

## 5. Two Level Systems in Glasses. Interaction, Interaction Stimulated Delocalization of energy.

In this lecture we approach the problem of energy relaxation.

Two level systems interact with each other because of the virtual exchange of them by photons (dipolar interaction) and phonons (elastic interaction). The first interaction is straightforward and its Hamiltonian can be expressed as

$$V_{TLS}^{dip} = \sum_{ij} \frac{\mathbf{i}_i \mathbf{i}_j - 3(\mathbf{i}_i \mathbf{n})(\mathbf{i}_j \mathbf{n})}{R_{ij}^3} S_i^z S_j^z, \quad \mathbf{n} = \frac{\mathbf{R}_{ij}}{R_{ij}}, \quad (4.1)$$

where  $\mu_i$  stands for the dipole moment of TLS  $i$ . Off diagonal term  $S^x \mu^x$  can be neglected because the off diagonal (transition) dipole moment  $\mu_x \sim \mu \Delta_0 / (\hbar \omega_D)$  estimated within the semiclassical approach is smaller in all relevant cases than the off-diagonal element  $\mu \Delta_0 / (E)$  caused by the direct effect of tunneling on wavefunctions. Eq. (4.1) is applicable up to the distances of order of resonant optical wavelength for photon energy of order of thermal energy

$$R < \lambda_T = \frac{hc}{kT} \sim 1cm. \quad (4.2)$$

In addition to the dipolar interaction there is elastic interaction of TLS. It also decreases with the distance as  $1/R^3$  and has the interaction constant (e. g. Burin, Natelson, Osheroff, Kagan, 1998)

$$U_{el}(R) \sim \frac{U_0}{R^3}, \quad U_0 \sim \frac{\gamma^2}{\rho v^2}. \quad (4.3)$$

Eq. (4.3) is applicable up to the distances of order of resonant phonon wavelength for photon energy of order of thermal energy

$$R < \lambda_T = \frac{h\nu}{kT} \sim 1\mu m. \quad (4.4)$$

Interaction constants of elastic and dipolar interactions are usually comparable to each other (except for vitreous silica where elastic interaction is stronger). This follows from the measurements of dielectric losses and internal friction (see previous lecture) which produce similar results.

How does the interaction affect the relaxation and decoherence of two level systems? First we start with the description of decoherence induced simultaneously by the interaction of TLS with each other and phonons following Black and Halperin and then turn to the direct interaction effect on the relaxation.

The decoherence rate of TLS can be estimated considering their phase change induced by the transition of surrounding TLS. Decoherence takes place when TLS phase fluctuation approaches unity. One can express that phase fluctuation as

$$\delta\Phi \sim \frac{\delta E(T_2)T_2}{\hbar} \sim 1 \quad (4.5)$$

and the solution of Eq. (4.5) gives us the definition of the decoherence rate.

Consider the phase fluctuation induced by TLS interaction. Since the transition rate of TLS (Eq. (3.3))

$$r = \frac{\Delta_0^2 E}{16\pi\hbar^4} \coth\left(\frac{E}{2kT}\right) \sum_{\mu} \frac{\gamma_{\mu}^2}{\rho v_{\mu}^5} = A\Delta_0^2 E \coth\left(\frac{E}{2kT}\right) \quad (4.6)$$

rapidly decreases with tunneling amplitude we can consider only most relevant thermal TLS having both energy and tunneling amplitude of order of the temperature

$$\Delta_0 \sim E \sim kT. \quad (4.7)$$

The density of thermal TLS is given by

$$n_T \sim P_0 T, \quad (4.8)$$

and their phonon-stimulated transition rate is given by

$$r = AT^3. \quad (4.9)$$

Then during the time  $t < 1/r$  the number of surrounding TLS which undergo transitions can be estimated as

$$n(t) = n_T r t. \quad (4.10)$$

The energy fluctuation associated with such changes for the interaction  $1/R^3$  can be estimated as the interaction with the neighboring TLS already transferred during the time  $t$

$$\delta E(t) = U_0 n_T r t. \quad (4.11)$$

Using this estimate in Eq. (4.5) one can define the decoherence rate as

$$\frac{1}{T_2} \sim \sqrt{r \frac{U_0 n_T}{\hbar}} = T^2 \sqrt{\frac{AP_0 U_0}{\hbar}}. \quad (4.12)$$

This result is applicable at relatively low temperature where

$$U_0 P_0 T > AT^3 \quad (4.13)$$

which takes place at

$$T < 200mK. \quad (4.14)$$

Measurements of the decoherence rate well below 100mK (Bayer and von Schikfus, 1988) however shows that the decoherence rate deviates from the predicted law  $T^2$  Eq. (4.12) but instead follows linear temperature dependence. This is possible when the relaxation rate depends linearly on the temperature. There exists the number of direct and indirect measurements showing linear temperature dependence of the TLS relaxation rate.

Therefore one can consider the question whether TLS interaction can lead to the alternative relaxation mechanism leading to the linear temperature dependence of TLS relaxation rate. We are going to consider this question now following earlier work by Burin and Maksimov (1989), Burin and Kagan (1995) and Burin, Kagan, Maksimov and Polishchuk, 1998.

First question one can raise is whether the spectral diffusion associated with neighboring TLS transitions (4.11) can itself induce the TLS relaxation. If we consider the typical thermal TLS Eq. (4.7) the maximum fluctuation of its energy  $TP_0U_0$  induced by TLS interaction is about factor of 1000 less than its energy. Therefore the spectral diffusion cannot induce its transition.

This is different if we consider TLS with energy and tunneling amplitude less than the spectral diffusion range  $TP_0U_0$ . For this TLS the energy fluctuations Eq. (4.10) inevitably bring this TLS to zero energy conditions. The rate of energy change can be estimated as

$$\frac{dE}{dt} \sim U_0 n_T r \quad (4.15)$$

and TLS having sufficiently small tunneling amplitude

$$\Delta_0 < \sqrt{U_0 n_T \hbar r} = \frac{1}{T_2} \quad (4.16)$$

are adiabatically slow compared to the rate of energy change. Those TLS undergo transitions defined by the Landau Zener mechanism. The transition rate is defined by the product of the number of level crossing per the unit time  $r$  multiplied by the Landau Zener probability of the non-adiabatic transition  $\Delta_0^2/|\hbar dE/dt|$  which yields

$$r_{sd} = \frac{\Delta_0^2}{U_0 n_T}. \quad (4.17)$$

This equation is derived more accurately in Appendix 4.1 attached as a separate pdf file. It can be understood as following. There exists the important characteristic scale of time, where the energy change during this time is equal to the inverse time multiplied by the Planck constant. This scale defines the energy uncertainty in our problem. Corresponding time coincides with the decoherence time  $T_2$ . Transitions occurs when the energy difference of two levels belongs to the energy domain  $(-\hbar/T_2, \hbar/T_2)$  and the probability of the transition is defined by the Landau Zener formula  $(\Delta_0 T_2/\hbar)^2$ . The probability of transition during the long time  $t$  is defined as the product of single transition probability multiplied by the number of occurrences of energy in the transition domain which is given by the product of number of opportunities to get there  $t/T_2$  and the probability that such an opportunity is realized  $\hbar/(U_0 n_T T_2)$ . The product of all three probabilities results in Eq. (4.17).

Can the spectral diffusion stimulate the relaxation of TLS with higher energy? The answer is yes if this TLS transfers together with some other TLS which compensates its energy for the total energy change during transition. Consider arbitrarily TLS with tunneling amplitude  $\Delta_0$  and energy  $E$  both smaller than the thermal energy. Assume that this is in its ground state. Then there can exist the neighboring TLS in excited state with energy  $E'$  such that

$$|E' - E| < U n_T. \quad (4.18)$$

This pair of TLS can undergo the collective transition where the given TLS is getting to its excited state, while the other one goes to its ground state due to the spectral diffusion. The transition amplitude for such joint transition can be expressed using the second order perturbation theory with respect to both TLS tunneling amplitudes as

$$V_p = \frac{U_0}{R^3} \frac{\Delta_0}{E} \frac{\Delta_0'}{E'}. \quad (4.19)$$

It is clear that the main contribution from neighboring TLS should be associated with the resonant TLS with  $\Delta_0'^2 \sim E'^2 \sim E$ , so one can set their density to be  $P_0 U_0 n_T$ . The closest TLS satisfying this condition is located at the distance defined by the equation

$$P U_0 n_T R^3 = 1. \quad (4.20)$$

The transition amplitude of two TLS at such distance is defined by Eq. (4.19)

$$V_p = U_0 n_T U_0 P_0 \frac{\Delta_0}{E}. \quad (4.21)$$

Under any conditions this value is smaller than the TLS dephasing amplitude  $\hbar/T_2$  as we will see below. Therefore one can use Eq. (4.17) for the spectral diffusion induced transition rate

$$r_{sd} = \eta \frac{\Delta_0^2}{E^2} T (P_0 U_0)^3, \quad (4.22)$$

where  $\eta$  is the dimensionless constant which can be large because of the phase volume factors like  $4\pi$ . This expression is applicable when it exceeds the transition rates of the given TLS induced by its interaction with phonons which has the form  $A\Delta_0^2 T$  Eq. (4.6). This works for TLS with sufficiently small energies

$$E < \sqrt{\frac{\eta(P_0 U_0)^3}{A}}. \quad (4.23)$$

Note that it is completely independent of phonons.

The typical TLS energy is given by the thermal energy  $E \sim kT$ . Until the temperature is sufficiently high

$$T > \sqrt{\frac{\eta(P_0 U_0)^3}{A}}. \quad (4.24)$$

the interaction stimulated relaxation is consistent with its main driving force which is phonon stimulated dynamics. This condition fails in the opposite limit of low temperature

$$T < \sqrt{\frac{\eta(P_0 U_0)^3}{A}}. \quad (4.25)$$

In this regime the correction to the relaxation rate of thermal TLS induced by the spectral diffusion becomes greater than the phonon stimulated relaxation rate, which is the origin of the spectral diffusion. At the same time in the temperature domain (4.25) pairs of most adjacent thermal TLS are no more adiabatic because the coupling strength of them Eq. (4.21) becomes equal to  $\hbar/T_2$ . However we can consider other thermal TLS separated from the given one by the distance corresponding to the crossover from nonadiabatic to adiabatic condition

$$\frac{U_0}{R^3} \sim \frac{\hbar}{T_2} = \sqrt{TP_0 U_0 r_T}, \quad (4.26)$$

where  $r_T$  is the thermal TLS transition rate. The number of such neighbors can be estimated as

$$N_{tr} \sim \frac{P_0 U_0 P_0 T U_0 T_2}{\hbar}, \quad (4.27)$$

and the number of transitions per the single quasi-period of the spectral diffusion  $r_T^{-1} = T_1$  is given by this number. Since this number should not exceed unity we got

$$T_2 = \frac{\hbar}{T(P_0 U_0)^2}. \quad (4.28)$$

Consistency between  $T_1$  and  $T_2$  Eq. (4.26) requires us to set the TLS relaxation rate to be

$$r_{sd} = \eta \frac{\Delta_0^2}{E^2} T (P_0 U_0)^3 \quad (4.29)$$

for all TLS including the thermal one. Thus we obtain that decoherence and relaxation rates are independent of phonons at very low temperature. This result can be used to explain experiments where these dependencies have been observed. We will show during the next lecture that this behavior can be understood as many-body delocalization of energy induced by the long-range interaction.

It is interesting to consider the same mechanism of relaxation and spectral diffusion for the arbitrary dimension  $d$  and interaction  $1/R^a$  with  $a \geq 3$ . In this case the characteristic scale of spectral diffusion can be estimated as

$$W_T \sim U_0 n_T^{a/d}. \quad (4.30)$$

The transition matrix element Eq. (4.19) for thermal excitations can be estimated as

$$V = U_0 (P_0 W_T)^{a/d} \quad (4.31)$$

And correspondingly the transition rate induced by the spectral diffusion reads

$$r_{sd} = \frac{V^2}{W_T} = \frac{U_0^2}{W_T} (P_0 W_T)^{2a/d}. \quad (4.32)$$

If the interaction with phonons leads to the standard  $AT^3$  behavior of the relaxation rate, then the relaxation induced by the spectral diffusion can compete with single phonon processes when  $a \leq 3d/2$ .

Eq. (4.32) is applicable only when the characteristic transition amplitude Eq. (4.31) is smaller than the energy uncertainty  $\hbar/T_2$ . When this condition is broken one should define the relaxation rate stimulated by the spectral diffusion differently. All neighboring TLS with energies  $E'$  and separated from the given TLS by the distance  $R$ , satisfying the conditions

$$\frac{U_0}{R^a} \geq \frac{\hbar}{T_2}, |E - E'| < W_T, \quad (4.33)$$

should be included into consideration for the transition of the pair. The number of such neighbors is given by

$$N_{sd} = P_0 W_T \left( \frac{U_0 T_2}{\hbar} \right)^{\frac{d}{a}}. \quad (4.34)$$

The number of transitions stimulated by any of them during the time  $t$  is given by the product  $(t/T_2)(\hbar/(W_T T_2))$ . The overall transition rate caused by the spectral diffusion can be expressed as

$$r_{sd} = N_{sd} \frac{1}{T_2} \frac{\hbar}{W_T T_2} \sim \frac{1}{T_2^{2-d/a}}. \quad (4.35)$$

Remember that the decoherence time  $T_2$  and relaxation time  $T_1$  can be expressed through each other as

$$\frac{W_T T_2}{\hbar} \left( \frac{T_2}{T_1} \right)^{\frac{a}{d}} \sim 1 \quad (4.36)$$

which results in the dependence  $T_2 \sim T_1^{d/(a+d)}$ . Accordingly the relationship between the transition rate stimulated by the spectral diffusion (4.35) and relaxation rate responsible for the spectral diffusion can be expressed as

$$r_{sd} \sim \left( \frac{1}{T_1} \right)^{\frac{2d-a}{a+d}}. \quad (4.37)$$

When the exponent in Eq. (4.37) is smaller than 1 the rate of transitions stimulated by the spectral diffusion becomes larger than the rate of transitions stimulating spectral diffusion at sufficiently small relaxation rate. This happens for the interaction  $1/R^a$  with exponent

$$a < 2d.$$

(4.38)

In this case the interaction leads to the relaxation independent on the interaction with the environment which is the consequence of the energy delocalization which will be addressed in the next lecture.

1. Burin, Natelson, Osheroff, Kagan, 1998
2. Black and Halperin