



The main message this short course is going to deliver to you is about the influence of the long-range interaction for relaxation and thermodynamics of systems with strong disordering at low temperature. Usually we can describe materials at low temperature in terms of elementary excitations (particles or quasi-particles). For example low temperature properties of metals can be described using the nearly ideal gas of conducting

electrons, for dielectrics we use quantized sound waves phonons which are the propagating deformation fields. Excitation of electrons in dielectrics requires the finite energy because the conducting band is separated from the valence band by the gap. If the system is in intermediate phase between metal and dielectrics (e.g. doped semiconductors) it might have gapless spectrum, but the density of conducting electrons is small and they can be localized in the random potential created by dopant impurities. Then elementary excitations can be represented by localized electron-hole pairs.

There exist other localized elementary excitations including two-level systems in amorphous solids which defines thermodynamics and kinetics properties there at $T < 1K$. Magnetic impurities in metals and dielectric realize localized spin excitations. More artificial example is the quantum bits (qubits), which are quantum two level systems supposed to be in quantum hardware. These two-level systems are controlled by external fields and they also interact with each other. The localization of elementary excitations is very important there because this is essentially equivalent to the conservation of the quantum information.

At low temperature only excitations with the energy comparable to the thermal energy are important. Their density decreases proportionally to the maximum energy kT . Accordingly they are separated by the long-distance increasing with decreasing temperature. Accordingly they can communicate to each only by means of the long-range interaction. Such interaction to my knowledge exists for all localized excitations. This is not very surprising because any excitation shows its presence responding to some external or internal perturbation, which can be expressed as electric, magnetic or elastic field. To interact with such a field the excitation should possess the corresponding moment (electric or magnetic dipole or elastic quadrupole). If it does not possess such moment there is no way to observe its effect so philosophically we cannot consider it as something existing in reality. All these moments of different excitations interact to each other by means of $1/R^3$ interaction. The exponent 3 in the distance dependence is the dimensionality of our world.

Universality of $1/R^d$ interaction is the consequence of its physical nature. This interaction is the consequence of the change of the zero-point vibration equilibria of gapless modes of environment induced by their coupling to given excitations. Energy of such mode is usually the function of the field derivatives like $\int dV \frac{(\nabla\varphi)^2}{8\pi}$ for electric

field or $\int dV A_{\alpha\beta\gamma\delta} \frac{\partial u_\alpha}{\partial x_\beta} \frac{\partial u_\gamma}{\partial x_\delta}$ for elastic field. The excitation also interacts with the field

gradient like $-\mu_\alpha \frac{\partial \varphi}{\partial x_\alpha}$ for electric dipole. The proper transformation of field

$\varphi \rightarrow \varphi + \phi(x)$ eliminates the linear interaction terms for different moments but leads to the additional energy proportional to the squared shift ϕ of the initial field. This term leads to the binary interaction of these moments. Since the simultaneous scaling transformation of fields and coordinates $\varphi \rightarrow \lambda\varphi$, $x \rightarrow \lambda x$, $\mu \rightarrow \lambda\mu$ change all terms in the energy by the factor λ^d the only distance dependence for the interaction of two moments satisfying this rule can be $\frac{\mu_1\mu_2}{R^d} \sim \frac{\lambda^{2d}}{\lambda^d} \sim \lambda^d$. I am not sure that this is the best derivation so I would be appreciate if somebody will advise me something better.

Anyway the long-range interaction seems to be unavoidable and therefore it needs to be studied. It strongly affects both kinetics and thermodynamics of elementary excitations. The strong kinetics effect is because it permits the excitation to hop to the very long distance. In general, $1/R^3$ interaction breaks down Anderson localization in the infinite system. At low but finite temperature it creates the universal mechanisms for relaxation and decoherence of excitations, which has essentially the many-body nature. This mechanism affects elastic, dielectric and conducting properties in amorphous solids and doped semiconductors and it can lead to the decoherence of interacting quantum bits. The target of the first part of the course is this important problem, i. e. the consideration of localization and delocalization in the presence of the long-range interaction. I will start with the review of the Anderson localization concept for the short range interaction. The localization and delocalization can be considered using the convenient and qualitatively transparent description based on resonant interactions. It uses the obvious fact that the particle is getting shared between two localized states with energy difference Δ coupled by the interaction V when this interaction exceeds energy difference. This is the resonant coupling and the delocalization can be described as the formation of infinite cluster of resonant couplings. To justify this concept and for educational purposes I am going to obtain the delocalization criterion also using perturbation theory approach in localized and delocalized regimes. Other important results of the localization theory including the dimension effect will also be discussed. This should give some introduction to the current status of the problem.

Then the concept of resonance interaction will be applied to the single particle and many-body localization problems following Levitov's and our works. All related lecture notes will be posted in this week.

Another fundamental effect of the long-range interaction is the quantitatively universal thermodynamics of elementary excitations. Due to the Coulomb interaction the density of localized electron states is universal in doped semiconductors and the density and interaction of two-level systems in amorphous solids are also almost identical in completely different materials possibly because of the long-range elastic interaction. There are plenty of things to discuss here including the Coulomb and dipole gaps in the density of states, universal sound attenuation, $1/f$ noise. Some of these problems were addressed in my earlier and recent work but I really do not know how much time would I have for this second part.

I wish to try to say fewer things, but in more accurate and clear manner. The topics under consideration are difficult and relatively new so I wish to give as many details as possible to help you to understand them. Particularly I wish to make this course clear for my graduate student whose future work is concerned with the development of similar concepts.

Since it is difficult to listen a lecture during 1.5 hours I would greatly appreciate questions, suggestions, objections, etc. I wish to have the remarkable time spend as discussion, which will help all us to make this course useful and productive in the maximum extent. Your comments and suggestions about notes which have plenty of both scientific and English mistakes would be greatly appreciated. Notes should be available in the PITP website. My e-mail is aburin@tulane.edu and I will do my best to respond any class related letter as soon as I can.

There are some problems at the end of notes. I would recommend the graduate students interested in the related topics to spend some time solving them. Would you have any questions about the problems please feel free to contact me by e-mail. Also I can be found in PITP.