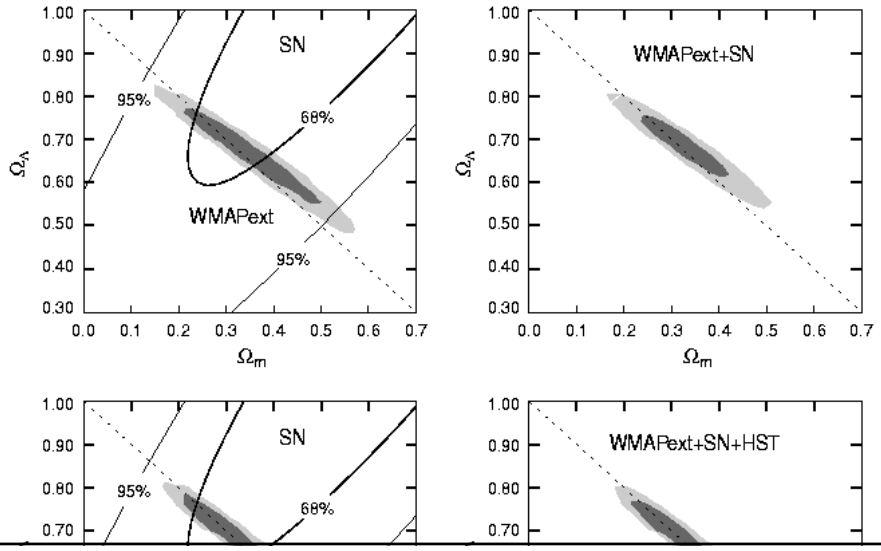
$R_{LS} < R_<$

$R_{LS} > R_>$

Sign of compactness ?



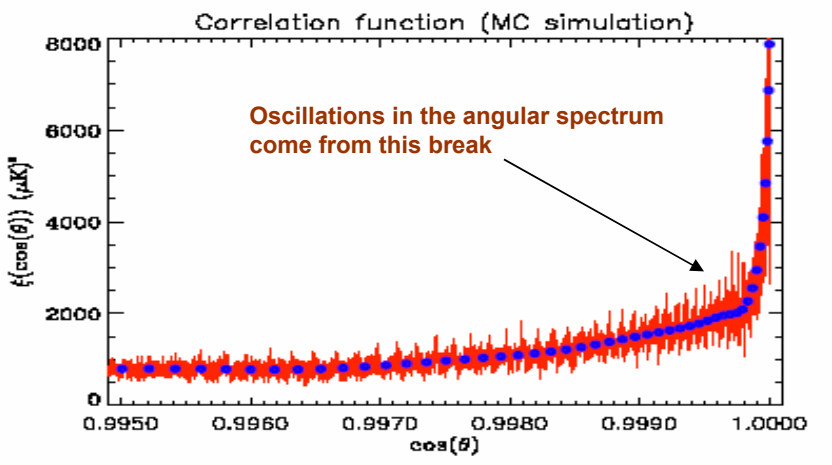
Closed, open and flat spaces



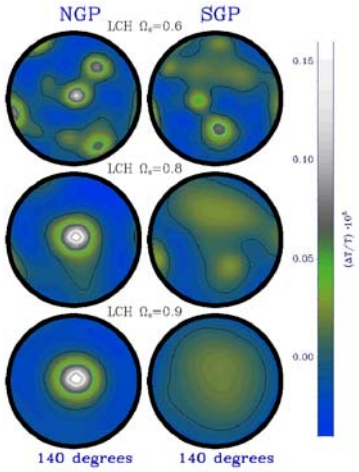
Perturbations in Compact space

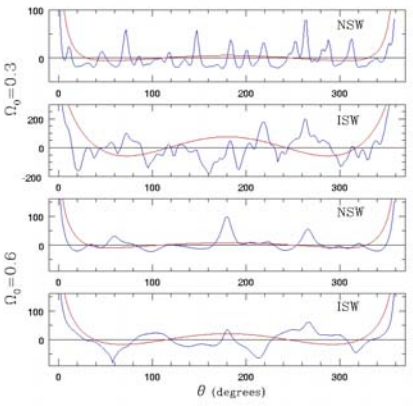
- Spectrum has the lowest eigenvalue
- Spectrum is discrete, hence statistics in general is anisotropic, especially at large scales.
- Statistical properties can be inhomogeneous.
- However, perturbations are Gaussian, thus in case of CMB temperature fluctuations are fully described by pixel-pixel correlation matrix $C_T(p,p')$

Correlation function of isotropic CMB

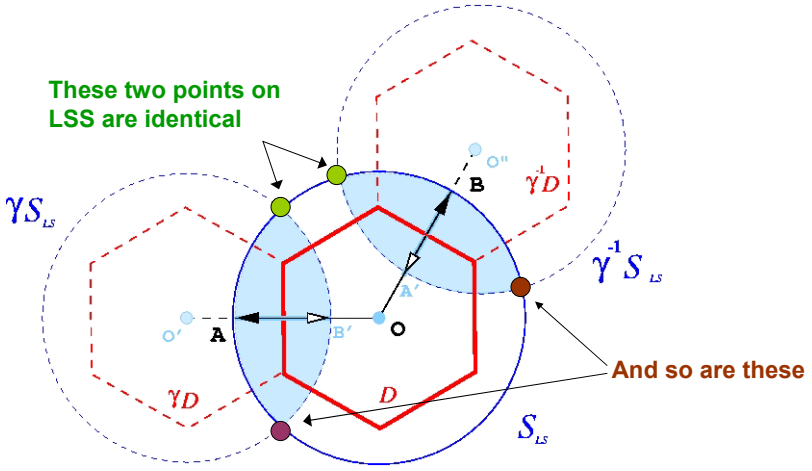


Pixel-pixel correlation with compact topology
(using method of images, BPS, Phys Rev D. 2000)



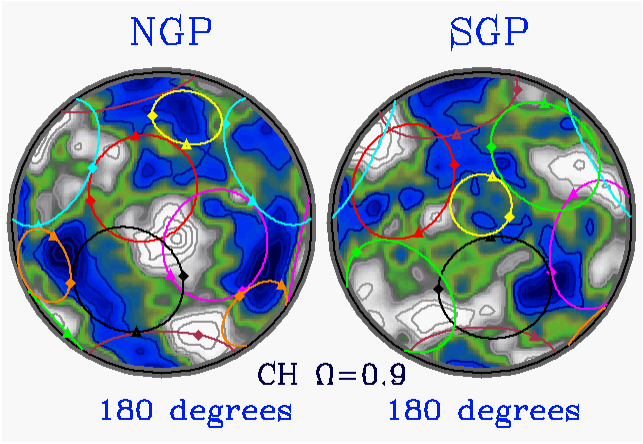


Example of strong correlation on last scattering surface

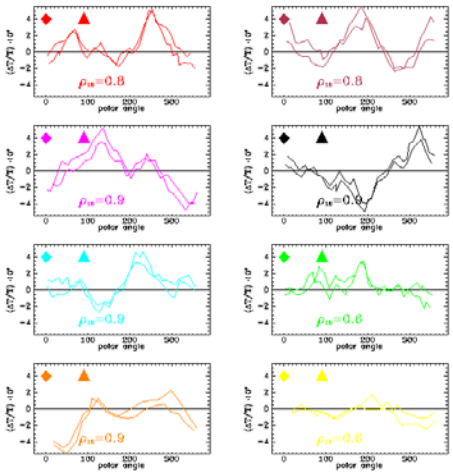


Correlated Circles

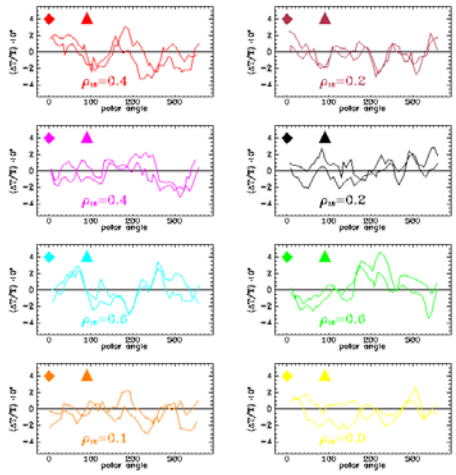
(after Cornish, Spergel, Starkman et al)



Temperature along the correlated circles (pure LSS signal)

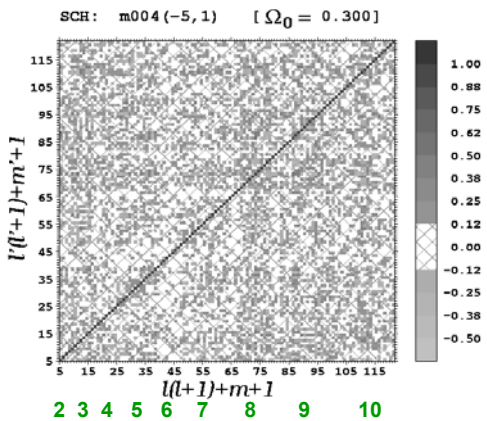


Temperature along the correlated circles (ISW modification)



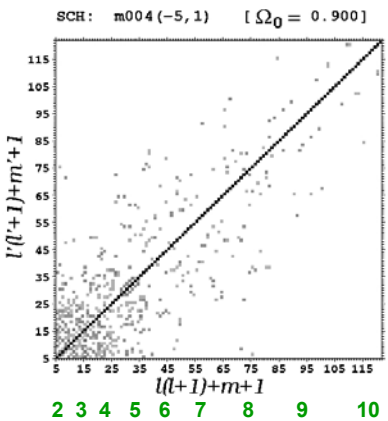
Inadequacy of isotropized C_l's: a_{lm} cross correlation

Very small space



2 3 4 5 6 7 8 9 10

Just a bit smaller than LSS

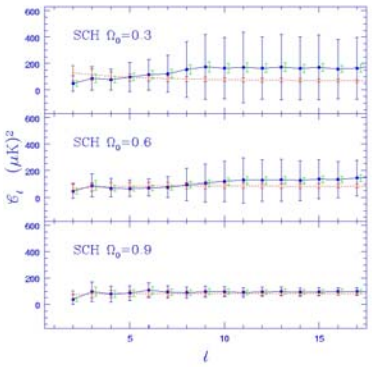


2 3 4 5 6 7 8 9 10

Compression to isotropic Cls is lossy
Inhanced cosmic variance of Cl's

$$C(\hat{q},\hat{q}') = C^{\mathrm{I}}(\hat{q},\hat{q}') + C^{\mathrm{A}}(\hat{q},\hat{q}')$$

$$\int d\Omega_{\hat{q}} \int d\Omega_{\hat{q}'} C^{\mathrm{A}}(\hat{q},\hat{q}')\, P_\ell(\hat{q}\cdot\hat{q}') = 0 .$$



$$\langle \tilde{C}_\ell \rangle = \frac{\ell(\ell+1)}{8\pi^2} \int d\Omega_{\hat{q}} \int d\Omega_{\hat{q}'} C(\hat{q},\hat{q}')\, P_\ell(\hat{q}\cdot\hat{q}') .$$

$$\mathrm{var}(\tilde{C}_\ell) \equiv \langle \tilde{C}_\ell^2 \rangle - \langle \tilde{C}_\ell \rangle^2$$

$$\frac{2\langle \tilde{C}_\ell \rangle^2}{2\ell+1} + \frac{\ell^2(\ell+1)^2}{32\pi^4} \int d\Omega_{\hat{q}_1} \int d\Omega_{\hat{q}_2} \left[\int d\Omega_{\hat{q}_3} C^{\mathrm{A}}(\hat{q}_1,\hat{q}_3)\, P_\ell(\hat{q}_2\cdot\hat{q}_3) \right]^2 .$$

Constraining the models from maps

- **Complete topological information is retained when comparison with data is done on map level**

$$\mathcal{L}(C_T) = \frac{1}{(2\pi)^{N_p/2} \|C_N + C_T\|^{1/2}} e^{-\frac{1}{2}\Delta^t (C_N + C_T)^{-1} \Delta} .$$

• Low res (Nside=32) maps contain most information, although special techniques as circle searching may benefit from finer pixalization. The cost – additional small scale effects which mask topological correlations.

• Main signal comes from effects, localized in space, e.q on LSS. But even integrated along the line of sight contributions retain signature of compact topology.

• Orientation of the space (and, possibly, position of observer) are additional parameters to consider. What is the prior for them ?

Conclusions

- Our analysis thus far demostrated that small Universes with V < VLS are failing to describe the temperature maps. Reason – complex correlation are not really observed (in line with circle finding results).
- This is despite the fact that it is not too difficult to fit the low l suppression of isotropized angular power spectrum.
- Currently we are incorporating the topology in our CMB analysis pipeline, with WMAP results expected soon
- The region near Otot=1 is rich with possibilities, with negatively or positively curved or flat spaces giving rise to distinct topological choices.