The UNCERTAINTY PRINCIPLE

The uncertainty principle is really just a fact about waves of any kind. In the case of quantum particles it says the following:

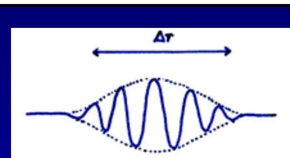
Suppose we localize the probability wave of a particle so that it is confined to a length of magnitude Δr (often called a "wave-packet" of size Δr). We say that the position is UNCERTAIN, because it can be anywhere in this region.

Then the uncertainty principle says that the momentum p of the particle is also uncertain- it is also smeared out, over a range Δp. The crucial result is that $\Delta r \sim h/\Delta p$

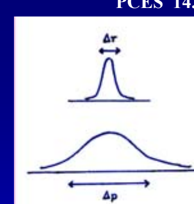
or $\Delta r\, \Delta p \sim h$ **where h is Planck's constant.**

Uncertainty Principle II

How can we understand the uncertainty principle? What it is saying is shown at right- if we want to make a wave-packet of spatial extent Δr, we can only do this by adding together waves of different λ, ie. different p.

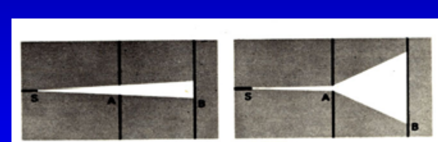


A wave-packet confined to a size Δr; the spread in wavelengths gives a spread Δp in momentum

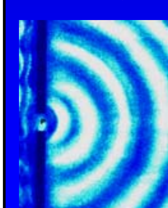


The net result is that a spread or uncertainty Δr in position means uncertainty Δp in momentum. The smaller is Δr, the larger is Δp (and vice-versa).

Actually we have met this already, it is merely a fact about waves. Thus diffraction effects are always larger if we confine a wave with a small hole. It is easy to show that a wave going through a hole of size d will spread an angle α by diffraction, where

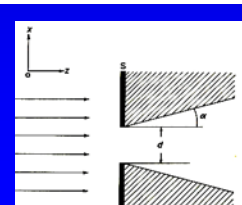


$$\alpha \sim \lambda/d = h/(p.\Delta r)$$



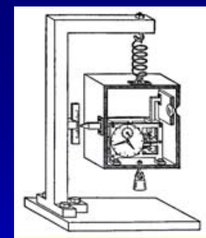
since d is just Δr here. The diffraction gives a momentum kick proportional to α, so that Δp ~ αp. We then just get the uncertainty principle back.

LEFT: if the wavelength is the same size or smaller than the hole, we get diffraction over all angles.
RIGHT: small wavelength gives small angle diffraction



Uncertainty Principle III

One can consider many examples of the uncertainty principle-. This was done in the early days of QM, particularly because Einstein objected strongly to it. When Bohr demonstrated that to contradict it would lead to a contradiction of QM with even General relativity (using the thought experiment at right) his opposition collapsed- the principle is now universally accepted.



Bohr's thought Experiment

Uncertainty Principle for SPIN

We saw on page 12.8 that we spin exists in discrete values or 'quantum numbers'. The uncertainty principle is very simple for spin-1/2 systems. As described on p. 12.8, an "up" spin is a superposition of left and right oriented spins. In QM one writes, eg., for an "up" spin:

$$\Psi_+ \sim (\phi_{->} + \phi_{<-})$$

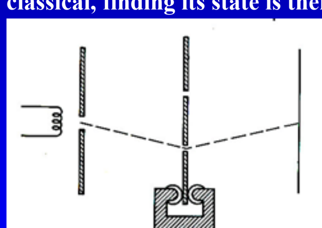
where the symbols → and ← mean spins oriented "left" and "right" respectively. This means an equal probability of 50% of finding the spin up system in a left or right state.

However this has a simple interpretation- there is an uncertainty principle for spin orientations. If we fix the orientation along one axis very tightly, then it becomes indeterminate in perpendicular directions. This is the analogue of the uncertainty principle governing position and momentum.

QUANTUM MEASUREMENTS: I

One of the most difficult points in QM is the idea of the measurement. Here I give you a simple approach to this- which depends on the assumption that some BIG system is ultimately doing the measurement, and that it behaves classically.

Essentially a measurement involves an interaction between the physical system of interest and a measuring apparatus. This establishes a correlation between the state of the system before the measurement interaction, and the state of the apparatus afterwards. It is assumed that because the apparatus is classical, finding its state is then simple.



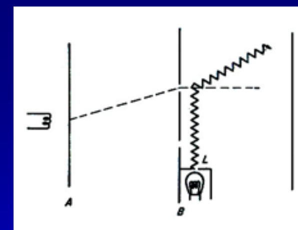
A 2-slit set-up where the plate with the slits can move up or down. Any scattering of the particle going through a slit, changing its direction, means exchange of momentum with the plate- which then recoils.

At left we see a simple example. We want to measure through which slit the particle passes. The 2-slit apparatus is set up so that it can move without friction between 2 rollers. If a particle goes through the bottom slit and back up to the screen, the plate containing the slits will recoil downwards. On the other hand it will recoil up if the particle goes through the upper slit. Then, by watching the motion of the plate containing the slits, we can see which slit the particle went through.

Actually this will destroy the interference pattern- see next slide.

QUANTUM MEASUREMENTS: II

The interaction with the measuring device also has an important connection with the uncertainty principle. Consider a slightly different way of measuring through which slit the particle passes, looking at it with photons. Two points are crucial:



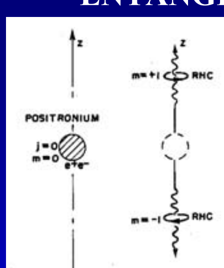
Using a microscope to see which slit the particle goes through. The particle is seen if photons scatter off it (into a microscope). But this causes the particles to recoil.

(i) To see which slit the particle goes through, we need photons of a short enough wavelength- if the wavelength is λ, we can't resolve the position at a finer scale than this (this is why light microscopes cannot resolve detail smaller than the wavelength of light).

However this light has a momentum, and in interacting with the particle it will give it a momentum kick of roughly the same size. As a result the particle acquires an uncertain momentum, so it no longer has a well-defined wavelength. If we work out the mathematical details we find the interference pattern is then smeared out completely because of this.

(ii) Actually this makes sense. If we can tell which slit the particle goes through, it follows logically that there can be no more interference pattern on the screen- interference only happens if the particle can go through both of them, without choosing a particular path.

ENTANGLEMENT between QUANTUM SYSTEMS



Positronium → 2 photons in state Ψ_{+-}

Consider a system of an electron & positron 'orbiting' each other- the overlap of their 2 wave-functions means they will eventually mutually annihilate, with emission of 2 photons. These photons must have equal & opposite momenta & spin, because the original system had zero momentum and spin, & these 2 quantities are conserved.

Such a state is shown at left. If we label the spin along the direction of photon propagation (usually called the 'helicity') by + & - , the the state shown can be written as

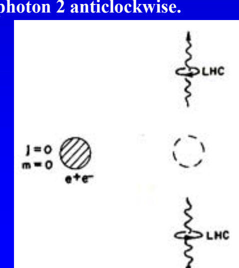
$$\Psi_{+-}(1, 2) = \phi_+(1)\ \phi_-(2)$$

where particle 1 is moving up, and particle 2 down.

However consider the state shown at right, which is:

$$\Psi_{-+}(1, 2) = \phi_-(1)\ \phi_+(2)$$

BELOW: positronium → photons in state Ψ_{-+} with photon 1 spinning clockwise along photon direction, & photon 2 anticlockwise.



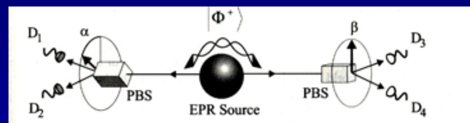
But QM uses all possible paths- so we will actually have a state like

$$\Psi \sim (\Psi_{+-} + \Psi_{-+}) = [\ \phi_+(1)\ \phi_-(2) + \phi_-(1)\ \phi_+(2)\ ]$$

Now in this state each photon has no definite spin- BUT we do have a definite quantum state! The state is such that the 2 spins must be opposite- they are "entangled".

The Einstein-Podolsky-Rosen (EPR) Paradox

!st detector, set to measure the a spin state at angle α



2nd detector, set to measure the spin state at angle β

We can now explain this paradox fairly easily. Suppose we have a state of 2 spins such that they must be opposite. We can write one such state as $\Psi = |++>$ which is a simple notation meaning they are both up. Another could be $|-->$ meaning they are both down; and we could have $(|++> + |-->)$. These are the 3 states talked about on the last slide.

Now we let the 2 spins separate by a large distance, & have 2 measuring systems to measure the spins at the 2 places. Suppose the measuring systems are measure if the spins are up or down. Then if the 1st measuring system finds spin 1 is up, we KNOW spin 2 will be down- & vice-versa. Apparently spin 2 must be then either up or down.

However now suppose at the very last instant we change our minds, & switch the 1st apparatus to measure whether spin 1 is → or ←. Now if we find spin 1 is → we KNOW spin 2 is ←; and vice-versa. The spins and apparatus are so far apart that no signal can travel between them in the time after we change our mind (unless it goes faster than light!). And yet by suddenly switching the 1st apparatus, QM says we change the possible quantum states that spin 2 can have. This is the EPR paradox (published in 1935), which led Einstein to argue that QM was not a complete theory of Nature.