Bandpowers & Boltzmann codes
AND
Lensing of the CMB

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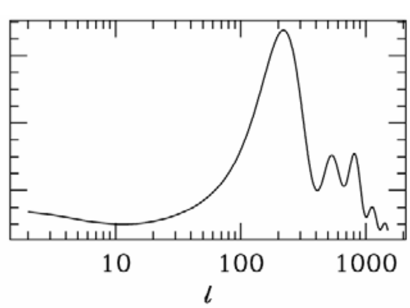
Bandpowers & Boltzmann codes

- Because of projection, baryon-photon inertia and lensing, the CMB spectra are smooth in ***l***.
  - HSSW (1995)
  - Plaszczynski & Couchot (2003)
  - Hu & Okamoto (2003)



You do not need >1000 numbers to describe this spectrum!

Savitsky-Golay filtering
(with a twist)

- Knowledge of N "bandpowers" is enough to reconstruct the spectrum (assuming smoothness).

$$\mathcal{C}_{B_i} \equiv \sum_{\ell} \frac{\ell(\ell+1)}{2\pi} C_\ell W_\ell^{(B_i)}$$

- Procedure: fit a low order polynomial to the bandpowers in a range of $\boldsymbol{l}$, and use the fit over that range.
- Surprisingly wide bandpowers (σ_l=15 or 40!) can be tolerated with sub-percent level accuracy in the reconstruction.
- Welch filters work well: $W_\ell^{(B_i)}(x = \Delta\ell/\sigma) = 1 - x^2$

Who cares?

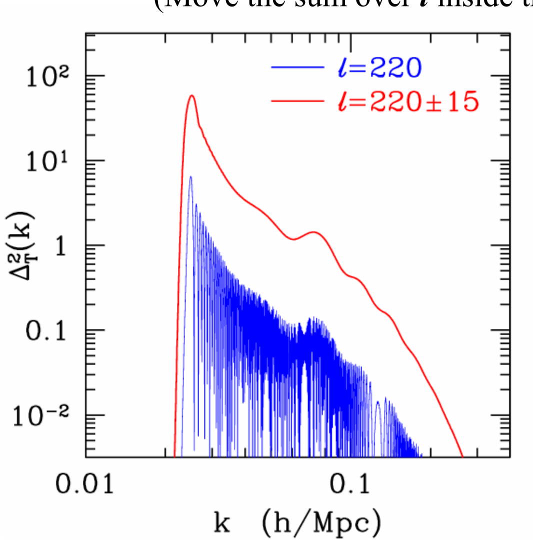
- The C_l are integrals of a primordial power spectrum times a transfer function.

$$C_\ell \sim \int \frac{dk}{k} P_{\rm init}(k) \Delta^2_{T\ell}(k)$$

- For an individual $\boldsymbol{l}$ mode, Δ^2 oscillates drastically (like j_l^2).
- Integrating, or storing, Δ^2 is thus expensive!

But integrating a bandpower is easy!

(Move the sum over ***l*** inside the integral …)



Reduces number of modes which need to be computed and/or stored!

Significantly speeds up a traditional hierarchy code with errors below 0.5% !

Hierarchy codes are very stable, and can now be “fast”.

Lensing of the CMB

Lensing by clusters of galaxies.

Lensing by large-scale structure.

Work done in collaboration with
Alexandre Amblard
and
Chris Vale.
Inspired by the APEX-SZ and Planck projects

The basic idea

We want to reconstruct the (projected) mass field using the CMB as our "sources" for lensing.
Three equations sum up the basic idea.

$$T(\theta) = \widetilde{T}(\theta - \delta\theta) \approx \widetilde{T}(\theta) - \delta\theta \cdot \nabla\widetilde{T}(\theta) \quad \text{(for small deflections)}$$

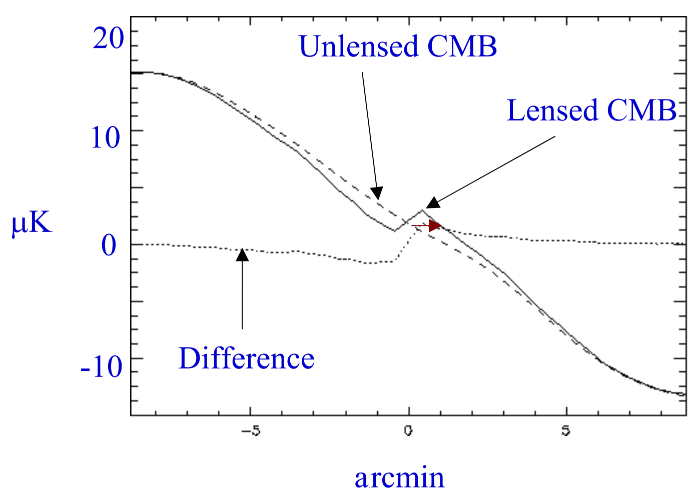
$$T(\theta) = \widetilde{T}_{y0} \times (\theta_y - \delta\theta_y) \quad \text{(for a pure gradient)}$$

$$\kappa(\mathbf{l}) = \frac{-i\mathbf{l}^2}{l_y}\delta\theta_y \quad \text{(for weak lensing)}$$

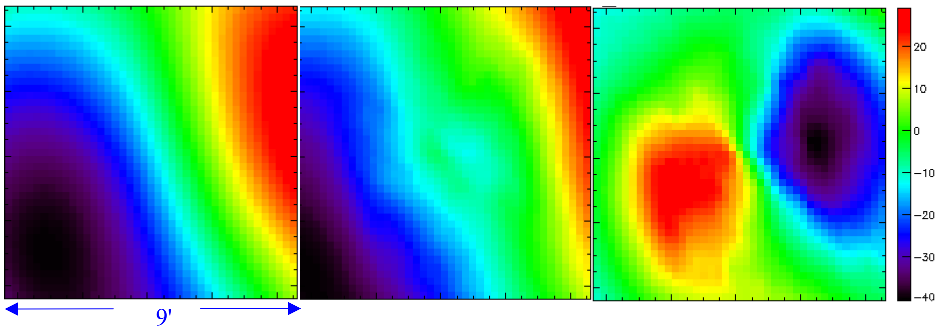
Seljak & Zaldarriaga 2000, ApJ, 538, 57
(with "improvements" by Wayne Hu)

Effect of lensing on the CMB

(A spherical 'cluster' at the origin)



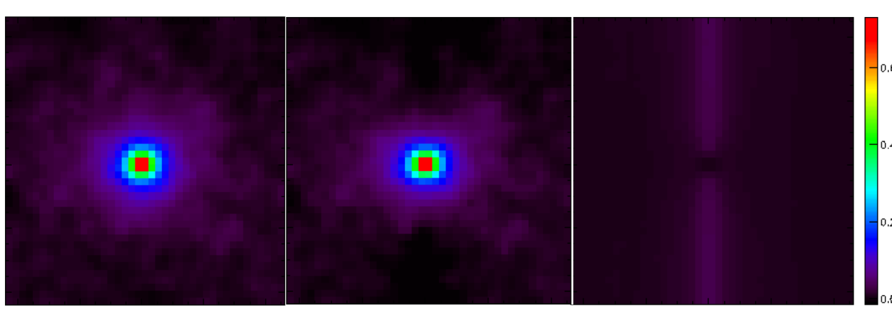
Lensing signal from the CMB



Unlensed Lensed Difference

Isolated halo lensing a gradient

(No noise, no beam, no foregrounds …)



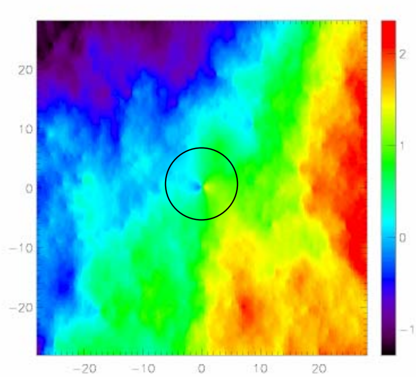
Cluster Reconstructed Difference

It works well for the toy model.

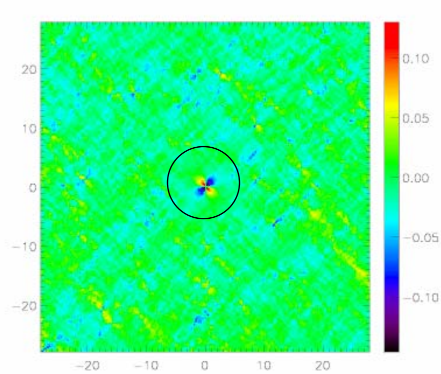
Some issues

- Seljak & Zaldarriaga assumed that one can approximate the CMB as a pure gradient.
  - This is manifestly not true, especially if we “mask” central regions.
- They assumed the deflection angle is the “signal” we need to reconstruct.
  - Since the unlensed CMB is not known, we always reconstruct deflection angle differences.
- The mass distribution is often complex.
  - Deflections from LSS are often bigger than from the cluster.
- SZ effect: thermal and kinetic.
  - The kinetic SZ is spectrally indistinguishable from, and highly correlated with, the lensing signal.
- Point sources and other foregrounds.
- Instrument effects: noise and beam.

The deflection field



Deflection field

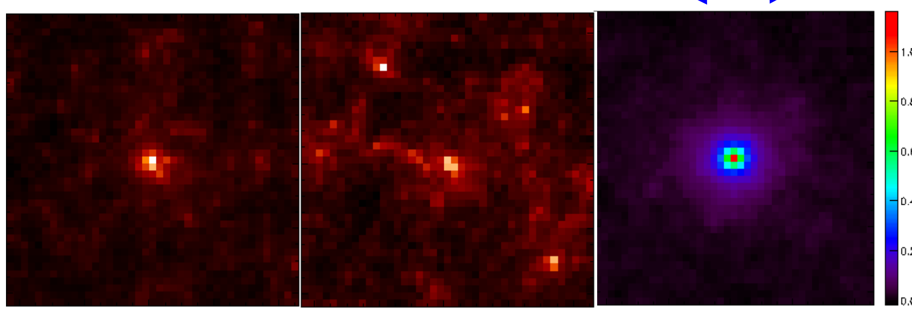


Gradient of deflection field

Cluster convergence profiles

(Linear color scale)

2 Mpc/h at z=1



9'

Isolated Cluster

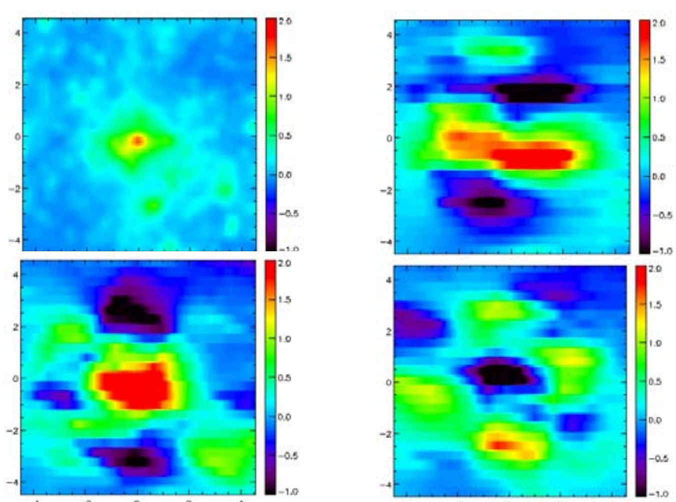
Typical Cluster

Stacked Average

The role of kSZ

No noise, thermal SZ or point sources, and beam is limited only by pixel size of 0.22'

CMB is a known pure gradient. kSZ for this cluster is almost a dipole.

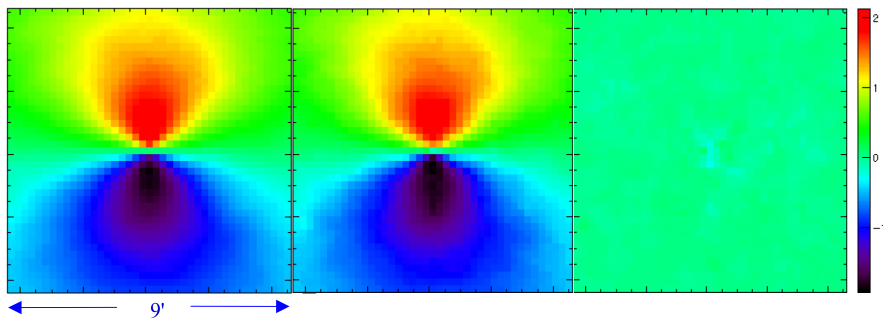


Best case stacked clusters

As before, but averaging 27,000 clusters found on the whole sky

$2x10^{14}<M<3x10^{14}$

$0.6<z<2$



Signal

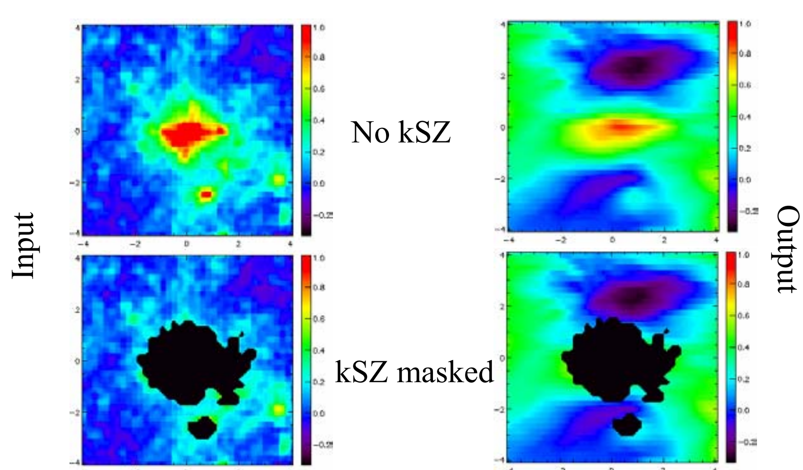
Signal + KSZ

Kinetic SZ
(averages down)

Can also mask the central regions of clusters

Beyond a pure gradient …

Known unlensed CMB gradient, no noise, no beam, …



Lensing by large-scale structure

(Want to reconstruct larger scale structures, and power spectra)

- We test the (optimal) divergence of temperature-gradient estimator
  - Hu 2001, ApJ 557, L79 & PRD 64, 083005
- The derivation is valid when:
  - Linear approximation, with small deflection.
  - Deflection field is Gaussian.
  - CMB anisotropies are Gaussian.
  - Reconstruction is noise dominated.

This estimator is also optimal for large angular scales, where it is equivalent to the maximum likelihood estimator (Hirata & Seljak 2003)

Testing the reconstruction

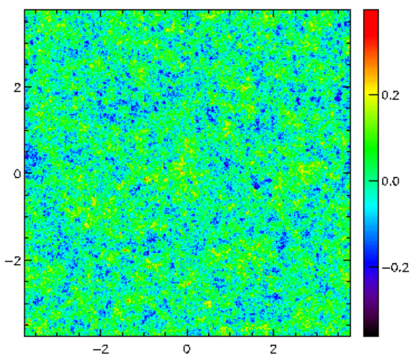
- We want to test this method on simulations to see how violations in the assumptions degrade performance.
  - Effects of signal dominance.
  - Effects of non-Gaussianity in lensing field.
  - Effects of non-Gaussianity in CMB (kSZ).
  - Foregrounds, noise, finite resolution, …

Model assumptions satisfied

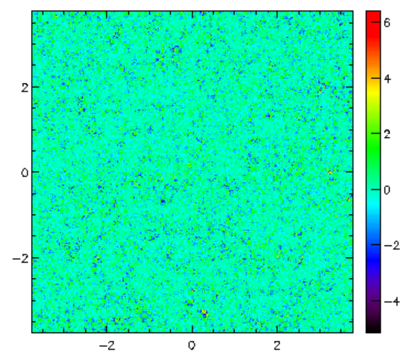
Nominal simulation : 0.8’ resolution, 30x30 degree field, 2 μK of noise

We used the Limber’s approximation to evaluate the integrated power spectrum (z > 2) along the line of sight with PD96 for the power spectrum and generated a Gaussian realization of this.

Map of κ



Map of κ_{est}



Raw maps (ie without filtering)

Model assumptions satisfied

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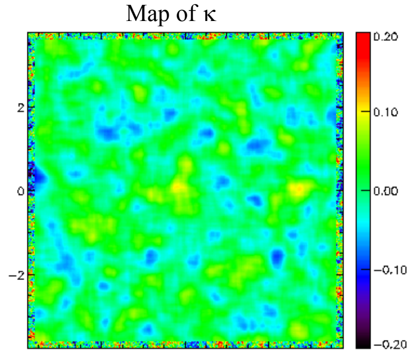

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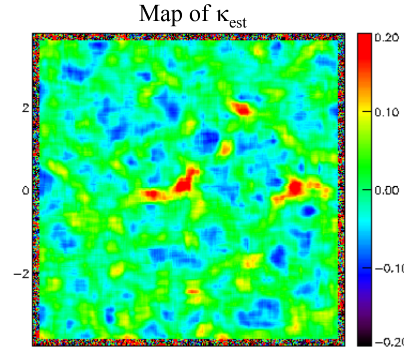


Filtered maps (θ>20’)

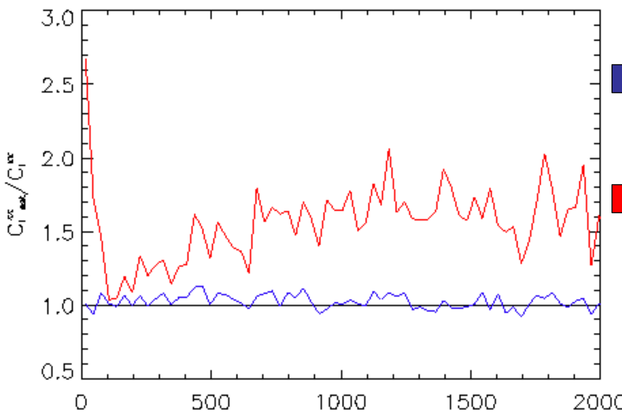
Power spectrum reconstruction

$$\langle \kappa^*_{est}(l')\kappa_{est}(l)\rangle = 2\pi\delta(l-l')(C_l^{\kappa\kappa} + \frac{N_l l^2}{4})$$

Spectrum of the estimated map

$$\langle \kappa^*(l')\kappa_{est}(l)\rangle = 2\pi\delta(l-l')C_l^{\kappa\kappa}$$

Cross-spectrum between real and estimated map.



Spectrum of the input map

The map is not biased

Noise can be difficult to estimate.

This bias is known

(but not widely appreciated)

$$\begin{aligned}
C_l^{2\mathrm{NG}} \quad = \quad & C_l^{\phi\phi}\left[(2\pi)^{-2}\int d^2 l_1 \mathbf{l}\cdot\mathbf{l}_1 C^{\Theta}_{|\mathbf{l}-\mathbf{l}_1|}\right]^2 \\
+ \quad & (2\pi)^{-4}\int d^2 l_1 \int d^2 l_2 \Big\{ C^{\phi\phi}_{|\mathbf{l}_1+\mathbf{l}_2|}[\mathbf{l}_1\cdot(\mathbf{l}_1+\mathbf{l}_2)C^{\Theta}_{l_1} \\
+ \quad & \mathbf{l}_2\cdot(\mathbf{l}_1+\mathbf{l}_2)C^{\Theta}_{l_2}][(\mathbf{l}+\mathbf{l}_2)\cdot(\mathbf{l}_1+\mathbf{l}_2)C^{\Theta}_{|\mathbf{l}+\mathbf{l}_2|} \\
- \quad & (\mathbf{l}_1+\mathbf{l}_2)\cdot(\mathbf{l}-\mathbf{l}_1)C^{\Theta}_{|\mathbf{l}-\mathbf{l}_1|}] \\
+ \quad & C^{\phi\phi}_{|\mathbf{l}-\mathbf{l}_1+\mathbf{l}_2|}[(\mathbf{l}-\mathbf{l}_1+\mathbf{l}_2)\cdot\mathbf{l}_2 C^{\Theta}_{l_2} \\
+ \quad & (\mathbf{l}-\mathbf{l}_1+\mathbf{l}_2)\cdot(\mathbf{l}-\mathbf{l}_1)C^{\Theta}_{|\mathbf{l}-\mathbf{l}_1|}] \\
& [-(\mathbf{l}-\mathbf{l}_1+\mathbf{l}_2)\cdot\mathbf{l}_1 C^{\Theta}_{l_1} \\
+ \quad & (\mathbf{l}-\mathbf{l}_1+\mathbf{l}_2)\cdot(\mathbf{l}+\mathbf{l}_2)C^{\Theta}_{|\mathbf{l}+\mathbf{l}_2|}]\Big\}
\end{aligned}$$

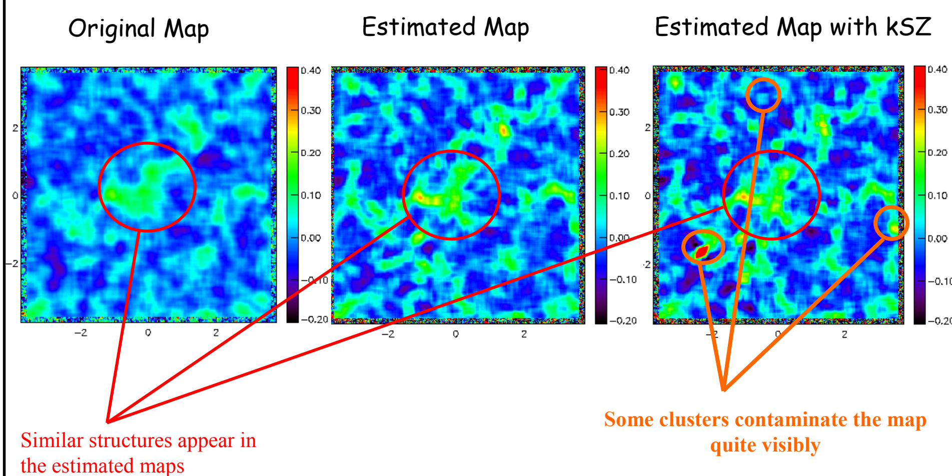
Included

Ignored!

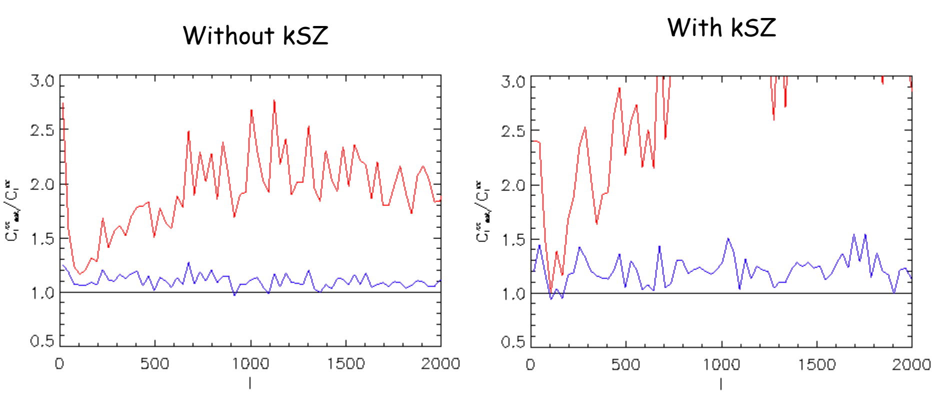
(Cooray & Kesden 2003)

Adding complexity

Now we add lensing by z<2 structure and kSZ



Power spectra

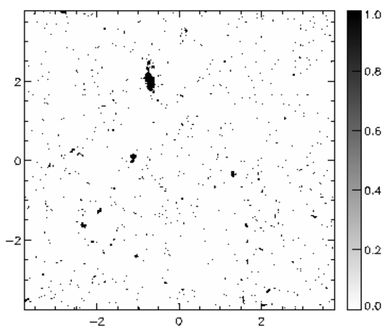


- The bias from the noise is even more difficult to determine (high red curves)
- The cross-spectrum is 10 to 20% higher than real spectrum

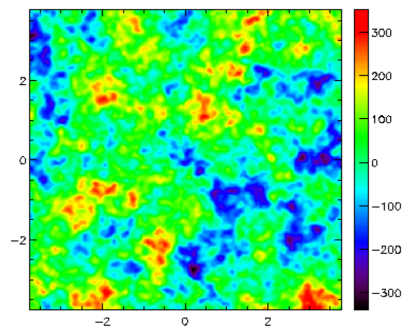
➡ The estimated map is biased

Corrective lensing?

Let's assume we have a map at another frequency which allows us to detect clusters via tSZ (S > 50 μK). Mask 1.4% of the pixels.

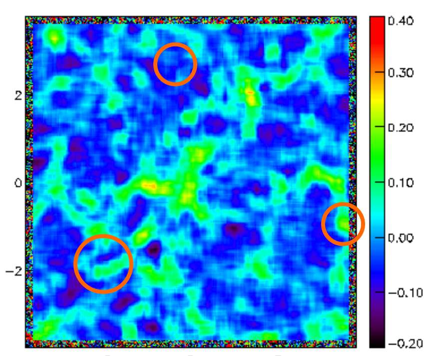


Mask used

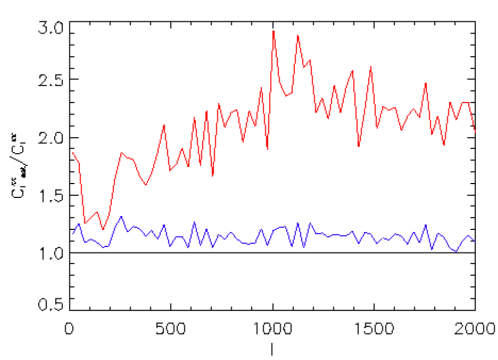


Interpolated Map

Results



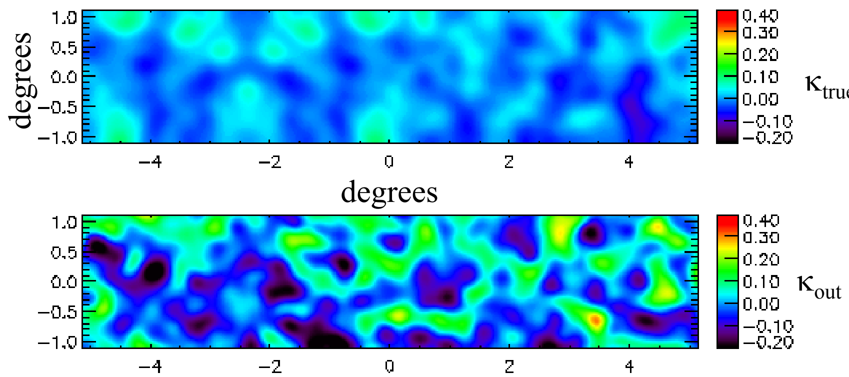
Some contamination from the SZ signal is reduced



The spectrum and cross-spectrum are coming back almost to spectrum without SZ component.

The situation for Planck

(Zoom in on central region of a 20x4 degree patch near the ecliptic pole)



5’ FWHM, 25 μK-arcmin

Lensed CMB + kSZ + noise

Simple reconstruction, smoothed to 20’

Conclusions

- Using bandpowers inside a hierarchy code can lead to significant speed-ups with little loss of accuracy.
- Bandpowers may be useful when storing grids or lookup tables (e.g. DASh).
- Cluster and large-scale structure reconstruction using the CMB as a source is fraught with difficulties.
- Existing estimators or the large-scale structure power spectrum are biased and noisy.
  - Work is underway to “fix” them.