Fractional Electrons in Liquid Helium?

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We argue that electrons in liquid helium bubbles are not fractional, they are in a superposed state.

PACS numbers: 72.10.-d, 67.80.Mg, 67.90.+z

In an analysis of several unexplained observations on the mobility of electron-inhabited "bubbles" in liquid helium, H. Maris has suggested that such bubbles can fission into two smaller daughter bubbles, each containing half of the original electron's wave function and each allowing the detection of fragments of the original electron, which has divided into two pieces that act as though they are fractions of the original particle.[1]

In this connection it is important to keep in mind two significantly different concepts of fractional fermions.

On the one hand, there are physical situations where each individual measurement yields a fractional result. Two examples are the field-theoretic models for fermions propagating across domain walls, as realized experimentally by solitons on polyacetylene;[2,3] and also the excitations in the fractional quantum Hall effect.[4] For these the fractional characteristics are sharp observables (quantum mechanical eigenvalues) without dispersion,[5] and the phenomena put into evidence previously unsuspected fermion fractionization.

Alternatively, the fraction is an expected value, not an eigenvalue. Then repeated measurements always yield either the full quantum numbers of the particle or a null result, and the "fraction" is just the probability of finding the full particle. This is characterized by a nonvanishing dispersion, which remains nonzero no matter how far apart the measurements are performed.

The phenomenon discussed by Maris belongs to the second class: fractional quantum numbers do not arise, only fractional expectations with non-vanishing dispersion. The reason for this is the following. Let the electron's wave function be presented as $\psi_+ = (\psi_1 + \psi_2)/\sqrt{2}$, where ψ_1 and ψ_2 are (normalized) wave functions peaked at the first and second daughter bubbles, respectively. The expected electron number localized around the first bubble is

$$n = \langle +|N_f|+\rangle = \int \mathrm{d}V\, f \psi_+^* \psi_+ \ .$$

Here f is a sampling function, which localizes the volume integral in the region of the first bubble. It is true that $n = 1/2$.

However, one must also look at the variance $(\Delta n)^2 = \langle +|N_f^2|+\rangle - n^2$. We remember that there exists another state $\psi_- = (\psi_1 - \psi_2)/\sqrt{2}$ that is almost degenerate with the state ψ_+. Retaining just these two states, we find:

$$(\Delta n)^2 = \langle +|N_f|+\rangle\langle +|N_f|+\rangle + \langle +|N_f|-\rangle\langle -|N_f|+\rangle - n^2 = |\langle +|N_f|-\rangle|^2$$
$$= \left|\int \mathrm{d}V\, f \psi_+^* \psi_-\right|^2 = \tfrac{1}{4} \ .$$

Thus no matter how far apart the two bubbles are taken, one cannot isolate a sharp fraction.

Indeed the effect is a standard quantum mechanical result for a superposed state. Here the superposition is in location: the electron is either in one bubble or the other, but before the measurement one cannot decide in which bubble it resides. While "half the electron's wave function" is in both bubbles, measurements will find a full electron in half the bubbles.

Finally, we observe that (as remarked by K. Canter) inasmuch as the helium bubble is stabilized by Pauli repulsion between the electron in the bubble and the orbital electrons in helium, for a bubble to exist its inhabitant must be identical with the electrons in the atoms, and cannot be just a fraction thereof.

Ultimately it is an experimental question whether split bubbles exist and whether they have any role in explaining the mobilities. We note that arguments have been presented that challenge, on dynamical grounds, the splitting.[6] While we do not assess the experimental data, we assert that fractional electrons do not belong in a theoretical description. Indeed, as Maris himself states, experiments require that each bubble acts as though it contains a full electron charge.

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REFERENCES

1. H. Maris, *J. Low Temp. Phys.*, **120**, 173 (2000); see also *Physics Today*, **53**, No. 11, 9 (2000) and *New Scientist*, No. 2260, 25 (2000).
2. R. Jackiw and C. Rebbi, *Phys. Rev. D* **13**, 3398 (1976).
3. W.-P. Su, J.R. Schrieffer, and A. Heeger, *Phys. Rev. Lett.* **42**, 1698 (1979); R. Jackiw and J.R. Schrieffer, *Nucl. Phys.* **B190** [FS3], 253 (1981).
4. R.B. Laughlin, H.L. Stormer, and D.C. Tsui, *Rev. Mod. Phys.* **71**, 863 (1999).
5. S. Kivelson and J.R. Schrieffer, *Phys. Rev. B* **25**, 6447 (1982); R. Rajaraman and J.S. Bell, *Phys. Lett.* **116B**, 151 (1982); R. Jackiw, A. Kerman, I. Klebanov, and G. Semenoff, *Nucl. Phys.* **B225** [FS9], 233 (1983).
6. V. Elser, eprint cond-mat/0012311.