

**Thomas Banks**

**Statement**

**and**

**Readings**



Quantum mechanics is widely regarded as mysterious, even among physicists. Yet no one has ever proposed a “more sensible” theory, which explains quantum mechanics in terms that its detractors find more easy to accept. Like special relativity it is a theory whose conclusions must be accepted because they conform to experimental reality, even though they seem to violate our intuition. With relativity we’ve come to accept that our intuitions about how velocities add have no more validity than Aristotle’s “obvious” conclusion that matter’s natural state of motion is to be at rest. Quantum mechanics is harder to accept because it violates the rules of classical logic that we long believed to be at the basis of all rational thought.

In my experience there are several simple ideas, all long extant in the literature, which help one to come to terms with the realities of quantum physics. The first is called *quantum logic*, a frightening term that obscures a rather simple idea: the mathematical formulation of the laws of logic, due to George Boole, contains within itself an obvious generalization. This mathematical formulation is simply the theory of a maximal set of commuting operators in Hilbert space, and once one has constructed it one cannot avoid the fact that there are Hermitian operators which do not commute with the chosen set, and that linear algebra provides us with a formula for a probability distribution for the eigenvalues of those operators, for each choice of sharp values of the maximal commuting set.

What is peculiar here is the appearance of probability in a situation which seems to have no uncertainty. We have measured, with the theorist’s absolute precision, the maximal amount of information that we can extract from the system. Probability’s original appearance in physics and gaming, was as a way of estimating the chances of something happening in a situation where we were ignorant of some of the details of the initial conditions. It plays a rather different, and much more intrinsic role here.

My main point though is that this mathematics is simply unavoidable. The non-commuting operators are there, even if one decides not to discuss them. Indeed, Koopman long ago showed how to reformulate classical mechanics as a special case of quantum mechanics. One simply introduces position and momentum as commuting quantum variables, each with its own canonical conjugate,  $\pi_{p,q} = -i\partial_{p,q}$ . If we take the quantum Hamiltonian to be the Liouville operator on phase space

$$H = i(\partial_p E \partial_q - \partial_q E \partial_p),$$

where  $E$  is the classical Hamiltonian function of the system, then, *as long as we agree not to talk about measurements of  $\pi_{p,q}$*  the predictions of this quantum system agree with the predictions of classical mechanics, with initial phase space probability density

$$\rho = \psi^*(p, q)\psi(p, q).$$

The probabilistic aspect of quantum mechanics here reduces to the conventional uncertainty in initial conditions. The italicized covenant prevents us from finding out about the behavior of the phase of the wave function. Nonetheless, the theory makes predictions about hypothetical measurements of operators that do not commute with the phase space coordinates.

So, the precise, and therefore mathematical, formulation of what we mean by *logic*, simply forces quantum mechanics, with its intrinsic probabilities, not tied to the uncertainties of initial conditions, upon our attention. The world has used it, even if we would rather not. Once one has accepted that *quantum mechanics is an intrinsically probabilistic theory* the “mystery of wave function collapse” becomes familiar, rather than mysterious. Our probabilistic theory of the weather is often unable to tell whether a hurricane will hit Galveston or New Orleans. Meteorologists routinely solve the equations, *and throw out that part of the solution that predicts a disaster in Galveston, when they see people suffering in a sports stadium in New Orleans*. This is called the method of conditional probabilities. Physical theories are a means for predicting the future. When we are forced to use probabilities in our equations, some of these predictions are inevitably wrong, because we consider probabilities for two mutually exclusive macroscopic events. For convenience, we then incorporate the observations of misery in New Orleans into our equations, by defining a new probability for future events, conditional upon the fact that the hurricane hit New Orleans. This is “an artificial rule imposed on the equations”, which “violates conservation of probability” (because the new conditional probability distribution predicts that New Orleans was hit with certainty). The “distribution collapses”, and if we regarded the distribution as a physical field, because it satisfies *e.g.* the Fokker-Planck equation, we could expend reams of paper and giga-bytes of tortured prose exploring the philosophical implications of the conditional probability rule.

Much of the confusion about QM in the literature comes from trying to attribute the same sort of reality to the quantum wave function that we are used to attributing to Maxwell’s electromagnetic fields. Real classical fields are coherent states of a huge number

of bosons, all in the same quantum state. They attain their reality (which means simply that we can make predictions about them, which have no uncertainties) because they are collective variables of macroscopic systems, whose quantum fluctuations are negligible for all practical purposes. Attempts to attribute the same sort of reality to wave functions of individual particles, as exemplified by sentences like, "The electron *is* its wave function, and satisfies a deterministic equation of motion", are conceptually wrong, because they try to hide the intrinsically probabilistic nature of QM.

Apart from the fact that probabilities are not only a consequence of uncertainty in initial measurements, QM differs from the equations of the weather in an important respect. The rule for computing probabilities involves superposition of amplitudes rather than of probabilities themselves. Some of the unease with wave function collapse has to do with the fact that this aspect of QM seems to disappear after a measurement is performed. Although this has long been understood by experts, the essence of it deserves more popular recognition. A measurement in QM consists, as von Neumann proposed long ago, of an interaction that puts a micro-variable like the state of an electron spin, into QM entanglement with *a collective coordinate of a macroscopic system called the measuring apparatus*. The phrase collective coordinate refers to some sort of an average value over a huge number of micro-states. Roughly speaking, if the system contains  $N$  atoms, there are  $e^{cN}$  microstates with the same value of the collective coordinate. In normal circumstances the microscopic state of the system wanders over this huge space of states, constantly changing its phase, without in any way altering the value of the collective coordinate. The wave function overlaps, which lead to the deviation of the quantum laws of probability from their classical counterparts, are doubly exponentially small (I use the phrase *doubly exponentially small* to denote numbers whose natural log is negative and of order  $10^n$  with  $n > 3$ ). For even modest values of  $N$   $10^4$ , the time that it would take to see the quantum interference effects is essentially the same number when expressed in Planck times, as in ages of the universe (the two units differ by a mere factor of  $10^{61}$ ). We call this effective erasure of quantum phase interference, *decoherence*.

In making these statements, we have completely neglected the coupling of the micro-system plus measuring apparatus to their vast external environment. In practice the decoherence due to coupling with the environment is an even larger effect, and makes it even harder to see interference between states of macroscopic collective variables. Environmental decoherence is one of the practical barriers to building a quantum computer. Only when the macrosystem is kept in its ground state throughout the course of the experiment, can

we hope to see quantum interference between states of collective coordinates. This has been achieved for currents in SQUIDs, for systems consisting of  $\sim 10^8$  atoms. Note that the quantum fluctuations of collective variables in the ground state are still small, but they vanish like powers, rather than exponentials of the number of atoms. The textbook explanation of why spreading of the wave function is negligible for basketballs or the moon, emphasizes these powers, while the exponential suppression due to decoherence is never mentioned.

As an example consider Schrödinger's infamous exercise in cruelty to animals. Once the intrinsically probabilistic interpretation of the wave function, and the simple facts of decoherence are taken into account, statements like "quantum mechanics predicts that the cat is alive and dead at the same time", are revealed to be silly misconceptions. Quantum mechanics simply predicts a probability that the cat will be found alive or dead in any given experiment, a prediction that can only be tested by subjecting many poor beasts to savage and useless experiments. Furthermore, *as a consequence of decoherence*, quantum mechanics predicts that once a particular cat is found dead, there is no probability that future experiments will find it alive. More precisely, that probability is ridiculously small, of order the probability for the atoms in the dead cat to spontaneously reassemble themselves to make the live cat.

These simple arguments show that our unease with QM is not so far removed from the unease Aristotle would have shown when confronted with the principle of inertia invented by Galileo and Newton. If we accept, following Darwin and Wallace, that the brain evolved, then the forces that shaped its evolution had to do with the behavior of collective coordinates of macroscopic systems. Watch that rock! Beware the jaws of that tiger! Jump for that low hanging fruit, but don't break your leg! These are the evolutionary pressures that led to the differential survival probability of those proto-humans who had the intelligence to formulate intuitions about some of the laws of physics. There was never a selection pressure for understanding that the distribution of electrons passing through two microscopically spaced slits, did not follow the same laws of probability that worked perfectly well for rocks and tigers. Indeed, our considerations above show that the likelihood that special relativistic corrections to Newtonian physics would come to our "evolutionary attention", is doubly exponentially larger than that for quantum mechanics. Perhaps this accounts for the remarkable longevity of even Nobel prize winning physicists' unease with the principles of quantum mechanics. About the only way I can think of of producing selection pressures that could lead to creatures with an intuitive understanding

of quantum mechanics, is to make the right to reproduce contingent on getting tenure in a physics department. The obvious political difficulties of such a proposal, are far from its worst aspect.

In summary, quantum mechanics is a radical departure from all previous physical theories because it abandons the hitherto sacred principles of classical logic for a more general, but less intuitive, set of rules. Yet the mathematical formulation of the rules of logic shows us that quantum mechanics is an unavoidable part of its structure. While we can artificially construct special systems, which allow one to avoid discussing non-commutative observables, one should not be surprised if nature has taken another path. The only puzzle is why our brains are not built to recognize the real quantum rules, and that is simply explained by the quantum theory of macroscopic objects. Quantum interference between states of the collective coordinates of such objects is, under almost all circumstances, an effect which is of order  $\exp(-10^{20})$  or smaller. Such doubly exponentially small numbers defeat even the the most precise experiments we can conceive, and were certainly irrelevant to the macroscopic events that shaped the evolution of our brains. This understanding of the mysteries of quantum mechanics does not require us to make any kind of mystical connection between consciousness and the fundamental laws of physics. It only requires us to understand that our consciousness is an evolved collective variable of a macroscopic system, which obeys the rules of local quantum theory.

# Locality and the classical limit of quantum systems

Tom Banks

*Department of Physics and SCIPP  
University of California, Santa Cruz, CA 95064  
E-mail: banks@scipp.ucsc.edu*

*and*

*Department of Physics and NHETC, Rutgers University  
Piscataway, NJ 08540*

## Abstract

I argue that conventional estimates of the criterion for classical behavior of a macroscopic body are incorrect in most circumstances, because they do not take into account the locality of interactions, which characterizes the behavior of all systems described approximately by local quantum field theory. The deviations from classical behavior of a macroscopic body, except for those that can be described as classical uncertainties in the initial values of macroscopic variables, are *exponentially* small as a function of the volume of the macro-system in microscopic units. Conventional estimates are correct only when the internal degrees of freedom of the macrosystem are in their ground state, and the classical motion of collective coordinates is adiabatic. Otherwise, the system acts as its own environment and washes out quantum phase correlations between different classical states of its collective coordinates. I suggest that it is likely that we can only achieve meso-scopic superpositions, for systems which have topological variables, and for which we can couple to those variables without exciting phonons.



# 1 Classical behavior in the non-relativistic quantum mechanics of particles

In standard texts on non-relativistic quantum mechanics the classical limit is described via examples and via the WKB approximation. In particular, one often describes the spreading of the wave packet of a free particle, and estimates it as a function of time and the particle mass  $M$ . There is nothing wrong with the mathematics done in these texts, but the implication that these estimates provide the basis for an understanding of why classical mechanics is such a good approximation for macroscopic objects is not correct and therefore misleading. In particular it leads one to conclude that the corrections to decoherence for a wave function describing a superposition of two different macroscopic states is power law in the mass. I would aver that this mistake forms part of the psychological unease that many physicists feel about the resolution of Schrödinger's cat paradox in terms of the concept of decoherence.

These estimates have also led to recent experimental proposals to demonstrate quantum superposition of states of variables which are "almost macroscopic". I will argue that no such demonstration is possible, without extreme care taken to keep the constituents of the macrosystem in their microscopic ground state. The essence of my argument is that the essential variable that controls the approach to the classical limit, is the number of localizable constituents of large quantum system. In a macroscopic material this would be something like the number,  $N$ , of correlation volumes contained in the sample. Away from critical points, the correlation volume is microscopically small, and we are roughly counting the number of atoms.

Indeed, all previous discussions also identify this number as the crucial parameter. These discussions identify a variety of collective classical variables, like the center of mass of the system, and note that the effective Lagrangian for these variables has a factor of  $N$  in it. For the center of mass, this is simply the statement that the mass is large. The traditional argument simply studies the quantum mechanics of these collective variables and estimates the corrections to classical predictions, which are typically power law in the large, extensive parameters. Estimates based on these ideas have led to the suggestion that plausible extensions of current experiments can reach the limit of quantum coherence for collective coordinates of systems with dimensions of millimeters. The failure to observe such correlations might be taken to mean that there is some fundamental error in applying quantum mechanics to macroscopic systems, as has been proposed by Penrose, Leggett and others.

The essential point of this paper is that "small" corrections to this collective coordinate

approximation completely invalidate this argument. It is not that the classical dynamics is not a good approximation to the quantum mechanics of the collective variables. What is not a good approximation is to neglect the back reaction of the collective variables on the huge set of other degrees of freedom in the macroscopic object. Locality ensures that external forces acting on the macroscopic body affect the collective coordinate through collective interactions with individual constituents, which then give rise to terms in the Hamiltonian coupling the collective coordinate to the constituents. In effect, different classical motions of the collective coordinate give rise to different, time dependent, Hamiltonians for the constituents. These extra terms are small, inversely proportional to powers of extensive parameters.

However, typical macro-systems have a finite microscopic correlation length. The wave function of the system is a sum of terms which are products of individual cluster wave functions for a localized microscopic subset of the constituents. This idea is the basis for approximate variational calculations like the Hartree-Fock or Jastrow approximations. As a consequence of the small corrections described in the previous paragraph, the individual cluster wave functions will be modified by a small amount and the overlap between wave functions for two different classical trajectories of the collective coordinate will be proportional to  $1 - \epsilon$  where  $\epsilon$  is a measure of the strength of the perturbation that leads to non-uniform motion of the center of mass. However, because the full many body wave function is a product of  $o(N)$  cluster wave functions, the overlap is of order  $(1 - \epsilon)^N$ , *which is exponentially small in the volume of the system measured in microscopic units*. In other words, *for a macroscopic body, different classical trajectories of a collective coordinate divide the system into different approximate super-selection sectors*.

One can argue, using the methods of quantum field theory and statistical mechanics, that the time that it takes to observe phase correlations between different approximate superselection sectors is of order  $10^{cN}$  where  $c$  is a constant of order one. This is true as long as one is in a regime where the density of states of the microscopic degrees of freedom is large, *i.e.* that the state of the system is a superposition of a densely spaced set of eigenstates, which behaves in a manner describable by statistical mechanics. Note that the ratio between the current age of the universe and the Planck time is a mere  $10^{61}$ , so that even for a moderately large system containing  $N \sim 10^3$  correlation volumes, this time is *so long that it is essentially the same number of Planck times as it is ages of the universe*. *No imaginable experiment can ever distinguish the quantum correlations between different states of the collective coordinates of a macro-system*. The extraordinary smallness of such double exponentials defeats all of our ordinary intuitions about ordinary physics. Over such long time scales, many counter-intuitive

things could happen. For example, in a hypothetical classical model of a living organism made of this many constituents, or in a correct quantum model, the phenomenon of Poincare recurrences assures that given (exponentially roughly) this much time, the organism could spontaneously self assemble, out of a generic initial state of its constituents. So much for Schrödinger's cat.

Another way of phrasing the same arguments comes from the vast literature on *decoherence*, which also introduces an important concept I have not yet emphasized. This is the fact that an approximate superselection sector is not a single state, but actually a vast ensemble of order  $10^{cN}$  states, which share the same value of the collective coordinate. In the decoherence literature, it is argued that rapid changes in the micro-state of a macroscopic environment wipe out the quantum phase correlations between *e.g.* states with two different positions of a macroscopic pointer, which have been put into a *Schrödinger's cat* superposition via interaction with some micro-system. Another way to state the conclusions of the previous paragraph is simply to say that the constituents of a macroscopic body serve as an environment, which serves to decohere the quantum correlations between the macro-states of collective coordinates. Unlike typical environments, which one might hope to eliminate by enclosing the system in a sufficiently good vacuum, the inherent environment of a macro-system cannot be escaped. The collective variables exist and behave as they do, because of the properties of the environment in which they are embedded. It is only when the macroscopic system is held in its ground state, during experiments in which the dynamics of the collective variables is probed, that conventional estimates of quantum coherence for the collective coordinate wave function are valid.

In this introductory section, I will fill in the argument that conventional estimates of quantum corrections to classical behavior are wrong, using standard ideas of non-relativistic quantum mechanics. In the remainder of the paper I will discuss the basis for these calculations in quantum field theory. This will also remove the necessity to resort to Hartree-Fock like approximations to prove the point directly in the non-relativistic formalism. As noted, the essential point of the argument is that we must take into account the fact that a macroscopic object is made out of a huge number, which generally I will take to be  $> 10^{20}$ , of microscopic constituents, in order to truly understand its classical behavior. I will argue that, as a consequence, the overlaps between states where the object follows two macroscopically different trajectories, as well as the matrix elements of all local operators<sup>1</sup> between such states, are of

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<sup>1</sup>In this context local means an operator which is a sum of terms, each of which operates only on a few of the constituent particles. A more precise, field theoretic, description will be given in the next section.

order

$$e^{-10^{20}}.$$

Consider then, the wave function of such a composite of  $N \gg 1$  particles, assuming a Hamiltonian of the form

$$H = \sum \frac{\vec{p}_i^2}{2m_i} + \sum V_{ij}(x_i - x_j).$$

Apart from electromagnetic and gravitational forces, the two body potentials are assumed to be short ranged. We could also add multi-body potentials, as long as the number of particles that interact is  $\ll N^2$ .

The Hamiltonian is Galilean invariant and we can separate it into the kinetic energy of the center of mass, and the Hamiltonian for the body at rest. The wave function is of the form

$$\psi(X_{cm})\Psi(x_i - x_j).$$

$\Psi$  is a general function of coordinate differences. In writing the Schrodinger equation we must choose  $N - 1$  of the coordinate differences as independent variables. If the particles are identical, this choice obscures the  $S_N$  permutation symmetry of the Hamiltonian. One must still impose Bose or Fermi statistics on the wave functions. This is a practical difficulty, but not one of principal. We now want to compare this wave function with the internal wave function of the system when the particle is not following a straight, constant velocity trajectory. In order to do this, we introduce an external potential  $U(x_i)$ . It is *extremely* important that  $U$  is not simply a function of the center of mass coordinate but a sum of terms denoting the interaction of the potential with each of the constituents. This very natural assumption is derivable from local field theory: the external potential must interact locally with “the field that creates a particle at a point”. So we assume

$$U = \sum u_i(x_i),$$

where we have allowed for the possibility, *e.g.* that the external field is electrical and different constituents have different charge.

To solve the external potential problem, we write  $x_i = X_{cm} + \Delta_i$  and expand the individual potentials around the center of mass, treating the remaining terms as a small perturbation. We then obtain a Hamiltonian for the center of mass, which has a mass of order  $N$ , as well as a potential of order  $N$ . The large  $N$  limit is then the WKB limit for the center of mass motion.

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<sup>2</sup>Or that the strength of  $k$  body interactions fall off sufficiently rapidly with  $k$  for  $k > N_0 \ll N$ .

The residual Hamiltonian for the internal wave function has small external potential terms, whose coefficients depend on the center of mass coordinate.

The Schrodinger equation for the center of mass motion thus has solutions which are wave functions concentrated around a classical trajectory  $X_{cm}(t)$  of the center of mass, moving in the potential  $\sum u_i(X_{cm})^3$ . These wave functions will spread with time in a way that depends on this potential. For example, initial Gaussian wave packets for a free particle will have a width, which behaves like  $\sqrt{t/Nm}$  for large  $t$ , where  $m$  is a microscopic mass scale. The fact that this is only significant when  $t \sim N$  is the conventional explanation for the classical behavior of the center of mass variable.

In fact, this argument misses the crucial point, namely that the small perturbation, which gives the Hamiltonian of the internal structure a time dependence, through the appearance of  $X_{cm}(t)$ , is not at all negligible. To illustrate this let us imagine that the wave function at rest has the Hartree-Fock form, an anti-symmetrized product of one body wave functions  $\psi_i(x_i)$ , and let us characterize the external potential by a strength  $\epsilon$ . In the presence of the perturbation, each one body wave function will be perturbed, and its overlap with the original one body wave function will be less than one. *As a consequence, the overlap between the perturbed and unperturbed multi-body wave functions will be of order  $(1 - \epsilon)^N$ .* This has the exponential suppression we claimed, as long as  $\epsilon \gg \frac{1}{N}$ . It is easy to see that a similar suppression obtains for matrix elements of few body operators. One can argue that a similar suppression is obtained for generalized Jastrow wave functions, with only few body correlations, but a more general and convincing argument based on quantum field theory will be given in the next section. Here we will follow through the consequences of this exponential suppression.

The effect is to break up the full Hilbert space of the composite object in the external potential, into *approximate super-selection sectors* labeled by macroscopically different classical trajectories  $X_{cm}(t)$  (microscopically different trajectories correspond to  $\epsilon \sim \frac{1}{N}$ ). We will argue that local measurements cannot detect interference effects between states in different super-selection sectors on times scales shorter than  $e^{10^{20}}$  (we leave off the obviously irrelevant unit of time). That is to say, for all *in principle purposes*, a superposition of states corresponding to different classical trajectories behaves like a classical probability distribution for classical trajectories. The difference of course is that in classical statistical physics one avers that *in principle* one could measure the initial conditions precisely, whereas in quantum mechanics the uncertainty is intrinsic to the formalism.

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<sup>3</sup>See [2] for a construction of such wave functions for the Coulomb/Newton potential.

The argument for the exponentially large time scale has two parts, each of which will be given in more detail below. First we argue that it takes a time of order  $N$ , for a local Hamiltonian to generate an overlap of order 1 between two different superselection sectors. Then we argue that most macroscopic objects are not in their quantum ground state. The typical number of eigen-states present in the initial state of the object, or that can be excited by the coupling to the time dependent motion of the collective coordinate is of order  $e^{cN}$ . These states are highly degenerate. The time dependent Hamiltonian induced by the coupling to the collective coordinate will induce a time dependent unitary evolution on this large space of states, with a time scale of order 1 (in powers of  $N$ ). Thus, there is a rapid loss of phase coherence between the two super-selection sectors, while the Hamiltonian is generating a non-trivial overlap between them. We would have to wait for the motion on the Hilbert space to have a recurrence before we could hope to see coherent quantum interference between two states with different macroscopic motions of the collective coordinate. The shortest recurrence time is of order  $e^{cN}$ .

Two paragraphs ago, I used the phrase *in principle* in two different ways. The first use was ironic; the natural phrase that comes to mind is *for all practical purposes*. I replace *in practice* by *in principle* in order to emphasize that any conceivable experiment that could distinguish between the classical probability distribution and the quantum predictions would have to keep the system isolated over times inconceivably longer than the age of the universe. In other words, it is meaningless for a *physicist* to consider the two calculations different from each other. In yet another set of words; the phrase “With enough effort, one can in principle measure the quantum correlations in a superposition of macroscopically different states”, has the same status as the phrase “If wishes were horses then beggars would ride”.

The second use of *in principle* was the conventional philosophical one: the mathematical formalism of classical statistical mechanics contemplates arbitrarily precise measurements, on which we superimpose a probability distribution which we interpret to be a measure of our ignorance. In fact, even in classical mechanics for a system whose entropy is order  $10^{20}$ , this is arrant nonsense. The measurement of the precise state of such a system would again take inconceivably longer than the age of the universe.

This comparison is useful because it emphasizes the fact that the tiny matrix elements between super-selection sectors are due to an entropic effect. They are small because a change in the trajectory of the center of mass changes the state of a huge number of degrees of freedom. Indeed, in a very rough manner, one can say that the time necessary to see quantum interference effects between two macroscopically different states is of order the Heisenberg recurrence time

of the system. This is very rough, because there is no argument that the order 1 factors in the exponent are the same, so the actual numbers could be vastly different. The important point is that for truly macroscopic systems both times are super-exponentially longer than the age of the universe.

The center of mass is one of a large number of *collective* or *thermodynamic* observables of a typical macroscopic system found in the laboratory. The number of such variables is a measure of the number of macroscopic moving parts of the system. As we will see, a system with a goodly supply of such moving parts is a good measuring device. Indeed, the application of the foregoing remarks to the quantum measurement problem is immediate. As von Neumann first remarked, there is absolutely no problem in arranging a unitary transformation which maps the state

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \otimes |Ready\rangle,$$

of a microsystem uncorrelated with the  $|Ready\rangle$  state of a measuring apparatus, into the correlated state

$$\alpha|\uparrow\rangle \otimes |+\rangle + \beta|\downarrow\rangle \otimes |-\rangle,$$

where  $|+/-\rangle$  are *pointer states* of the measuring apparatus. If we simply assume, in accordance with experience, that the labels  $+/-$  characterize the value of a macroscopic observable in the sense described above, then we can immediately come to the following conclusions

- 1. The quantum interference between the two pieces of the wave function cannot be measured on time scales shorter than the super-exponential times described above. The predictions of quantum mechanics for this state are identical *in principle* (first usage) to the predictions of a classical theory that tells us only the probabilities of the machine reading  $+$  or  $-$ . *Like any such probabilistic theory* the algorithm for interpreting its predictions is to condition the future predictions on any actual measurements made at intermediate times. This is the famous “collapse of the wave function”, on which so much fatuous prose has been expended. It no more violates conservation of probability than does throwing out those weather simulations, which predicted that Hurricane Katrina would hit Galveston.
- 2. One may worry that there is a violation of unitarity in this description, because if I apply the *same* unitary transformation to the states  $|\uparrow\rangle \otimes |Ready\rangle$  and  $|\downarrow\rangle \otimes |Ready\rangle$ , individually, then I get a pair of states whose overlap is not small. This seems like a violation of the superposition principle, but this mathematical exercise has nothing to do with physics, for at least two reasons. First the macro-states labeled by  $+/-$  are not

single states, but huge ensembles, with  $e^N$  members. The typical member of any of these ensembles is a time dependent state with the property that time averages of all reasonable observables over a short relaxation time are identical to those in another member of the ensemble. The chances of starting with the identical  $|Ready\rangle$  state or ending with the same  $|+/-\rangle$  states in two experiments with different initial micro-states, is  $e^{-N}$ . Furthermore, and perhaps more importantly, the experimenter who designs equipment to amplify microscopic signals into macroscopic pointer readings, *does not* control the microscopic interaction between the atoms in the measuring device and *e.g.* the electron whose spin is being measured. Thus, in effect, every time we do a new measurement, whether with the same input micro-state or a different one, it is virtually certain that the unitary transformation that is actually performed on the system is a different one.

For me, these considerations resolve all the *angst* associated with the Schrödinger's cat paradox. Figurative superpositions of live and dead cats occur every day, whenever a macroscopic event is triggered by a micro-event. We see nothing remarkable about them because quantum mechanics makes no remarkable predictions about them. It never says "the cat is both alive and dead", but rather, "I can't predict whether the cat is alive or dead, only the probability that you will find different cats alive or dead if you do the same experiment over and over". Wave function collapse and the associated claims of instantaneous action at a distance are really nothing but the the familiar classical procedure of discarding those parts of a probabilistic prediction, which are disproved by actual experiments. This is usually called the use of conditional probabilities, and no intellectual discomfort is attached to it.

It is important to point out here that I am not claiming that any classical probability theory could reproduce the results predicted by quantum mechanics. John Bell showed us long ago that this is impossible, as long as we insist that our classical theory obey the usual rules of locality. My claim instead is that the correct philosophical attitude toward collapse of the wave function is identical to that which we invoke for any theory of probability. In either case we have a theory that only predicts the chances for different events to happen, and we must continuously discard those parts of the probability distribution, which predicted things that did not occur. The considerations of this paper show that when we discard the dead cat part of the wave function after seeing that the cat is alive, we are making mistakes about future predictions of the theory that are in principle unmeasurable.

We are left with the discomfort Einstein expressed in his famous aphorism about mythical beings rolling dice. Those of us who routinely think about the application of quantum mechanics



to the entire universe, as in the apparently successful inflationary prediction of the nature of Cosmic Microwave Background temperature fluctuations, cannot even find comfort in the frequentist's fairy tale about defining probability "objectively" by doing an infinite number of experiments. Probability is a guess, a bet about the future. What is it doing in the most precisely defined of sciences? I will leave this question for each of my readers to ponder in solitude. I certainly don't know the answer.

Finally, I want to return to the spread of the wave packet for the center of mass, and what it means from the point of view presented here. It is clear that the uncertainties described by this wave function can all be attributed to the inevitable quantum uncertainties in the initial conditions for the position and velocity of this variable. Quantum mechanics prevents us from isolating the initial phase space point with absolute precision. These can simply be viewed as microscopic initial uncertainties in the classical trajectory  $X_{cm}(t)$ . In the WKB approximation, the marginal probability distributions for position and momentum are Gaussian, and there is a unique Gaussian phase space distribution that has the same marginal probabilities.

If we wait long enough these uncertainties would, from a purely classical point of view, lead to macroscopic deviations of the position from that predicted by the classical trajectory we have expanded around. The correct interpretation of this is that our approximation breaks down over such long time scales. A better approximation would be to decide that after a time long enough for an initial microscopic deviation to evolve into a macroscopic one, we must redefine our super-selection sectors. After this time, matrix elements between classical trajectories that were originally part of the same super-selection sector, become so small that we must declare that they are different sectors.

Thus instead of, in another famous Einsteinian phrase, complaining that the moon is predicted to disappear when we don't look at it (over a time scale power law in its mass), we say that quantum mechanics predicts that our best measurement of the initial position and velocity of the moon is imprecise. The initial uncertainties are small, but grow with time, to the extent that we cannot predict exactly where the moon is. Quantum mechanics *does* predict, that the moon has (to an exponentially good approximation) followed some classical trajectory, but does not allow us to say which one, a long time after an initial measurement of the position and velocity.

Of course, if the constituents of the macroscopic body are kept in their ground state during the motion, then we must treat the wave function of the center of mass with proper quantum mechanical respect, and the predictions of quantum interference between different classical

trajectories should be verifiable by experiment. This is clearly impossible for the moon. In a later section, I will discuss whether it is likely to be true for mesoscopic systems realizable in the laboratory.

### 1.1 Bullets over Broad-slit-way

To make these general arguments more concrete, let's consider Feynman's famous discussion of shooting bullets randomly through a pair of slits broad enough to let the bullets pass through. The bullet moves in the  $x$  direction, and we assume the initial wave function of the center of mass of the bullet is spread uniformly over the  $y$  coordinate distance between the slits. Then subsequent to the passage through the slits, the wave function of the center of mass is, to a good approximation, a superposition of two Gaussian wave functions, centered around the two slit positions. A conventional discussion of this situation would solve the free particle Schrodinger equation for this initial wave function and compare the quantum mechanical probability distribution a later times, with a classical distribution obtained by solving the Liouville equation for a free particle, with initial position and momentum uncertainties given by some positive phase space probability distribution whose marginal position and momentum distributions coincide with the squares of the position and momentum space wave functions.

In the latter calculation, the term in the initial probability distribution coming from the overlap of the Gaussians centered at the two different slits is of order  $e^{-(\frac{L}{w})^2}$ , where  $L$  is the distance between the slits and  $w$  their width. Liouville evolution can lead to uncertainty about which slit the particle went through in a time of order  $\frac{2ML}{\hbar}$ , just as in the quantum calculation. However, it gives rise to a different spatial distribution of probability density, with no interference peaks. Thus, for such times, the interference terms and exact Schrodinger evolution give a different result from classical expectations with uncertain initial conditions.

Now, let us take into account the fact that the two branches of the center of mass wave function must be multiplied by wave functions of the internal coordinates, which are in different super-selection sectors. As long as the micro-state is a superposition of internal eigenstates coming from a band with exponentially large density of states, it would be highly unnatural to assume that the micro-state in the top slit is simply the space translation of that in the bottom slit. The probability for this coincidence is  $e^{-cN}$ . It then follows from our previous discussion that we will have to wait of order a recurrence time in order to have a hope that the interference term in the square of the Schrodinger wave function is not exponentially small. The difference between quantum evolution of the center of mass wave function, and classical evolution with

uncertain initial conditions is completely unobservable, except perhaps at selected instants over super-exponentially long time scales.

## 2 Quantum field theory

I will describe the considerations of this section in the language of relativistic quantum field theory. *A fortiori* they apply to the non-relativistic limit, which we discussed in first quantization in the previous section. They also apply to cutoff field theories, with some kind of spatial cutoff, like a space lattice. The key property of all these systems is that the degrees of freedom are labeled by points in a fixed spatial geometry, with a finite number of canonical bosonic or fermionic variables per point. The Hamiltonian of these degrees of freedom is a sum of terms, each of which only couples together the points within a finite radius<sup>4</sup> In the relativistic case of course the Hamiltonian is an integral of a strictly local Hamiltonian density.

Let us first discuss the ground state of such a system. If the theory has a mass gap, then the ground state expectation values of products of local operators fall off exponentially beyond some correlation length  $L_c$ . If  $d$  is the spatial dimension of the system, and  $V$  is a volume  $\gg L_c^d$ , define the state

$$|\phi_c, V\rangle,$$

as the normalized state with minimum expectation value of the Hamiltonian, subject to the constraint that

$$\langle \phi_c, V | \int_V d^d x \phi(x) / V | \phi_c, V \rangle = \Phi_c.$$

Let  $N = V/L_c^d$ . One can show, using the assumption of a finite correlation length, that these states have the following properties

1. The quantum dynamics of the variable  $\Phi_c$  is amenable to the semi-classical approximation, with expansion parameter  $\propto 1/N$ .

2. The matrix elements of local operators between states with different values of  $\Phi_c$  satisfy

$$\langle \Phi_c, V | \phi_1(x_1) \dots \phi_n(x_n) | \Phi'_c, V \rangle \sim e^{-cN},$$

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<sup>4</sup>Various kinds of exponentially rapid falloff are allowed, and would not effect the qualitative nature of our results.

where  $n$  is kept finite as  $N \rightarrow \infty$ .

3. The interference terms in superpositions between states with different values of  $\Phi_c$  remain small for times of order  $N$ . This follows from the previous remark and the fact that the Hamiltonian is an integral of local operators. This remark is proved by thinking about which term in the  $t$ -expansion of  $e^{-iHt}$  first links together the different superposition sectors with an amplitude of order 1. One needs terms of order  $N$  in order to flip a macroscopic number of local clusters from one macro-state to another. This term is negligible until  $t \sim N$  in units of the correlation length. For many systems, there is a technical problem in this argument, because the Hamiltonian is unbounded, but it is intuitively clear that a cutoff at high energy should not affect the infrared considerations here.

4. However, there is another important phenomenon occurring, on a much shorter time scale. The microscopic degrees of freedom are evolving according to the microscopic Hamiltonian, perturbed by the time dependent term due to the motion of the collective coordinate. In a typical situation, the macroscopic object is not in its quantum ground state, but rather in some micro-state that is a superposition of many eigenstates from an energy band where the density of states is, according to quantum field theory, of order  $e^{N^5}$ . The evolution in this subspace of states is qualitatively like that of a random Hamiltonian in a Hilbert space of this dimension. It leads to thermalization and loss of quantum coherence, through rapid changes of relative phase[3]. Initial quantum correlations will reassert themselves only once a recurrence time, and the shortest recurrence time is  $o(e^N)$ .

In the language of the previous section, *averages of local fields over distances large compared to the correlation length are good pointer observables, whenever the system is in a typical state chosen from an ensemble where the density of states is  $o(e^N)$* . It is only when a macro-system is in its ground state, and the motion of the collective coordinates is adiabatic, relative to the gap between the ground state and the region of the spectrum with exponential density of states, that conventional estimates of power law (in  $N$ ) time scales for seeing quantum coherence are valid.

Typical field theories describing systems in the real world contain hydrodynamic modes like phonons, with very low energies, and one would have to consider frequencies of collective coordinate motion lower than these hydrodynamic energy scales in order to observe quantum

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<sup>5</sup>In reality, the system is unavoidably coupled to an environment, and is not in a pure state. If nothing else, soft photon emission will create such an environment. However, since our point is that the macroscopic system decoheres itself, we can neglect the (perhaps numerically more important) environmental decoherence.

coherence for the macroscopic observables over power law time scales. Certainly the motion of the moon is not in such an adiabatic regime.

To define an actual apparatus, we have to assume that the quantum field theory admits bound states of arbitrarily large size. Typically this might require us to add chemical potential terms to the Hamiltonian and insist on macroscopically large expectation values for some conserved charge. The canonical example would be a large, but finite, volume drop of nuclear matter in QCD. We can repeat the discussion above for averages over sub-volumes of the droplet.

Of course, in the real world, the assumption of a microscopically small correlation length is not valid, because of electromagnetic and gravitational forces. Indeed, most real measuring devices use these long range forces, both to stabilize the bound state and for the operation of the machine itself. I do not know how to provide a mathematical proof, but I am confident that the properties described above survive without qualitative modification<sup>6</sup>. This is probably because all the long range quantum correlations are summarized by the classical electromagnetic and gravitational interactions between parts of the system<sup>7</sup>. It would be desirable to have a better understanding of the modification of the arguments given here, that is necessary to incorporate the effects of electromagnetism and (perturbative) gravitation. One may also conclude from this discussion that a system at a quantum critical point, which has long range correlations not attributable to electromagnetism or gravitation, would make a poor measuring device, and might be the best candidate for seeing quantum interference between “macroscopic objects”. Of course, such conformally invariant systems do not have large bound states which could serve as candidate “macroscopic objects”.

Despite the mention of gravitation in the previous paragraph, the above remarks do not apply to regimes in which the correct theory of quantum gravity is necessary for a correct description of nature. We are far from a complete understanding of a quantum theory of gravity, but this author believes that it is definitely not a quantum field theory. In a previous version of this article [1] I gave a brief description of my ideas about the quantum theory of gravitation. I believe that it gets in the way of the rest of the discussion, and I will omit all but the conclusions.

In my opinion, the correct quantum theory of gravity has two sorts of excitations, something

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<sup>6</sup>In the intuitive physics sense, not that of mathematical rigor.

<sup>7</sup>Recall that the Coulomb and Newtonian forces between localized sources are described in quantum field theory as quantum phase correlations in the wave function for the multi-source system.

resembling conventional particles, and black holes. A given region of space-time supports only a finite amount of information, and a generic state of that region is a black hole. Black holes have very few macroscopic moving parts, and do not make good measuring devices. Low entropy states in the region can be described in terms of particles with local interactions, and are approximable for many purposes by local field theory. I have explained how the general principles of field theory lead to an understanding of approximately classical measuring devices.

The exponential approach to the classical limit allows us to understand why these conclusions will not be changed in the quantum theory of gravity. Systems describable by local field theory over a mere  $10^3 - 10^4$  correlation volumes already have collective variables so classical that their quantum correlations are unmeasurable. The fact that there exist energy scales orders of magnitude below the Planck scale, when combined with these observations, show us that practically classical systems can be constructed without the danger of forming black holes.

On the other hand, these same considerations show us that *exactly classical* observables in a quantum theory of gravity must be associated with infinite boundaries of space-time. This observation is confirmed by existing string theory models, and has profound implications for the construction of a quantum theory of gravity compatible with the world we find ourselves in.

### 3 Proposed experiments

#### 3.1 Schrödinger's drum

I was motivated to rewrite this article for publication, by a number of papers, which propose experiments to observe quantum correlations for the observables of a mesoscopic system[4]. In its simplest form the system consists of two dielectric membranes, suspended in a laser cavity. By tuning the laser frequencies, it is claimed that one can cool the motion of the translational collective coordinates of the two membranes, which are coupled through the laser modes, down to their “steady state ground state”. The ground state can be engineered to be a superposition of two different relative positions for the membranes. The system has been dubbed *Schrödinger's Drum*[4]. Although state of the art experiments can not yet reach the ground state splitting, it is plausible that it can be reached in the near future.

The membranes are about one millimeter square and 50 nano-meters thick. Typical phonon

energies are thus of order  $10^{-4}$  eV<sup>8</sup>. In the analysis of the proposed experiments [5] it is argued that the collective coordinates of the two membranes in a laser cavity has a pair of classically degenerate ground states, which are split by  $10^{-10}$  eV. It is then argued that by tuning the laser frequencies, one can cool the collective coordinate system down to temperatures below this splitting. The true ground state is a superposition of two classical values for the collective coordinates, and it is claimed that one can observe the entanglement of these two states.

At first glance, one might assume that the extremely low temperature of the collective coordinates means that the considerations of our analysis are irrelevant. However, on closer scrutiny it becomes apparent that the whole process of laser cooling, depends crucially on the coupling between the collective coordinates and a source of “mechanical noise”. The latter is treated as a large thermal system with a temperature (in the theoretical analysis  $10^{-7} - 10^{-6}$  eV)<sup>9</sup>. The analysis of [5] uses a quantum Langevin equation to describe the way in which energy is drained from the collective coordinates into the reservoir of mechanical noise. This might be<sup>10</sup> perfectly adequate for showing that the temperature of the collective coordinates can indeed be lowered, but it does not give an adequate account of quantum phase coherence.

The very fact that the source of mechanical noise can be modeled as a system obeying the laws of statistical mechanics, implies that the collective coordinate is coupled to a large number of other degrees of freedom, in a regime where the density of states of these degrees of freedom is exponentially large. Our analysis applies, and there should be no phase coherence between superselection sectors of the noise bath. If we take the correlation length in the membranes to be 100 nano-meters, then  $e^N \sim e^{10^{12}}$ , and there is no hope of seeing quantum coherence. *The failure to see quantum correlations in these experiments is not an indication that quantum mechanics breaks down for macro-systems, but simply a failure to understand that the approximate two state system of the membrane collective coordinates, suffers decoherence due to its coupling to the system which cools it down to the quantum energy regime.*

Indeed, the collective coordinates are coupled to the the system that provides the mechanical noise. The very fact that it is permissible to describe this system by statistical mechanics shows that it has a huge reservoir of states through which it is cycling on a microscopic time

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<sup>8</sup>Here I refer to the phonons of internal sound waves on each membrane, rather than the phonons associated with the relative motion of the two membranes.

<sup>9</sup>I suspect that when applied to actual experiments with mm. size membranes, the source of this noise is excitation of sound waves on the membrane, and the temperature is even higher.

<sup>10</sup>The book [6] referred to in [5] suggests that the quantum Langevin treatment is adequate only when the noise reservoirs are collections of oscillators with linear couplings to the collective coordinates. It is not clear to me that this is the case. The collective coordinates are really zero wave number phonons of the individual membranes, and I would have guessed that they are coupled non-linearly to the shorter wavelength modes.

scale. The coupling of the collective coordinates to these states, which is necessary for the cooling process, also washes out phase correlations between different classical states of the collective modes.

### 3.2 Josephson's flux

By contrast, an older experiment[7] seems to illustrate the fact that when the microscopic degrees of a macro-system can be kept in their ground state, the standard analysis of the quantum mechanics of collective coordinates is correct. This experiment consists of a Josephson junction, with a flux condensate composed of  $o(10^9)$  Cooper pairs. By appropriate tuning, one can bring the system to a state where there is resonant tunneling between a degenerate pair of quantum levels of the Landau-Ginzburg order parameter. The author's argue, correctly I believe, that because of the superconducting gap, and because their external magnetic probes couple directly to the order parameter, they can keep the system in its quantum ground state. They verify the level repulsion of a two state quantum system, when two classically degenerate states are connected by a tunneling transition. This experiment truly achieves a quantum superposition of macro-states. The recurrence time scale  $e^{10^9}$  of a typical state of this many microscopic constituents is irrelevant to the analysis of this experiment.

There is a hint here of what is necessary in order to approach macroscopic superpositions, and it echoes an insight that has already appeared in the quantum computing literature. Kitaev[8] has emphasized that *topological order parameters* may be essential to the construction of a practical quantum computer. Quantum Hall systems and superconductors have such order parameters. In the rather abstract language of quantum field theory, what would appear to be necessary is a system whose low energy dynamics is described by a *topological field theory*. In plain terms, this is a system whose localized excitations are separated from excitations of a selected set of topological variables by an energy gap. From a practical point of view, what we need is a system in which this gap is large enough so that one can carry out experiments, which do not excite states above the gap.

Macroscopic systems will generically have phonon excitations, with energies that scale like the inverse of the largest length scale in the macroscopic body. However, as the case of the Josephson junction shows, it may be possible to devise probes of the system, which couple directly to the topological order parameters, without exciting mechanical oscillations. If that is the case, we are in a regime which is properly analyzed by the conventional collective coordinate quantum mechanics. For appropriately mesoscopic systems, and with sufficient



protection against decoherence by coupling to a larger environment, we can achieve quantum coherence for appropriate macroscopic order parameters.

This does not change the main burden of this article, which is that for typical macroscopic objects, quantum coherence is *superexponentially* unlikely, and cannot be observed over any experimentally realizable time scale. It does however, confirm the insight of Kitaev, that there may be a topological route to practical quantum computation.

## 4 Conclusions

I suspect the material in this paper is well understood by many other physicists, including most of those who have worked on the environmental decoherence approach to quantum measurement. If there is anything at all new in what I have written here about quantum measurement, it lies in the statement that a macroscopic apparatus of modest size serves as its own “environment” for the purpose of environmental decoherence. In normal laboratory circumstances, the apparatus interacts with a much larger environment and the huge recurrence and coherence times become even larger. Nonetheless, there is no reason to suppose that a modestly macroscopic apparatus, surrounded by a huge region of vacuum, with the latter protected from external penetrating radiation by thousands of meters of lead, would behave differently over actual experimental time scales, than an identical piece of machinery in the laboratory.

The exception to this kind of self-decoherence that we have identified, seems to involve topological variables of systems like superconductors and quantum Hall materials. These are systems with an interesting finite dimensional Hilbert space of quasi-degenerate ground states, separated from the rest of the spectrum by a substantial gap. In addition, one must have probes which can couple directly to the topological variables, without exciting low energy phonon degrees of freedom (which are present in any macroscopic object). For such systems, one might expect to be able to create robust superpositions of states of collective variables of macroscopic systems. Kitaev has argued that these may be the key to quantum computing.

The essential point in this paper is that the corrections to the classical behavior of macroscopic systems are exponential in the size of the system in microscopic units. This puts observable quantum behavior of these systems in the realm of recurrence phenomenon, essentially a realm of science fiction rather than of real experimental science. When a prediction of a scientific theory can only be verified by experiments done over times super-exponentially longer than the measured age of the universe, one should not be surprised if that prediction is counter-intuitive

or “defies ordinary logic”.

Quantum mechanics does make predictions for macro-systems which are different than those of deterministic classical physics. Any time a macro-system is put into correlation with a microscopic variable - and this is the essence of the measurement process - its behavior becomes unpredictable. However, these predictions are indistinguishable from those of classical statistical mechanics, with a probability distribution for initial conditions derived from the quantum mechanics of the micro-system. It is only if we try to interpret this in terms of a classical model of the micro-system that we realize something truly strange is going on. The predictions of quantum mechanics *for micro-systems are* strange, and defy the ordinary rules of logic. But they do obey a perfectly consistent set of axioms of their own, and we have no real right to expect the world beyond the direct ken of our senses, which had no direct effect on the evolution of our brains, to ”make sense” in terms of the rules which were evolved to help us survive in a world of macroscopic objects.

Many physicists, with full understanding of all these issues, will still share Einstein’s unease with an intrinsically probabilistic theory of nature. Probability is, especially when applied to non-reproducible phenomena like the universe as a whole, a theory of guessing, and implicitly posits a mind, which is doing the guessing. Yet all of modern science seems to point in the direction of mind and consciousness being an emergent phenomenon; a property of large complex systems rather than of the fundamental microscopic laws. The frequentist approach to probability does not really solve this problem. Its precise predictions are only for fictional infinite ensembles of experiments. If, after the millionth toss of a supposedly fair coin has shown us a million heads, and we ask the frequentist if we’re being cheated, all he can answer is “probably”. Neither can he give us any better than even odds that the next coin will come up tails if the coin toss is truly unbiased.

I have no real answer to this unease, other than “That’s life. Get over it.” For me the beautiful way in which linear algebra generates a new kind of probability theory, even if we choose to ignore it and declare it illogical<sup>11</sup>, is some solace for being faced with a question to which, perhaps, my intrinsic makeup prevents me from getting an intuitively satisfying answer. On the other hand, I believe that discomfort with an intrinsically probabilistic formulation of fundamental laws is the only “mystery” of quantum mechanics. If someone told me that the fundamental theory of the world was classical mechanics, with a fixed initial probability

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<sup>11</sup>One can easily imagine an alternate universe, in which a gifted mathematician discovered the non-commutative probability theory of quantum mechanics, and speculated that it might have some application to real measurements, long before experimental science discovered quantum mechanics.

distribution, I would feel equally uncomfortable. The fact that the laws of probability for micro-systems don't obey our macroscopic "logic" points only to facts about the forces driving the evolution of our brains. If we had needed an intuitive understanding of quantum mechanics to obtain an adaptive advantage over frogs, we, or some other organism, would have developed it. Perhaps we can breed humans who have such an intuitive understanding by making the right to reproduce contingent upon obtaining tenure at a physics department. Verifying the truth of this conjecture would take a long time, but much less than time than it would take to observe quantum correlations in a superposition of macro-states.

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