THE DS/DS CORRESPONDENCE

w/ MOHSEN ALISHAHIHA
EVA SILVERSTEIN
DAVID TONG

040725
+ WORK IN PROGRESS

PARTLY BASED ON:

AK 0305192

GOAL: HOLOGRAPHIC DESCRIPTION OF DE SITTER SPACE

WHY? DARK ENERGY!

70 % OF THE UNIVERSE ARE MADE OF THAT STUFF

→ CAN WE UNDERSTAND IT ?

THERE ARE EVEN PROPOSALS OUT THERE (HOGAN) FOR EXPERIMENTAL IMPLICATIONS OF HOLOGRAPHY DURING INFLATION

BASED ON "COUNTING"

(BOUSSO'S VERSION OF $S \leq A/4G$)

WE WANT: HOLOGRAPHIC DESCRIPTION, NOT JUST BOUND.

ALSO: QUANTUM GRAVITY IN DS IS ALSO FRONTIER IN OUR (MIS) UNDERSTANDING OF THE "BASICS"

(NO S-MATRIX... WHAT ARE OBSERVABLES?)

RECALL: AdS/CFT

GENERAL STATEMENT:

GRAVITY + OTHER FIELDS ON AdS_d M_5, L $M_5 L >> 1$ $\sim$ CFT ON $(d-1)$ DIMENSIONAL BOUNDARY GEOMETRY.

FIELDS $\longleftrightarrow$ OPERATORS

$(M_5 L)^3 \longleftrightarrow N^2 \sim N_{DOF}$

$g_{\mu\nu} \longleftrightarrow T_{\mu\nu}$

+UV COMPLETION

FOLLOWS FROM ISOMETRIES AND THE FACT THAT BOUNDARY CORRELATION FUNCTIONS ARE THE CLOSEST TO S-MATRIX WE HAVE

MALDACENA'S DUALITY IS A PARTICULAR EXAMPLE!

d-dim: IIB SUGRA

UV-COMPLETED BY

IIB-STRING THEORY

BACKGROUND: $AdS_5 \times S_5$

---

d-1 dim: $\mathcal{N}=4$ SYM

SU(N) GAUGE GROUP

$(M_5 L)^3 \sim N^2$

$L \sim (g^2_{YM} N)^{1/4}$

GUIDING PRINCIPLES:

- HORIZON COMPLEMENT.

  ⇝ PHYSICS BEHIND HORIZONS IS REDUNDANT

  ⇝ dS IS A "HOT TIN CAN!"

- EXISTENCE OF FUTURE INFINITY w/ $ds^2 \sim e^{2t} ds^2_{SPAT.} - dt^2$

  QUESTIONABLE

  DON'T MESS WITH TIME

  ⇝ POINCARE RECCURENCES

  ⇝ METASTABILITY OF STRINGY dS

→ HOLOGRAPHIC DESCRIPTION OF METASTABLE DS SHOULD FOLLOW NATURALLY.

THE PROPOSAL, v.1 ($dS_d \rightsquigarrow dS_5$)

GRAVITY + OTHER FIELDS ON dS_d

$M_5, L.$

$LM_5 \gg 1$

+ UV-COMPLETION

$\sim$

GRAVITY ON dS_{d-1}

+ 2 CFT's CUTOFF AT SCALE $1/L$

+ UV COMPLETION

$M_4^2 = M_5^3 L$

IN PARTICULAR:

GRAVITY ON dS_d ~ GRAVITY + CFT on dS_{d-1}

S

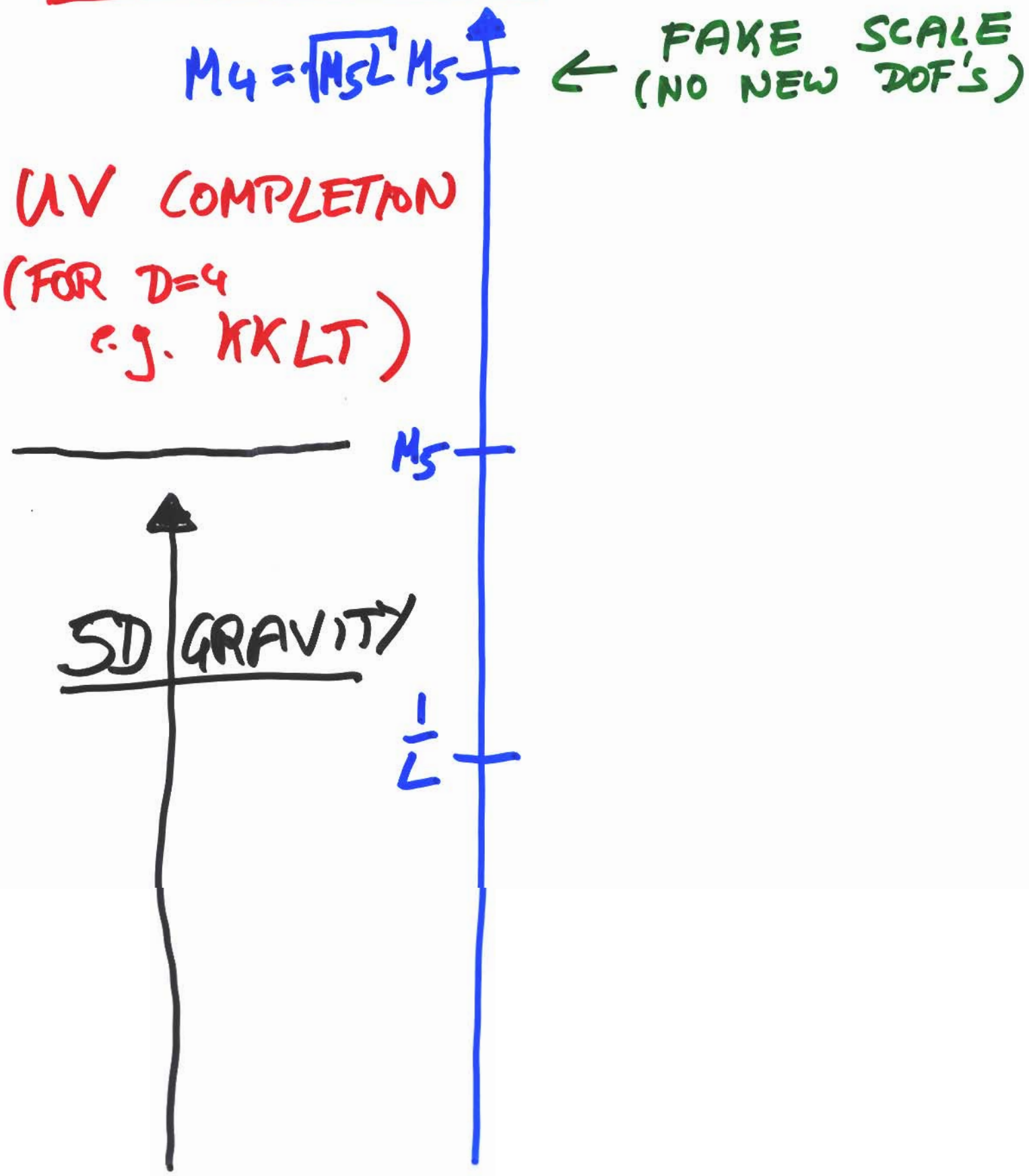
GRAVITY + CFT on dS_2 ..... ~ GRAVITY + CFT ON dS_{d-2}

S

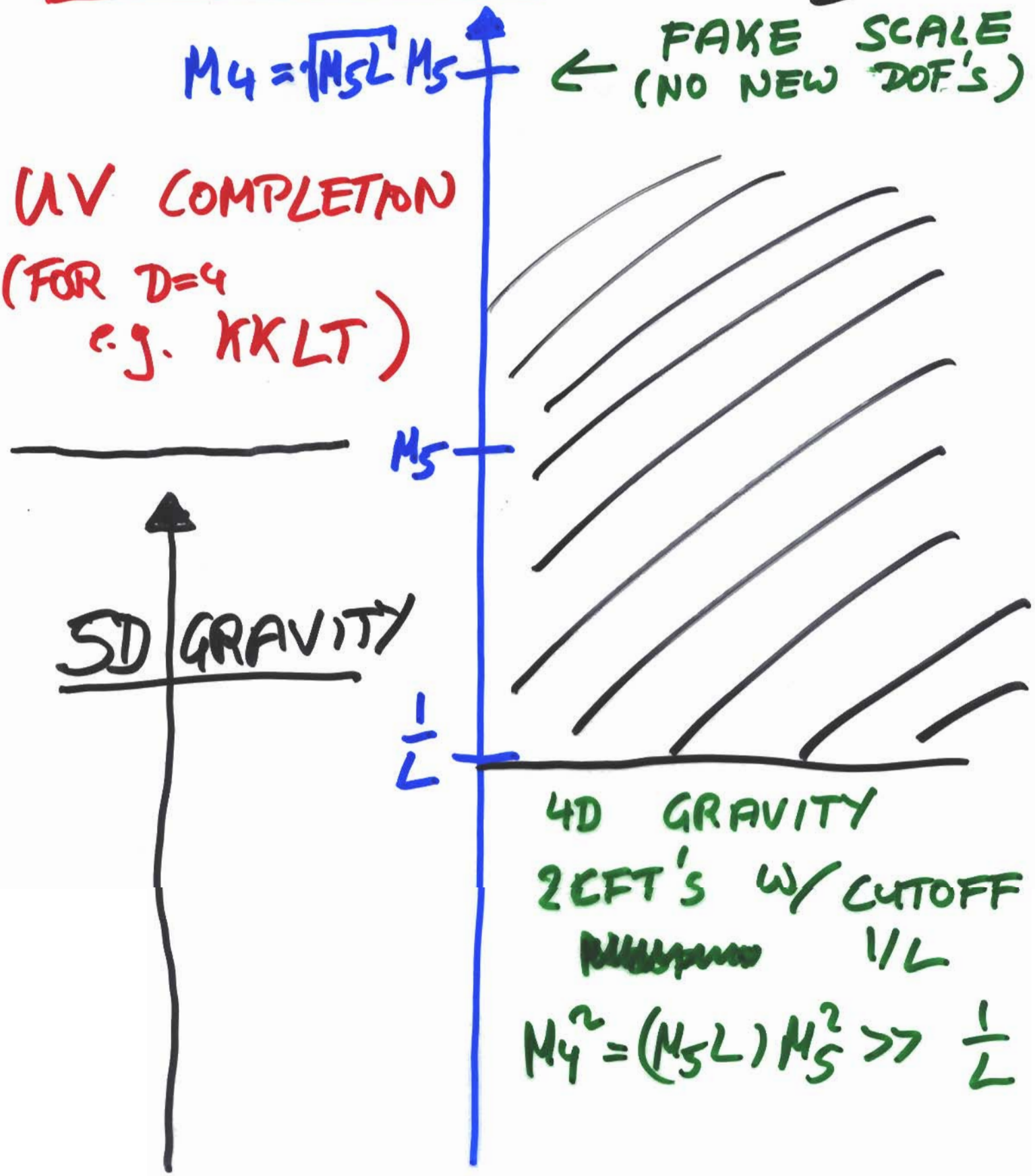
GRAVITY + CFT on dS_1

THESE ARE SOLVABLE, UV COMPLETE QUANTUM GRAVITIES

THE SCALES: (RS OR DS/OS)



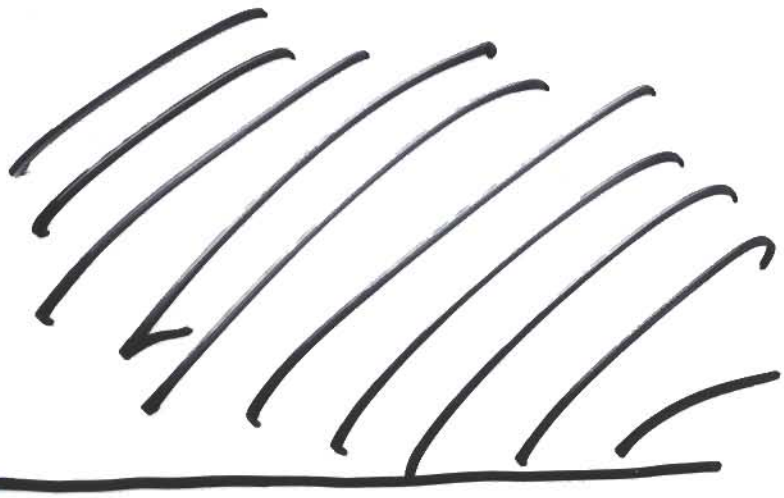
THE SCALES: (RS OR DS/DS) V.1



THE SCALES: (RS ON. 25/05)

$M_4 = \sqrt{M_5 L}\, M_5$ ← FAKE SCALE (NO NEW DOF's)

UV COMPLETION (FOR D=4 e.g. KKLT)



M_5

4D GRAVITY w/ $M_4^2 = M_5^3 L$

2 CFTs

$\frac{1}{L}$

CONFORMAL INVARIANCE BROKEN ABOVE 1/L DUE TO COUPLING TO GRAVITY

$\mathcal{L} = \mathcal{L}_{CFT_1} + \mathcal{L}_{CFT_2} + M_4^2 (R - 2\Lambda)$

VALID UP TO M_5!

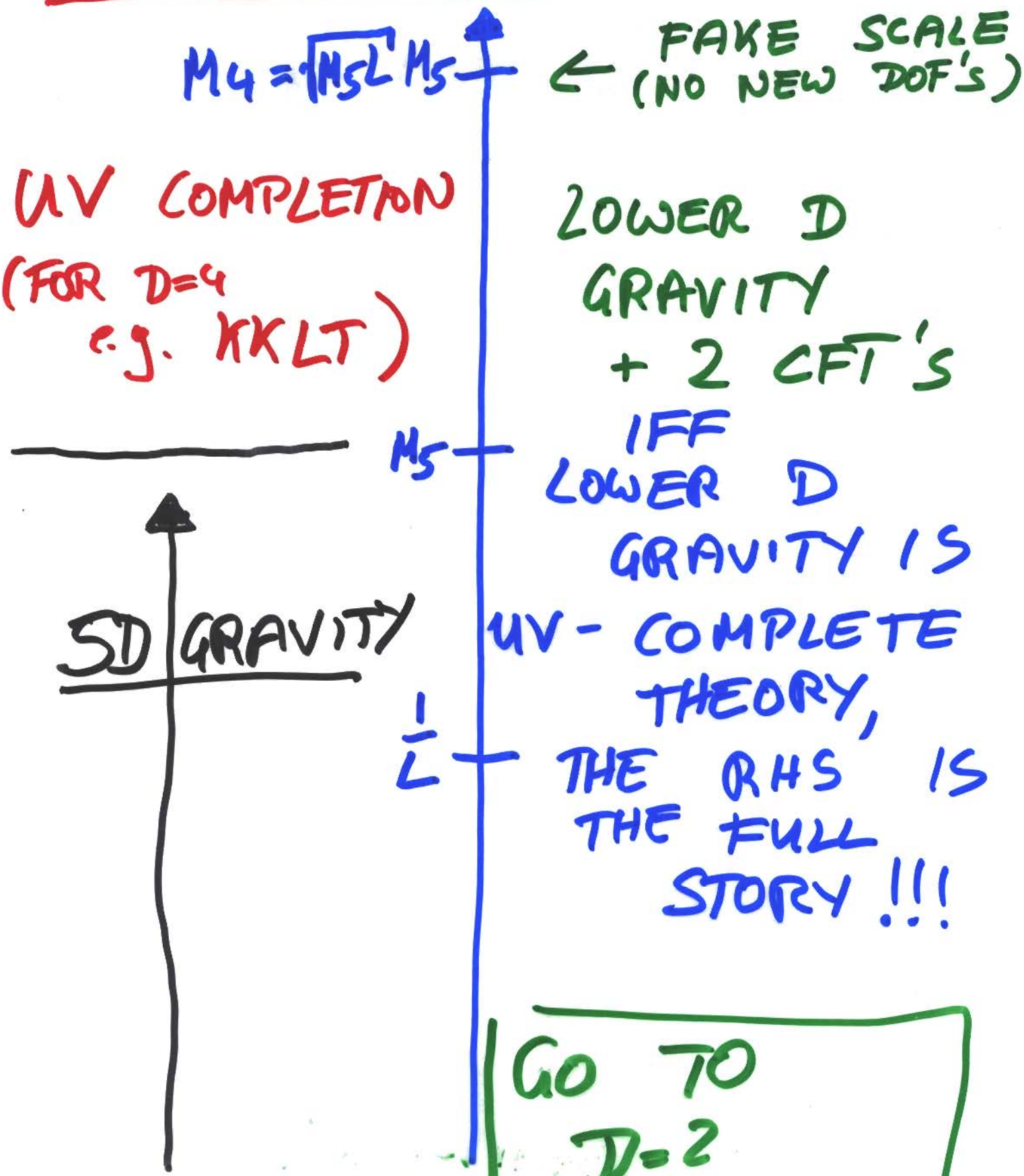
+ COUPLINGS OF CFT_1/CFT_2

5D GRAVITY

DO CLASSIC. GRAVITY

⇒ READ OFF FT PROPERTIES ABOVE 1/L

THE SCALES: (RS OR ~~DS~~)



WHAT IS QUANTUM GRAVITY IN 0+1 DIMENSIONAL dS_1 + CFT ?

EXAMPLE: THE RELATIVISTIC FREE POINT PARTICLE

$$S = \int d\tau \sqrt{(\dot{X})^2} \cong$$

$$\cong \int d\tau \underbrace{\sqrt{g} g^{\tau\tau}}_{N^{-1/2}} \partial_\tau X \partial_\tau X + \Lambda \sqrt{g}$$

$$= \int d\tau \; N^{-1/2} (\partial_\tau X)^2 + \Lambda N^{1/2}$$

$$\Lambda = 0 \Rightarrow \boxed{H = 0}$$

HAMILTONIAN CONSTRAINT

GRAVITY IN 1D = QM + CONSTRAINT.

dS/dS ACTION IN 0+1
HAS THE SAME "GENERIC"
FORM W/

$$(\dot{X})^2 \to \mathcal{L}_{CFT_1} + \mathcal{L}_{CFT_2} + \dots$$

BOTTOMLINE:

QUANTUMGRAVITY IN DS
≈ QM W/ HAMILTONIAN
CONSTRAINT

→ CLOSED SYSTEM

$|\Psi\rangle$ DOES NOT EVOLVE

WHAT ARE THE OBSERVABLES?

NOTE THAT A CLOSED TIME REPARAMETRIZATION INVARIANT QM HAS BEEN PROPOSED AS THE CORRECT FRAMEWORK FOR QUANTUMCOSMOLOGY BEFORE! (HARTLE, ...)

THERE: MOTIVATION FROM MINI SUPER SPACE

SPACE OF SPACES

TRUNCATED TO FRW-LIKE SCALE FACTOR .... NOT REALLY VALID, BUT TOY MODEL

WE GET A SIMILAR SYSTEM AS A HOLOGRAPHIC DUAL !!

SO WHAT ARE THE OBSERVABLES?

DECOHERENCE FUNCTIONALS

(HARTLE; HOROWITZ & MALDACENA)

PROJECTION OPERATORS:

$$\{P_{\alpha_k}(t_k)\}\ ;\ \sum P_{\alpha_k}(t_k) = 1$$

"ALTERNATIVE HISTORIES"

$$C_{\alpha\beta} = P_{\alpha_n}(t_n) \cdots P_{\alpha_1}(t_1)$$

$$\alpha = \alpha_1 \alpha_2 \alpha_3 \cdots \alpha_n$$

BUT HOW CAN ONE DERIVE (AT LEAST VERSION 1) OF THIS DUALITY?

REVIEW: AdS/CFT APPLIED TO RS

CUT OFF CFTS + GRAVITY $\sim$ CUT OFF SPACES

RS- ACTION: 5d GRAVITY + CC

$$S = M_5^3 \int d^5x \sqrt{g} \,(R - 2\Lambda)$$

$\Lambda < 0$, $\lambda > 0$

$$- \lambda \int d^5x \, \delta(r) \sqrt{g_I}$$

BRANE

GROUNDSTATE = GEOMETRY
↳ MAXIMAL SYMMETRIC SLICES

$$ds^2 = e^{2A} ds^2_{4d} + dr^2$$

ds^2_{4d}: dS_4, H_4 or AdS_4

NO BRANE: BULK IS AdS_5 WRITTEN IN dS_4, H_4, AdS_4 SLICING.

⇒ DUAL CFT NATURALLY DEFINED ON dS_4, H_4, AdS_4

BRANE: CUT AND PASTE (2 COPIES)

↳ CUTOFF GEOMETRY → TRUNCATE CFT

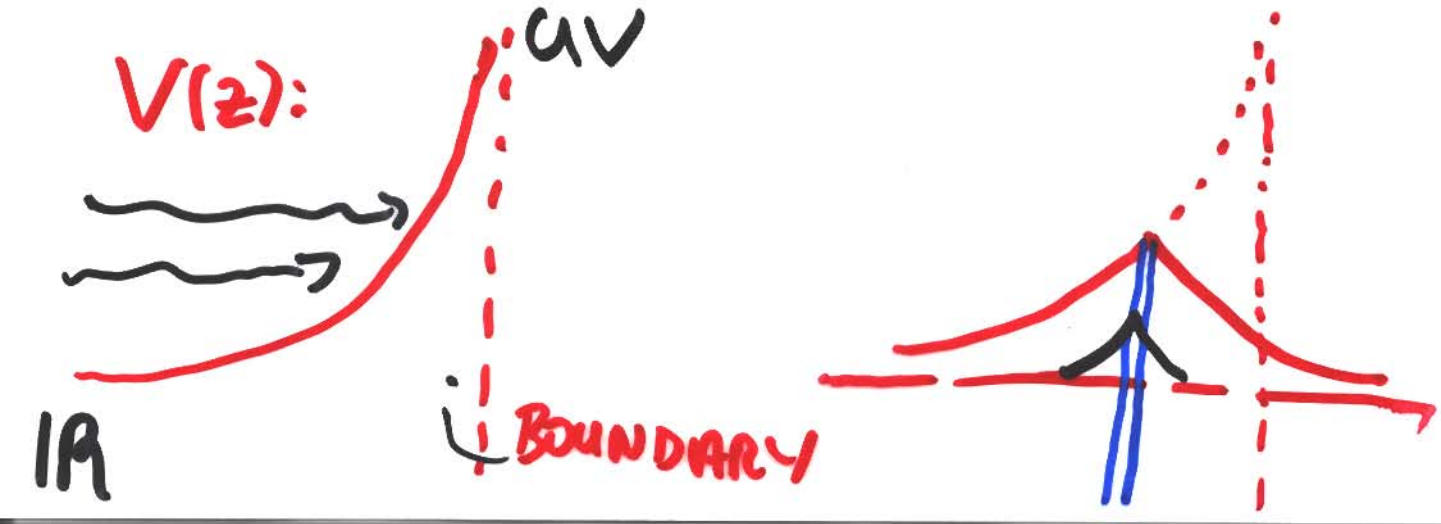
$$e^A = \begin{cases} L/\epsilon \sinh(r/L) & dS_4 \\ e^{r/L} & \text{FOR } M_4 \\ L/\epsilon \cosh(r/L) & AdS_4 \end{cases}$$

SMALL FLUCTUATIONS: $g_{\mu\nu} + h_{\mu\nu}$

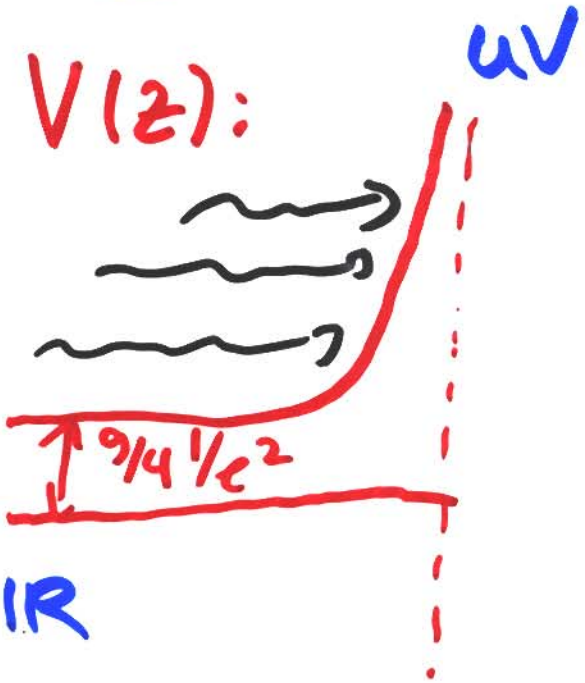
EINSTEIN: 2nd ORDER DEQ FOR $h_{\mu\nu}$

→ REWRITE AS SCHRÖDINGER EQUATION W/ $V(z)$ → VOLCANO

e.g.: M_4 RS



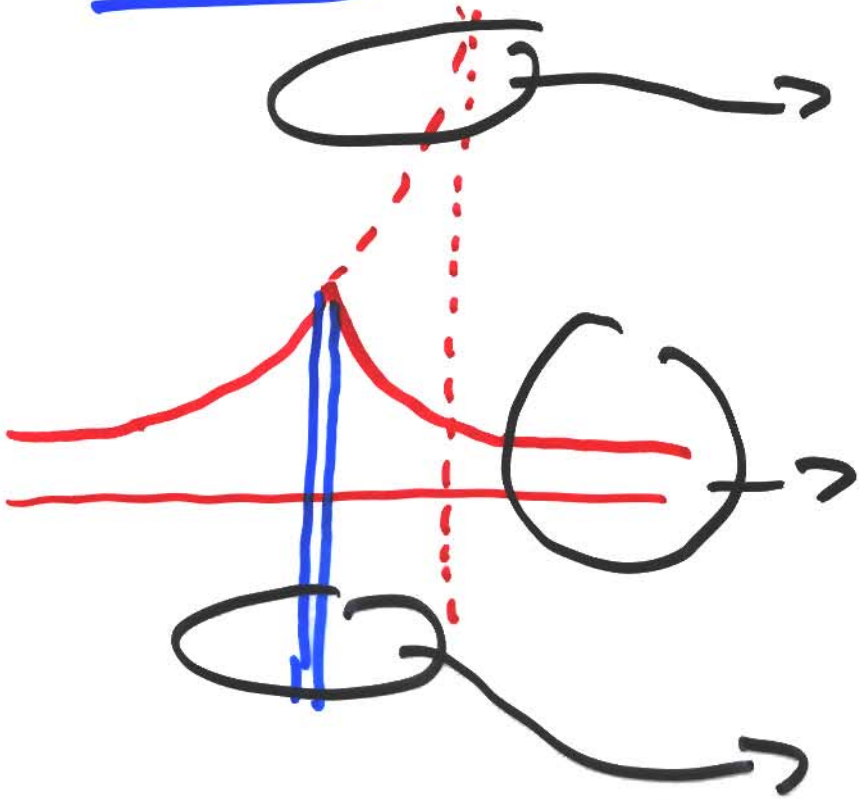
dS_4 - SLICING:



CFT ON dS_4

USE UV/IR

+ BRANE:

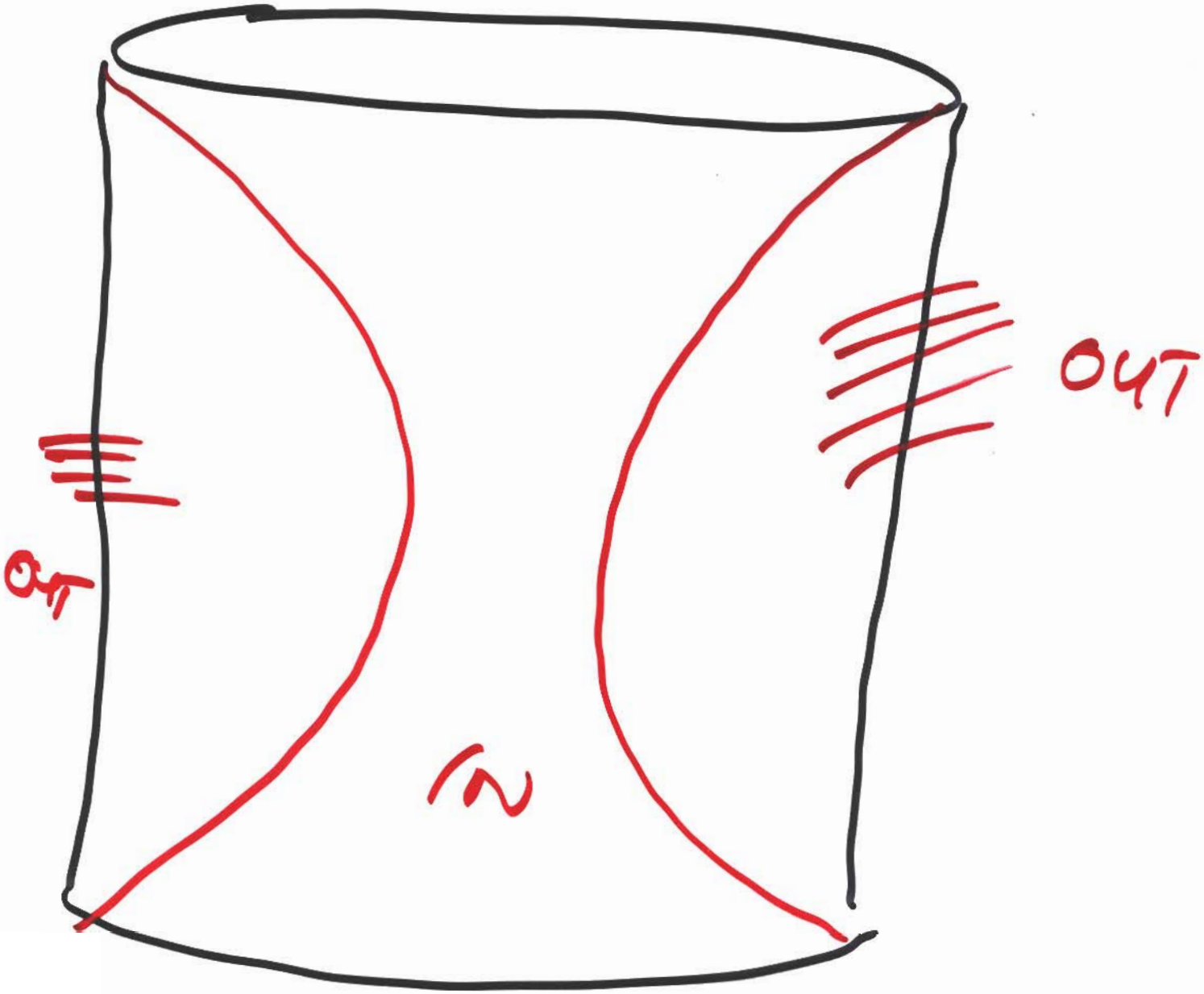


UV DEGREES OF FREEDOM CUT-OUT

2nd COPY OF CFT

LOCALIZED GRAVITON $M_4^2 = \ell M_5^3$

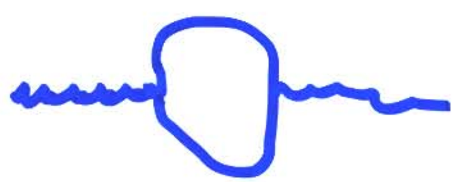
IN TERMS OF GEOMETRY:



* 2

$$M_4^2 = M_5^3 \cdot L$$

GEOMETRIC FORMULA

$$M_4^2 = \frac{1}{G_N} = \frac{1}{G_N|_{BARE}} + $$ 

$\frac{1}{G_N|_{BARE}}$ " 0 $\quad$ diagram " $N_{DOF} \cdot \text{CUTOFF}^2$

$$\Rightarrow \boxed{N_{DOF} = M_5^3 L^3} \quad \left(\frac{1}{L}\right)^2$$

$M_5^3 L^3$ ALSO "HOLOGRAPHIC" BULK COUNT

$AdS_5 \times S_5$: $M_5^3 L^3 = N^2$ ✓

$N_{DOF} \sim (M_5 L)^3$

ALSO AGREES WITH

$\langle TT \rangle$ CORRELATOR

ONE GETS USING

AdS/CFT

DICTIONARY...

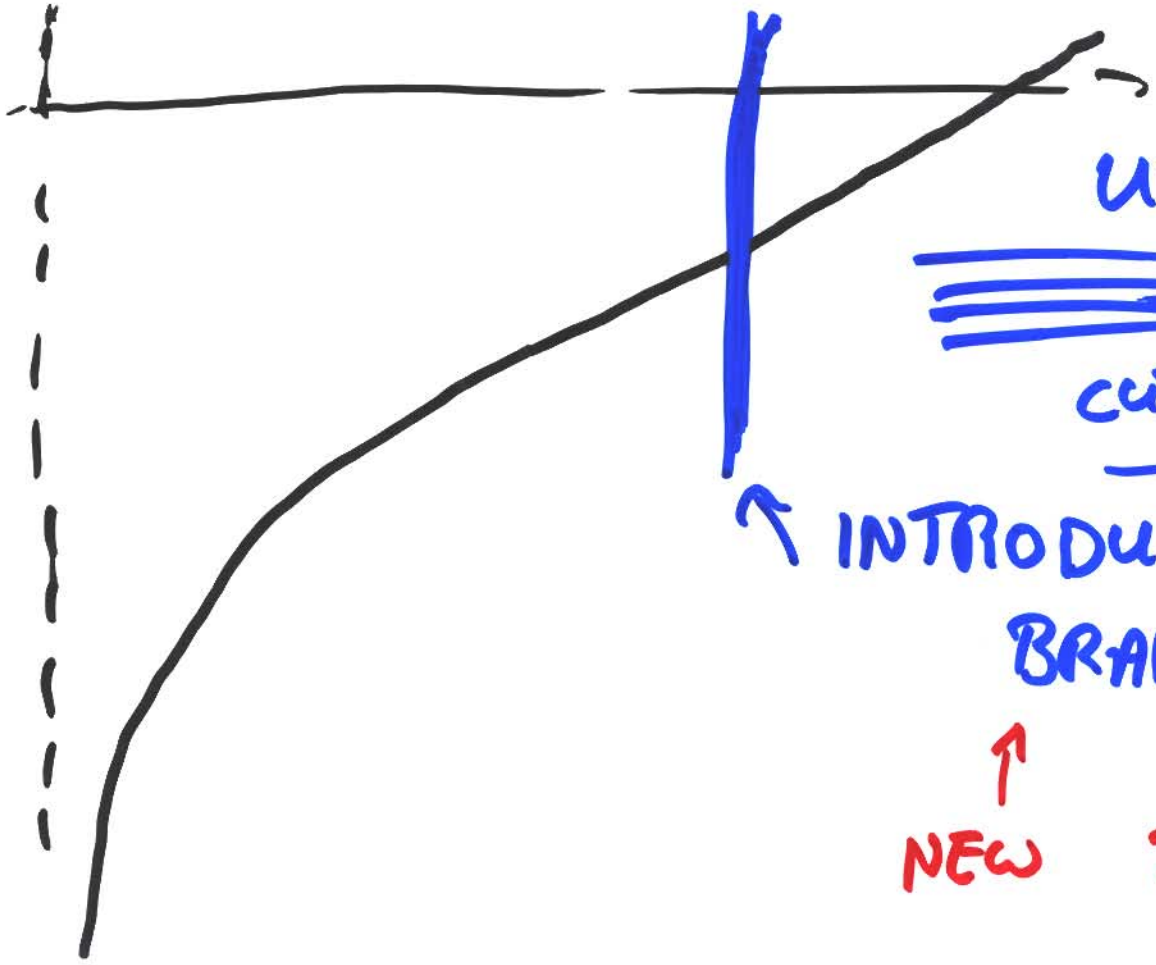
WHAT IS THE "NEW"

PHYSICS THAT COMES

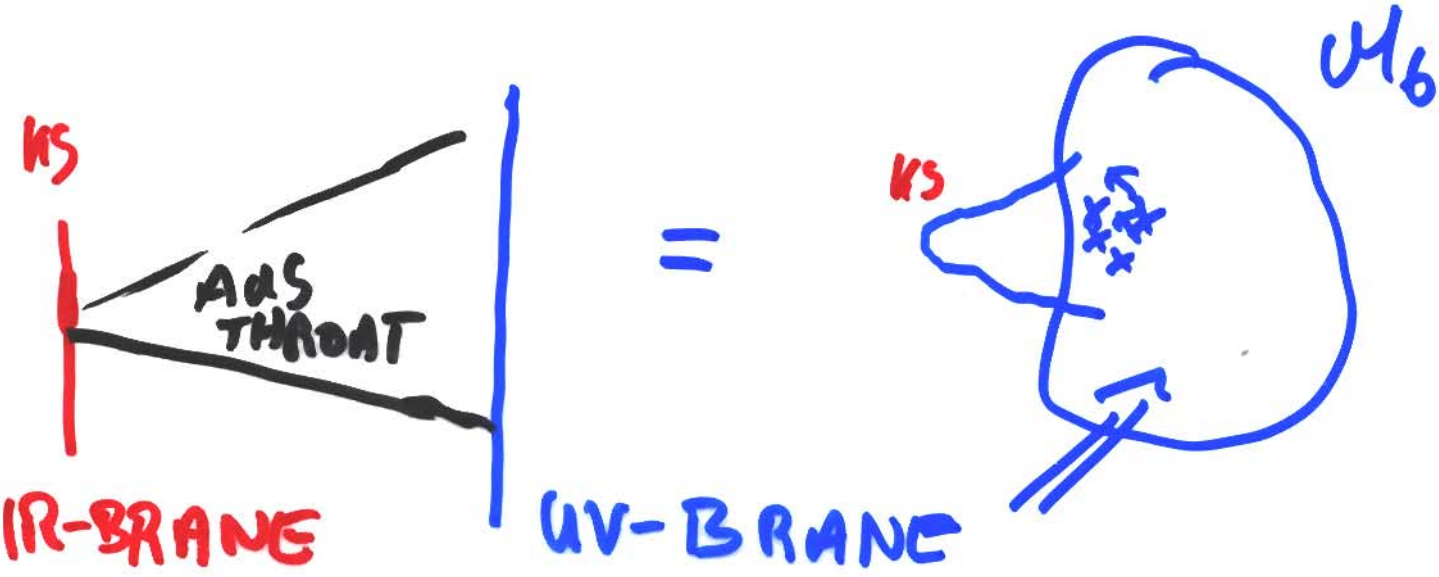
IN AT THE CUTOFF

SCALE $1/L$ ($\ll M_5 \ll M_4$)?

RECALL: $A(r) \sim \log(\sinh r)$



E.G. VERLINDE, ... , KKLT



IN ADDITION:

STRONG GRAVITY AT $1/L$,
MATTER ENHANCED

$$S_{GRAVITY} =$$

$$= \int M_4^2 \left(R + \frac{R^2}{1/L^2} + \frac{R^4}{1/L^4} + \ldots \right)$$

+ CFT's TALK... !

QUESTION:

CAN WE FIND A THEORY
WITH UV-BRANE
THAT DOES NOT INTRODUCE
NEW DOF's AT $1/L$?

CONFORMAL INVARIANCE STILL BROKEN AT $1/L$, DUE TO GRAVITY AT $M_4^2 = M_5^3 L$ W/ LARGE N_{DOF}.

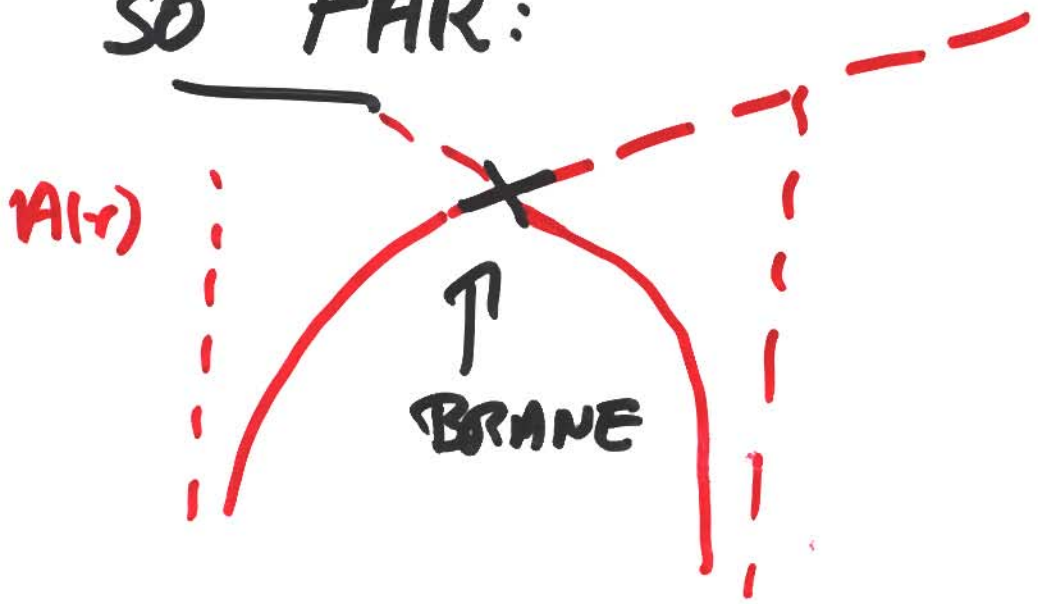
BUT ACTION IS

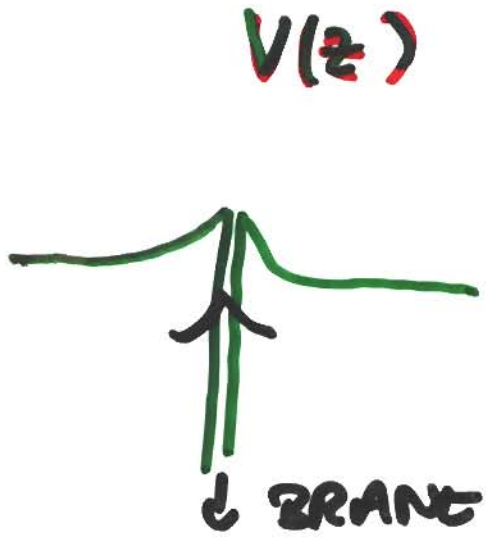
$$\int \sqrt{g}\left(\mathcal{L}_1 + \mathcal{L}_2 + M_4^2(R - 2\Lambda)\right)$$

UP TO M_5. (+ COUPL. OF $\mathcal{L}_1$ & $\mathcal{L}_2$)

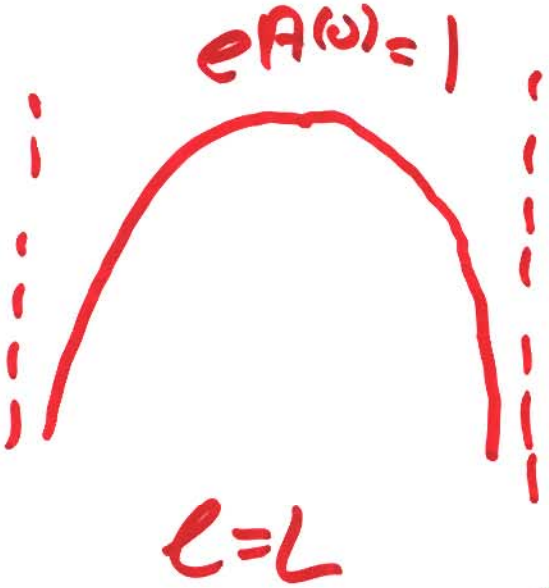
→ UV-BRANE NEEDS TO BE INTRINSIC PART OF THE WARPED GEOMETRY

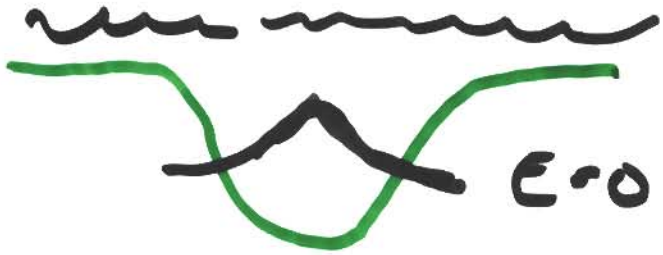
SO FAR:





SMOOTH OUT





e.g. $A = k\log\frac{\ell}{\epsilon}\cos\left(\frac{\pi}{L}\right)$

THIS GEOMETRY HAS BUILT IN UV-BRANE!

BUT WHAT IS THIS?

$$ds^2 = \cos^2\omega \, ds^2_{dS_4} + d\omega^2$$

IS $\underline{dS_5}$!!!!

(WICK ROTATE

$$ds^2_{\mathbb{R}_5} = \cos^2\theta \, ds^2_{\mathbb{R}_4} + d\theta^2 \,)$$

→ dS_5 HAS BUILT IN UV-BRANE (AS DOES ANY dS_d)

→ AT $E < \frac{1}{L}$ GRAVITY ON dS_5 IS DUAL TO GRAVITY ON $\underline{dS_4 + 2\ CFT's}$ (ARGUMENT BY TAUTOLOGY)

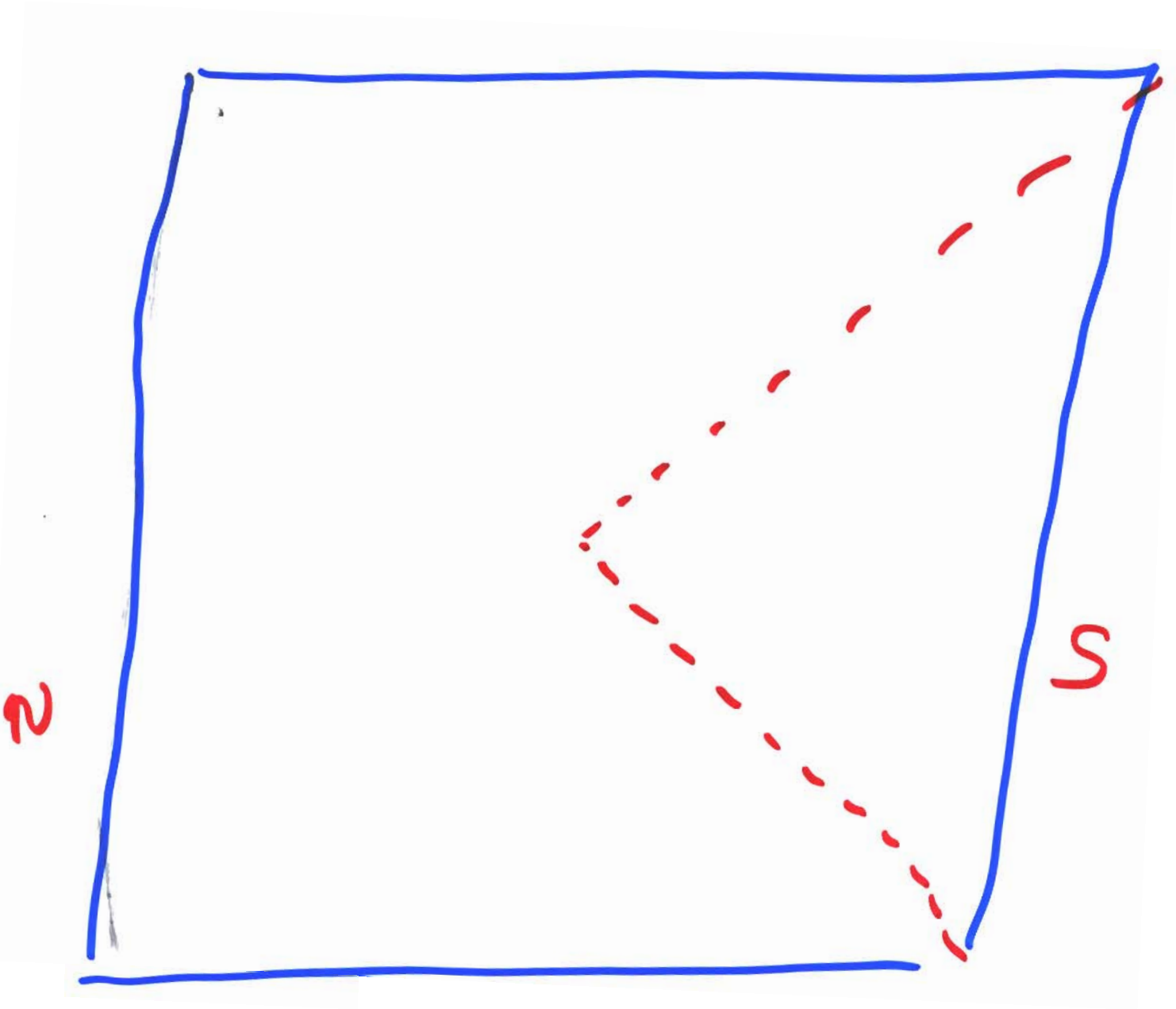
TAUTOLOGY :

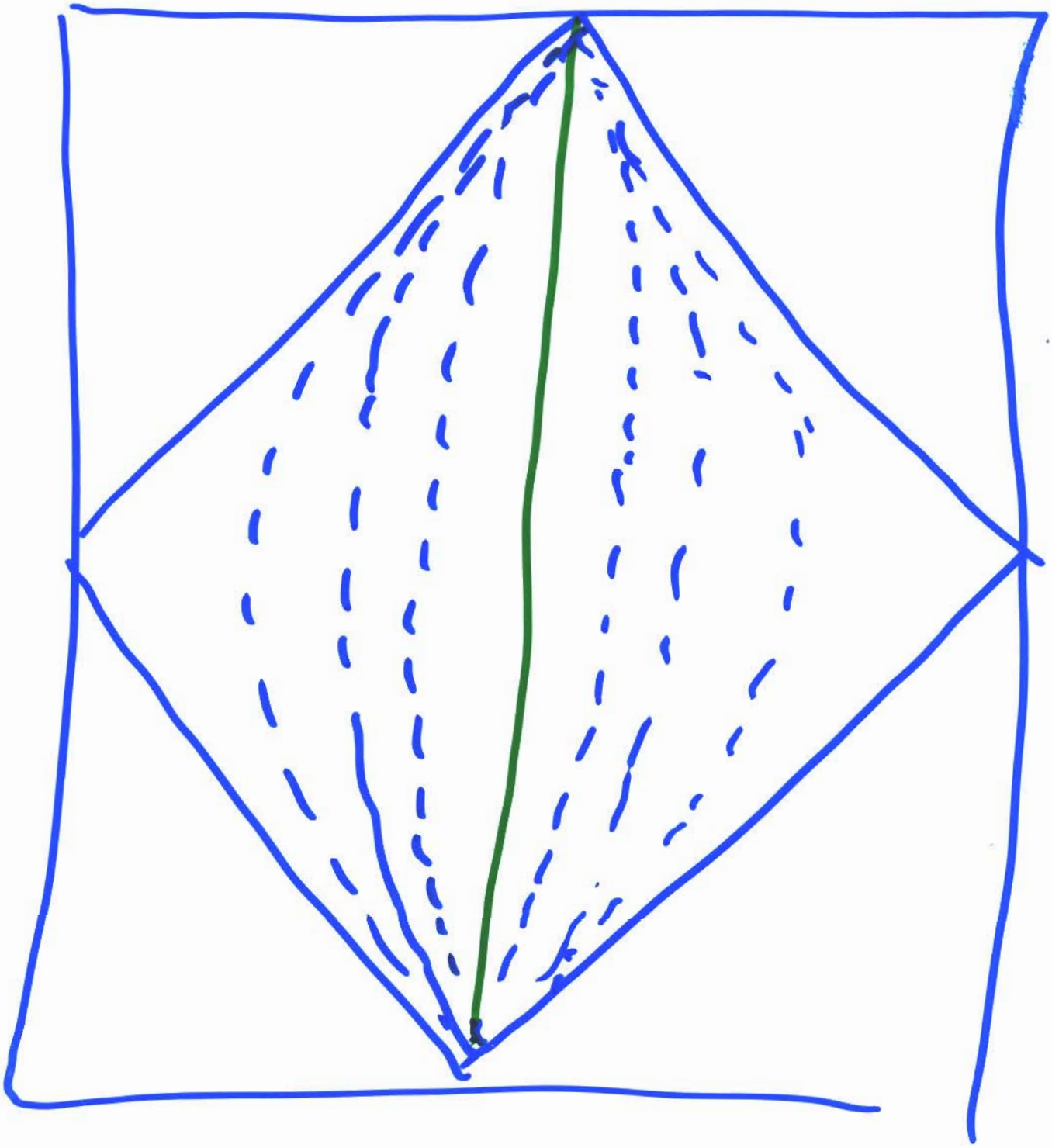
RS w/ AdS_{BULK} + UV-BRANE
+ DS SLICES
HAS IDENTICAL GEOMETRY
TO DS_5 AWAY FROM
THE BRANE ... SAME PHYSICS

CAN E.G. BE CHECKED BY
INTRODUCING PROBE BRANES

IN DS-CASE UV-BRANE
IS INTRINSIC
TO GEOMETRY

$\Rightarrow$


N
S



Os

SO FAR:

USING RS- AdS/CFT TYPE DUALITY ARGUED THAT GRAVITY ON DS_D DUAL TO GRAVITY ON DS_{D-1} + CFT

CAN BE ESTABLISHED UP TO SCALES $1/L$ TO THE EXTEND THAT RS/CFT WORKS (OR IR/UV)

GRAVITY IN THE BULK VALID UP TO $M_5 \gg 1/L$

USE BULK GRAVITY

(+ SCALARS , + GAUGE FIELDS)

TO LEARN ABOUT

THE PROPERTIES OF

THE D-1 DIM. THEORY

IN THE ELUSIVE

$1/L < E < M_5$

REGIME.

DOES A CONSISTENT PICTURE EMERGE?

CONFORMAL ANOMALY!

(FOLLOWING HENNINGSON & SKENDERIS)

$$\langle T_\mu{}^\mu \rangle = \frac{a}{16\pi^2}\tilde{R}^2 - \frac{c}{16\pi^2}W^2$$

$$W^2 = R_{ijke}^2 - 2R_{ij}^2 + \frac{1}{3}R^2$$

$$\tilde{R}^2 = R_{ijke}^2 - 4R_{ij}^2 + R^2$$

MEASURE DOF'S!

REGULATED ACTION NOT CONFORMALLY INVARIANT

BUT THIS IS KNOWN HOW TO GET IN AdS!

HENNIGSON-SKENDERIS:
(FEFFERMAN-GRAHAM)

AdS-RECIPE:

- CALCULATE ACTION ON SOLUTION, e.g.

$$S = \int d^{d+1}x \sqrt{g} R \sim \int d^{d+1}x \sqrt{g}$$

∞: INFINITE VOLUME

- CUTOFF AT $e^r \sim 1/\varepsilon$

TERMS $\sim 1/\varepsilon^p$: CANCEL BY COUNTERTERMS

(OR KEEP AS THEY ARE IN RS ... E.G M_{Pl}^9)

LOG ε TERMS IN EVEN (FT) $d+1$ $\Rightarrow$ ANOMALY

e.g.:

AdS_5 IN FLAT SLICING

$$\int \sim \int e^{4r} \sim e^{4r}$$

$\rightarrow$ NO ANOMALY

IN AdS SLICING:

$$\int \sim \int [\cosh(r)]^4 dr = \sinh(4r) + 8\sinh(2r) + 12r$$

ANOMALY!!!

THIS WAS ALL IN AdS

... HOW ABOUT dS?

IN (CONFORMAL) Z- COORDINATE:

$$ds^2_{dS} = \frac{1}{\cosh^2(z)}\left(ds^2_{dS-1} + dz^2\right)$$

$$ds^2_{AdS} = \frac{1}{\sinh^2(z)}(\quad) = \frac{1}{\tanh^2(z)}\, ds^2_{dS}$$

CHANGE OF VARIABLES:

$$g_{\mu\nu} \to \tanh^2(z)\, g_{\mu\nu}$$

TAKES dS TO AdS !

GRAVITY? $M^{d-2}\int\sqrt{g}R \to M^{d-2}\int\sqrt{g}Rf^{d-2}$

POSITION DEPENDENT M_{Pl}!

GRAVITY STRONG AT BOUNDARY (= UV-BRANE)

SCALAR:

$$\int (\partial\Phi)^2 + m^2\Phi^2$$

$$\to \int (\partial\Phi)^2 + m^2(z)\,\Phi^2$$

POSITION DEPENDENT MASS.

REMARK: NICELY CAPTURES PHYSICS OF EACH ONE OF THE 2 CFT'S BY MAPPING TO STANDARD AdS (WITH UNUSUAL ACTION

$\cosh^{-1}: -\infty \to \infty$

$\sinh^{-1}: -\infty \to 0$

GRAVITY: (EVALUATE ACTION ON SOLUTION)

$$a = c = \theta \; ?$$

FOR ANY GRAVIT. ACTION NOT INVOLV. Riju

LIKE 2d QUANT. GRAVITY = STRING THEORY WORLDSHEET

TAKE ANY (?)
(C) FT WITH CENTRAL CHARGE c

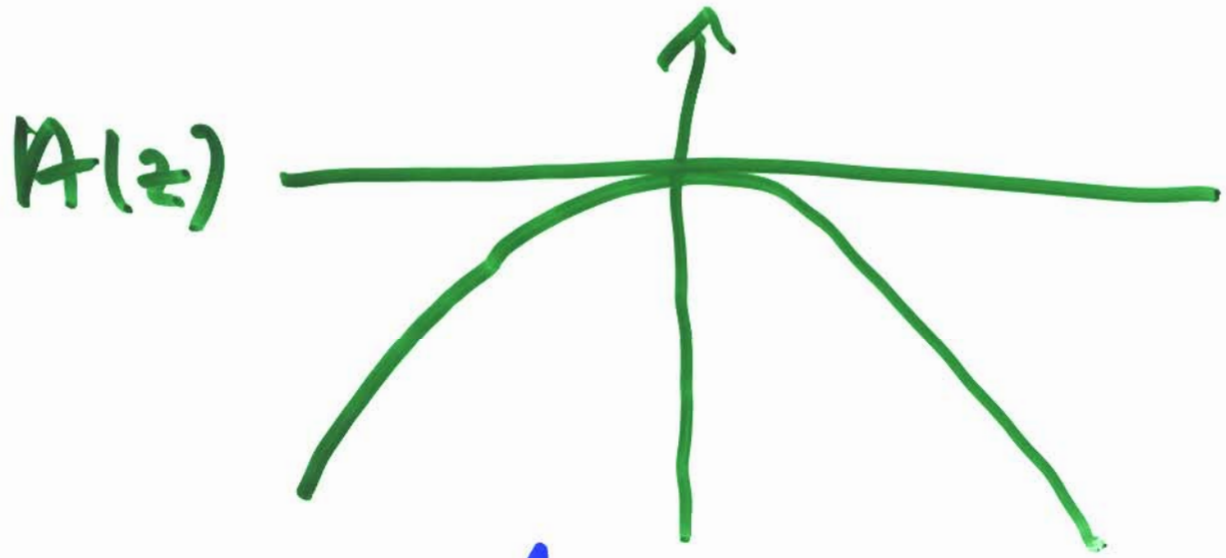
→ IF $c \neq 26$ LIOUVILLE MODE DOESN'T DECOUPLE, FULL MATTER + GRAVITY ALLWAYS $C = 0$ CFT.

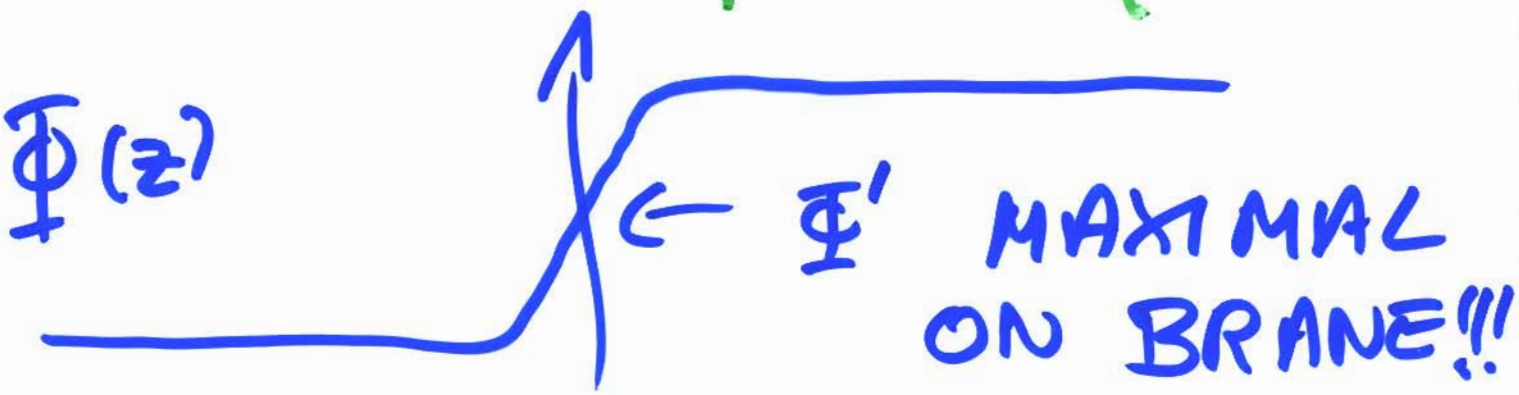
WE FIND THE SAME HAPPENING IN dS/CS IN ANY D....

TRIVIAL?

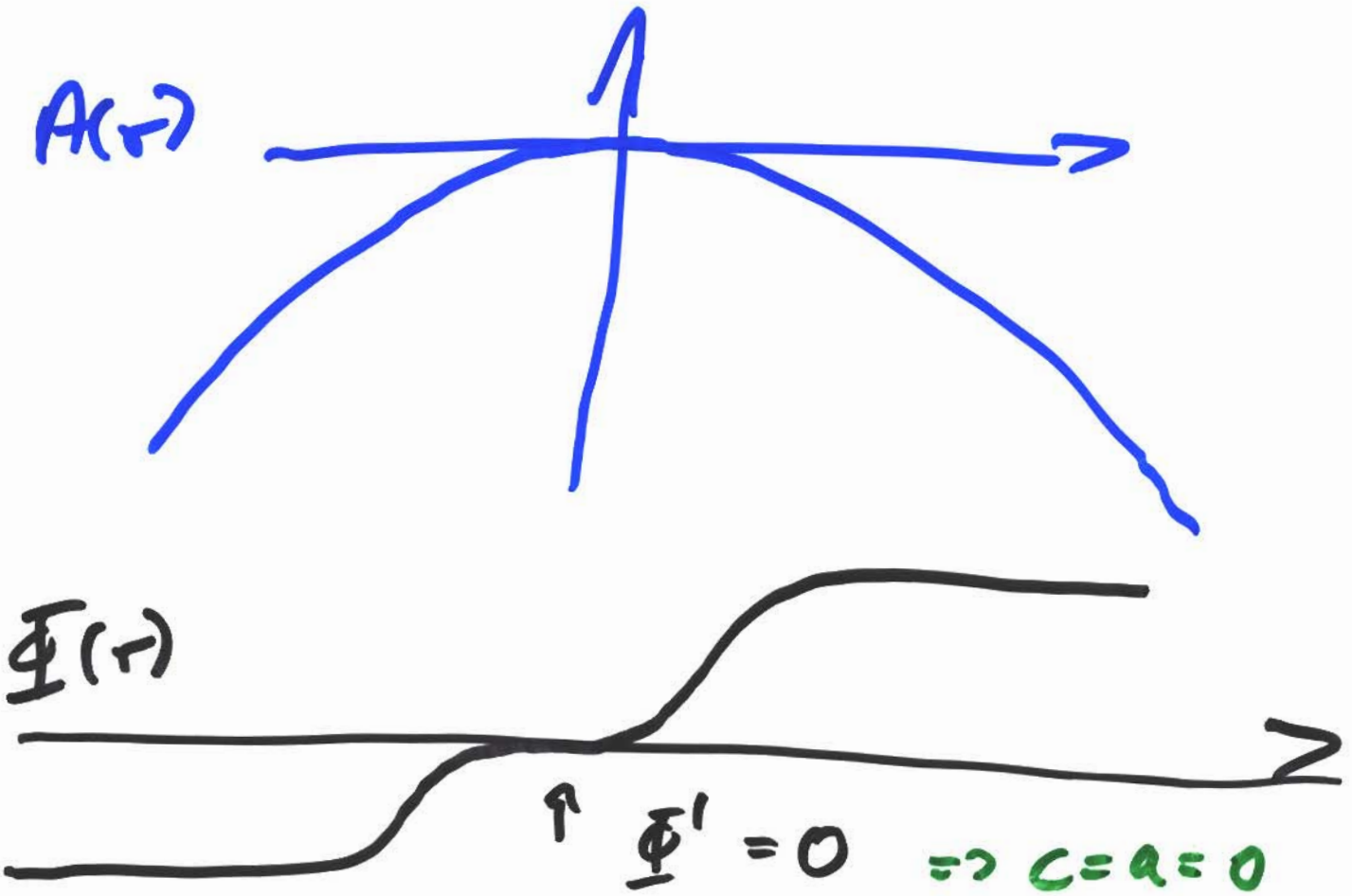
COMPARE TO GENERIC SMOOTH RS-WALL (BUILT FROM SCALAR)

$a = c \sim \Phi'(\text{WALL})$





BUT: CAN FINETUNE SCALAR POTENTIAL TO GET



WHAT IS Φ'? IRRELEVANT OPERATOR!!

IF NOT FINETUNED IT DOMINATES THE UV!

INTERPRETATION:

2d GRAVITY CAN'T REALLY HANDLE EVERYTHING. 2d MATTER SHOULD AT LEAST BE A UV-COMPLETE THEORY TO MAKE $C=0$ SYSTEM

GENERIC IRRELEVANT OPERATOR LEADS TO UNCONTROLLED UV-BEHAVIOR

$$\Rightarrow C \neq 0$$

dS/dS HAS $\underline{C=0}$

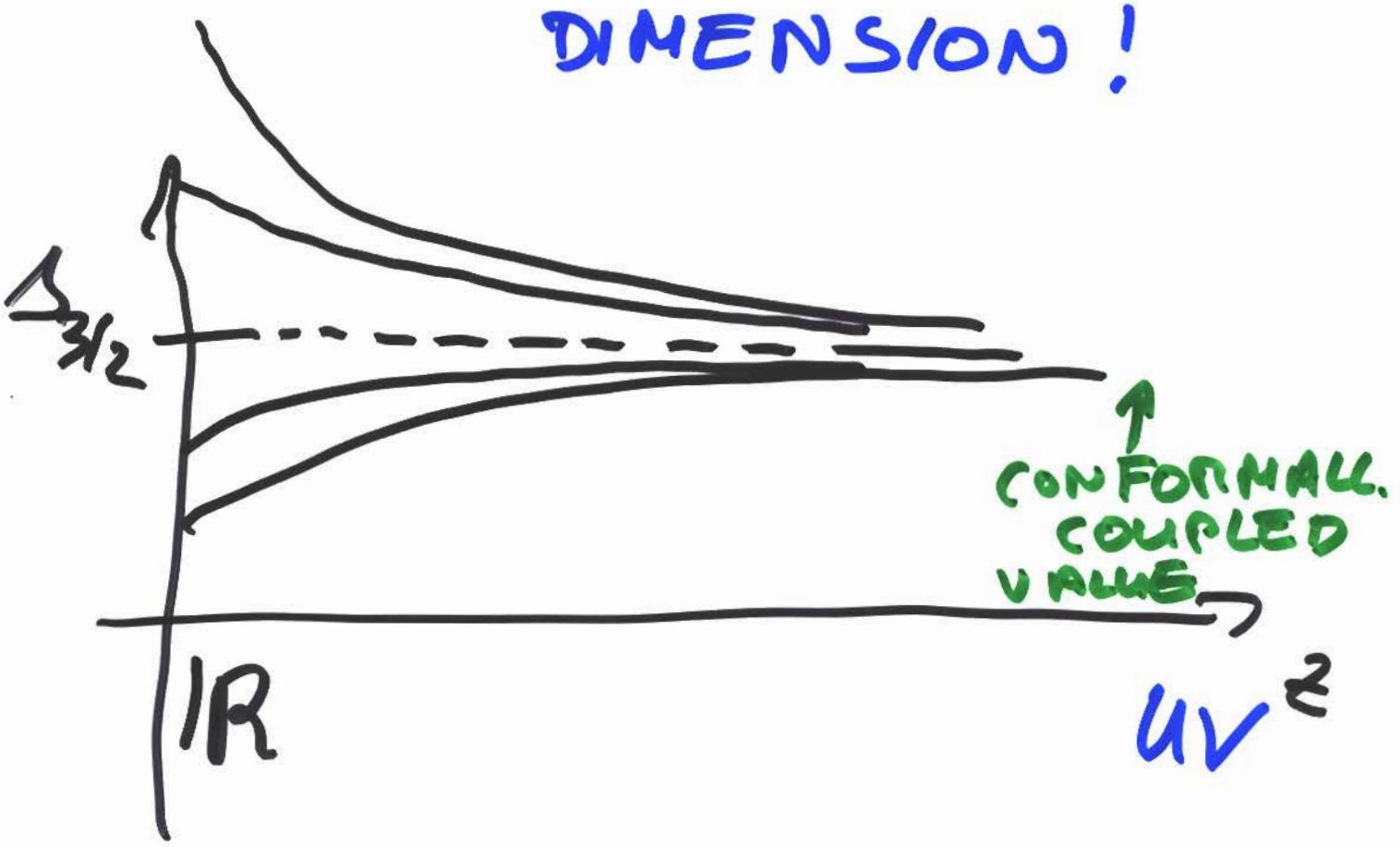
$\Rightarrow$ WELL DEFINED

$(\subset_{?})$ FT ABOVE $\frac{1}{L}$

HOW ABOUT SCALARS?

POSITION DEPENDENT MASS

=> SCALE DEPENDENT DIMENSION!



AT SHORT DISTANCES

$\Delta x \ll L \qquad (E \gg 1/L)$

$\langle OO \rangle \sim \frac{1}{\Delta x^3}$ ← RIGHT BEHAVIOR OF Sd PROPAGATOR

INTERPRETATION:

GRAVITATIONAL DRESSING

ALSO WELL KNOWN FROM 2d LIOUVILLE

$$\mathcal{O}_i \to e^{\alpha_i \xi} \mathcal{O}_i$$

↑ LIOUVILLE MODE

SUMMARY OF 2nd PART:

PROPERTIES OF FT CAN BE MAPPED OUT SYSTEMATICALLY USING CONFORMAL TRAFO TO ADS

CONSISTENT STORY EMERGES WHERE LOWER D GRAVITY DRESSES THE CFT INTO "UNIVERSAL" CLOTHES

$c=0, \quad \Delta = d/2 - 1$

AN EXPLICITE EXAMPLE WOULD BE DESIRABLE

(SEE SILVERSTEIN FOR KK/LT CONTEXT)

CONCLUSIONS:

WHAT I DIDN'T (AND WON'T) TELL YOU ABOUT.

OTHER SPACETIMES:

BUBBLE/BUBBLE DUALITY (VACUUM DECAY OF dS_D MAPS TO SIMILAR DECAY IN dS_{D-1}) KULT ✓

INFLATION/INFLATION

INFLATION ~ EVOLUTION ~ FINAL dS
w/ MATTER OR RADIATION

IS (POSITION-DEPENDENT)

RG-FLOW

BH/BH - CORRESPONDENCE

$S\text{-}dS_d$ BLACKHOLE

$\parallel$

$S\text{-}dS_{d-1}$ BLACKHOLE

+ HAWKING RADIATION

$\langle T_{\text{HAWKING}}{}^{M}_{M} \rangle = 0$

($\uparrow c=0$)

AS IT SHOULD

FIRST EXPLICITE SOLUTION FOR A BRANE-WORLD BLACK-HOLE

OPEN QUESTIONS:

A) EXAMPLES WITH SOME CHECKS

B) IMPRINTS ON CMB ?

C) FLAT SPACE ?
(FINAL STEP IN HHCT)

$C \neq 0$

Ndof's GROWS IN UV

$$ds^2 = \frac{1}{e^{zz}}(ds^2_{dS_{d-1}} + dz^2)$$