Towards Quantum Superpositions of a Mirror

D. Bouwmeester

7 Pines, May 9, 2010

$$\left|\Psi\right\rangle = \alpha\left|\mathrm{UCSB}\right\rangle + \beta\left|\mathrm{Leiden}\right\rangle$$

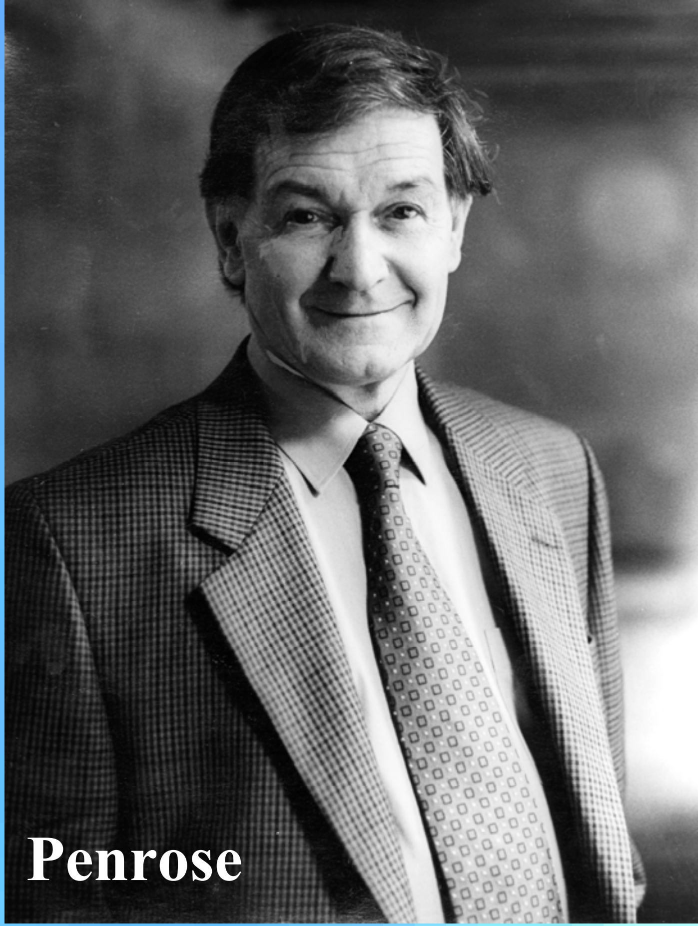


Oxford 2001

Towards Quantum Superpositions of a Mirror

William Marshall,[1,2] Christoph Simon,[1] Roger Penrose,[3,4] and Dik Bouwmeester[1,2]

Twistor Theory



Twistor Theory

$V^a \in M$ (Minkowski space) with components (V^0, V^1, V^2, V^3)

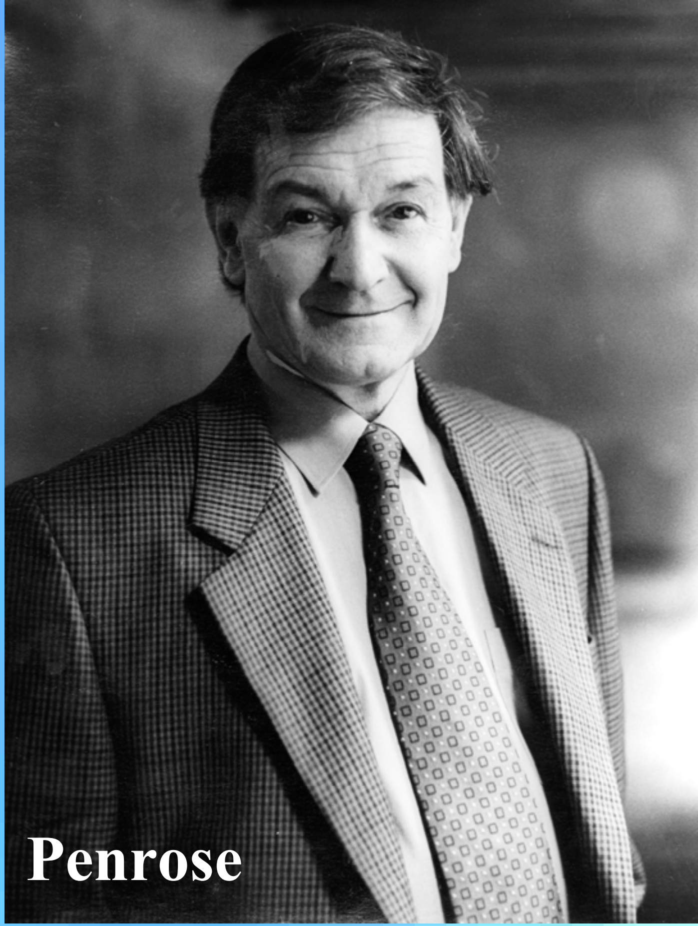
$$\Rightarrow V^{AA'} = \begin{bmatrix} V^{00'} & V^{01'} \\ V^{10'} & V^{11'} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} V^0 + V^3 & V^1 + iV^2 \\ V^1 - iV^2 & V^0 - V^3 \end{bmatrix}, \quad \text{Det} = 4-\text{interval}$$

$$V^{AA'} \to \tilde{V}^{AA'} = t^A{}_B V^{BB'} \overline{t}_{B'}{}^{A'}, \text{ where } \begin{bmatrix} t^0{}_{0'} & t^0{}_{1'} \\ t^1{}_{0'} & t^1{}_{1'} \end{bmatrix} \in SL(2,C), \text{and } \overline{t}_{B'}{}^{A'} = \overline{t^A{}_B}$$

$SL(2,C) \to L_+^\uparrow$ (Lorentz group) is 2-1 isomorphism.

example 1: Lorentz boost in z-direction: $t = \begin{bmatrix} e^{\frac{\varphi}{2}} & 0 \\ 0 & e^{-\frac{\varphi}{2}} \end{bmatrix}$

example 2: Rotation through φ in the x-y plane: $t = \begin{bmatrix} e^{\frac{i\varphi}{2}} & 0 \\ 0 & e^{-\frac{i\varphi}{2}} \end{bmatrix}$



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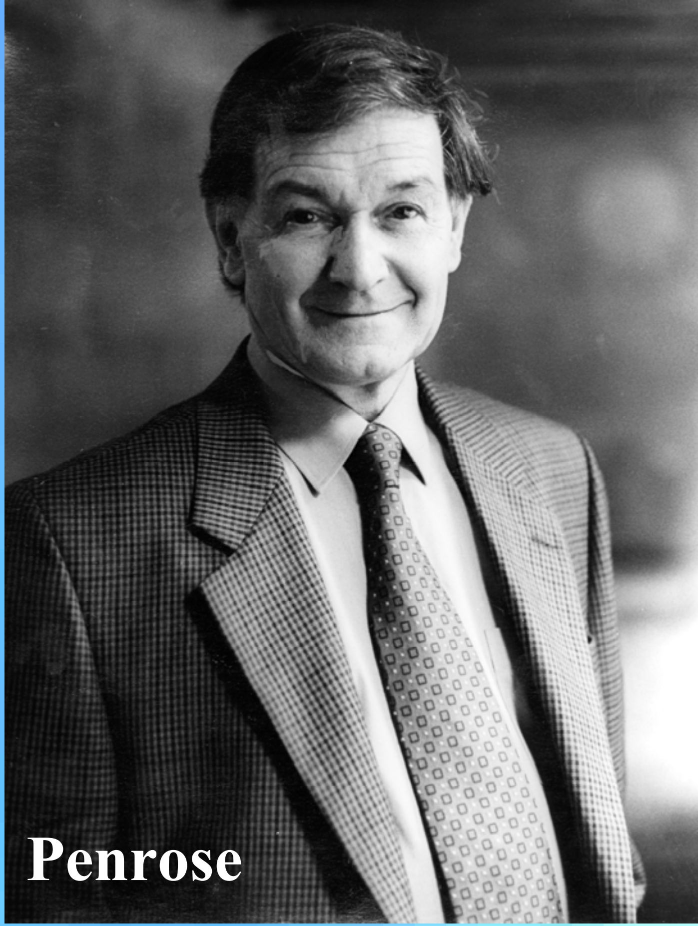

Penrose

Note: rotation by 2π gives -I in $SL(2,C)$, rotation by 4π gives I in $SL(2,C)$,

Twistor Theory

if $\det V^{AA'} = 0$, $\begin{bmatrix} V^{00'} & V^{01'} \\ V^{10'} & V^{11'} \end{bmatrix} = \begin{bmatrix} \alpha^0\bar{\alpha}^{0'} & \alpha^0\bar{\alpha}^{1'} \\ \alpha^1\bar{\alpha}^{0'} & \alpha^1\bar{\alpha}^{1'} \end{bmatrix} = \alpha^A\bar{\alpha}^{A'}$

Note:

spinor α^A determined up to phase factor by this construction

$\Rightarrow$ intrinsic quantum mechanical features

Note:

SL(2,C) acts on spinor α^A

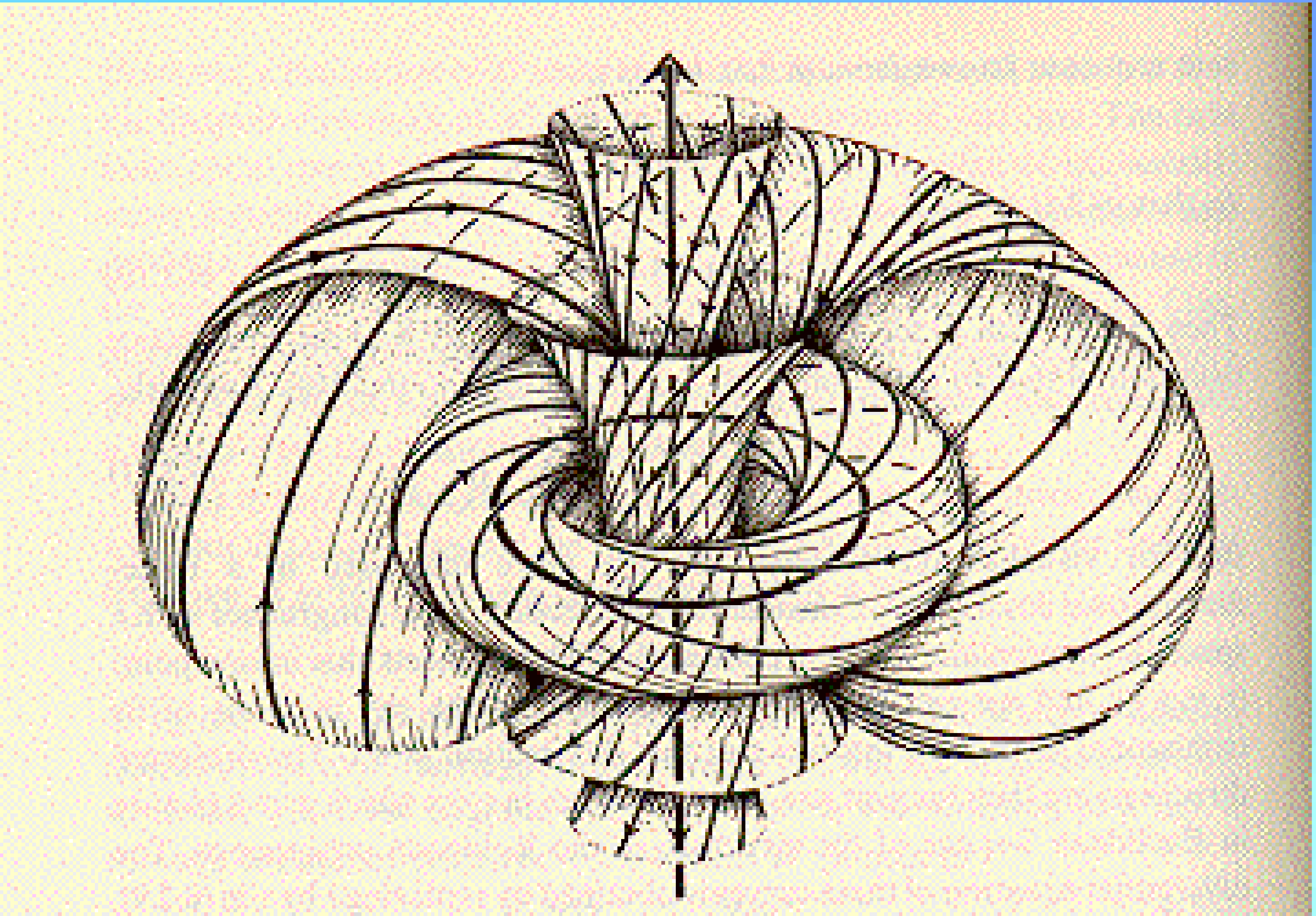
$\Rightarrow$ intrinsic fermionic properties

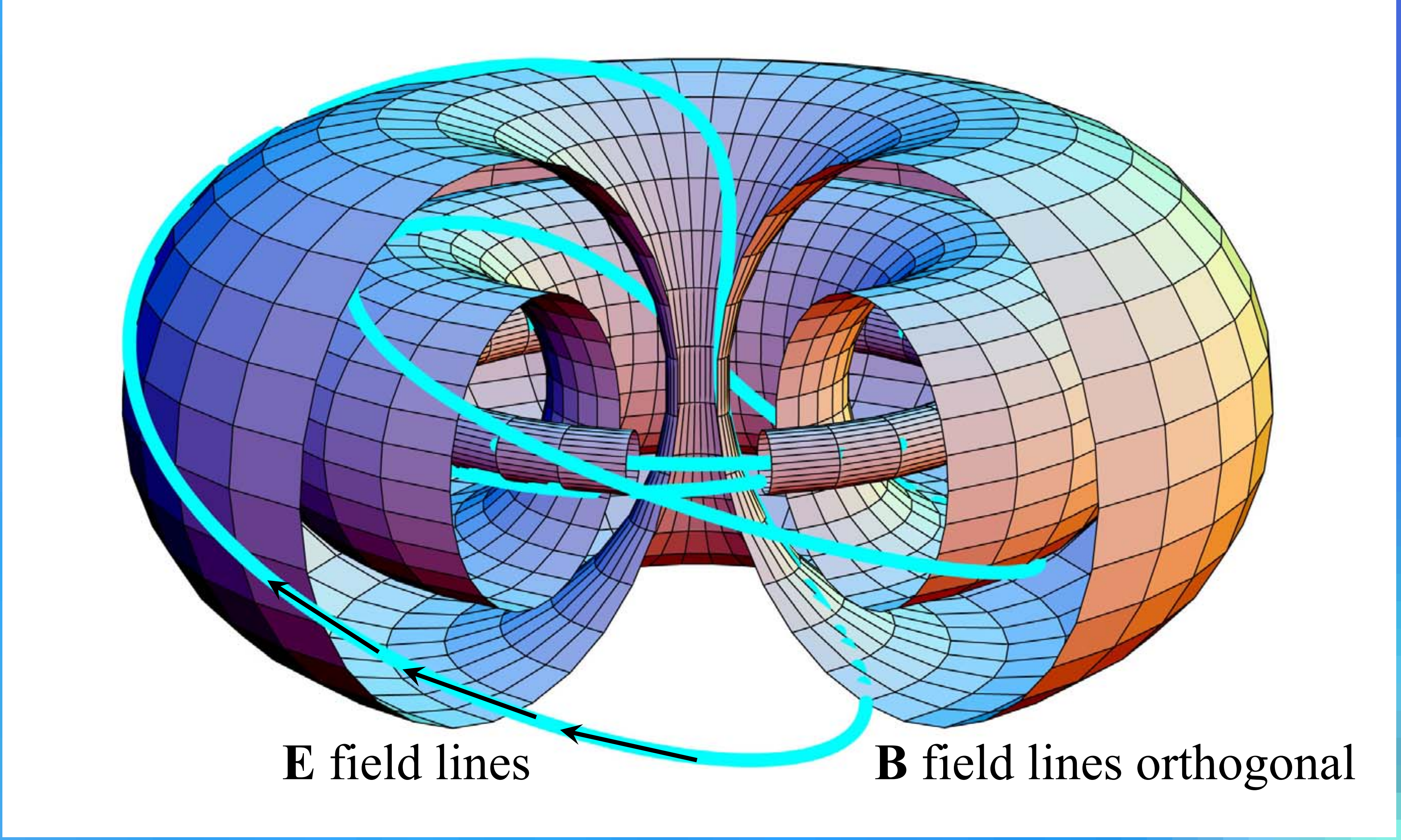
A twistor $Z^\alpha = \left(\omega^{A}, \pi_{A'}\right) \in$ T(4-complex dimensional twistor space) defines a spinor field $\Omega^A(x)$ in M by $\Omega^A(x) = \omega^{A} - ix^{AA'}\pi_{A'}$

$\Omega^A(x) = 0$ defines planes in complexified compactified Minkowski space. If this plane intersects with real M the resulting line is a null geodesic and Z^α is called null .

For Z^α non-null we can get a visualization of the twistor by drawing the null geodesics corresponding to null twistors that are "orthogonal" to it $\Rightarrow$ Robinson congruence of null geodesics.

Robinson congruence: visualization of a (non-null) twistor:





KNOTS OF LIGHT

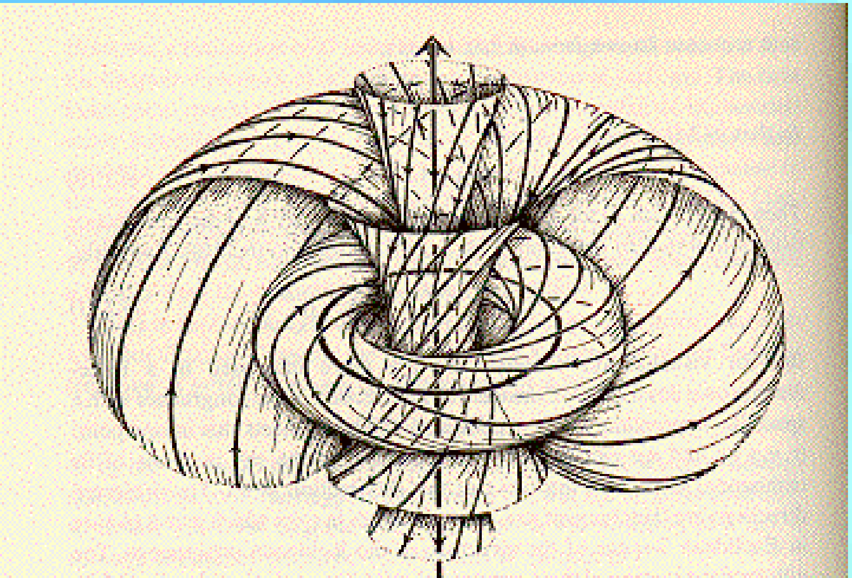
A.F. Ranada and J.L. Trueba, Phys. Lett. 232 A, 25 (1997).

William Irvine and Dirk Bouwmeester, Nature Physics, September 2008

Twistor theory

Space-time is a secondary concept and has:

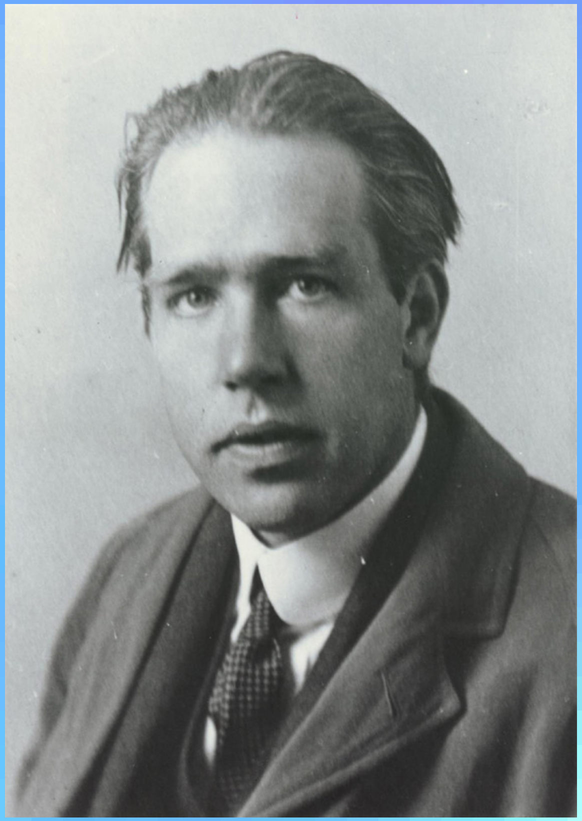
- **naturally 3 space - 1 time coordinates**
- **intrinsic fermionic properties**
- ***intrinsic quantum mechanical properties***
- **should be considered as complexified (and compactified and conformally invariant)**
- **has elementary solutions to wave equations that are related to Robinson congruences**



Quantum Mechanics

Niels Bohr
Copenhagen interpretation:

The wavefunction $|\Psi\rangle$ is not to be taken seriously as describing a quantum level physical reality, but is to be regarded as merely referring to our knowledge of the system.



Quantum Measurements

Zurek (and others):

Environment Induced Decoherence

Caldeira-Leggett model (and others) assumes a linear coupling between the position of the system and a bath of harmonic oscillators

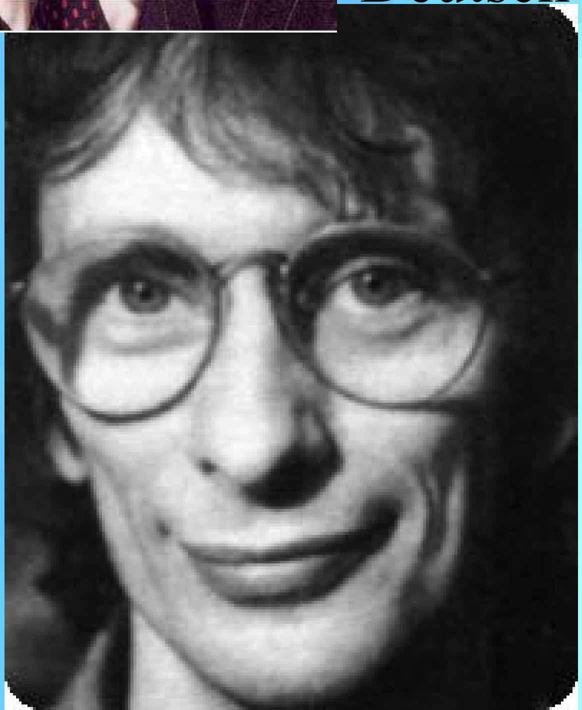
Stamp (and others) considers coupling to spin bath

The wavefunction $|\Psi\rangle$ is a representation of a *real* physical state.

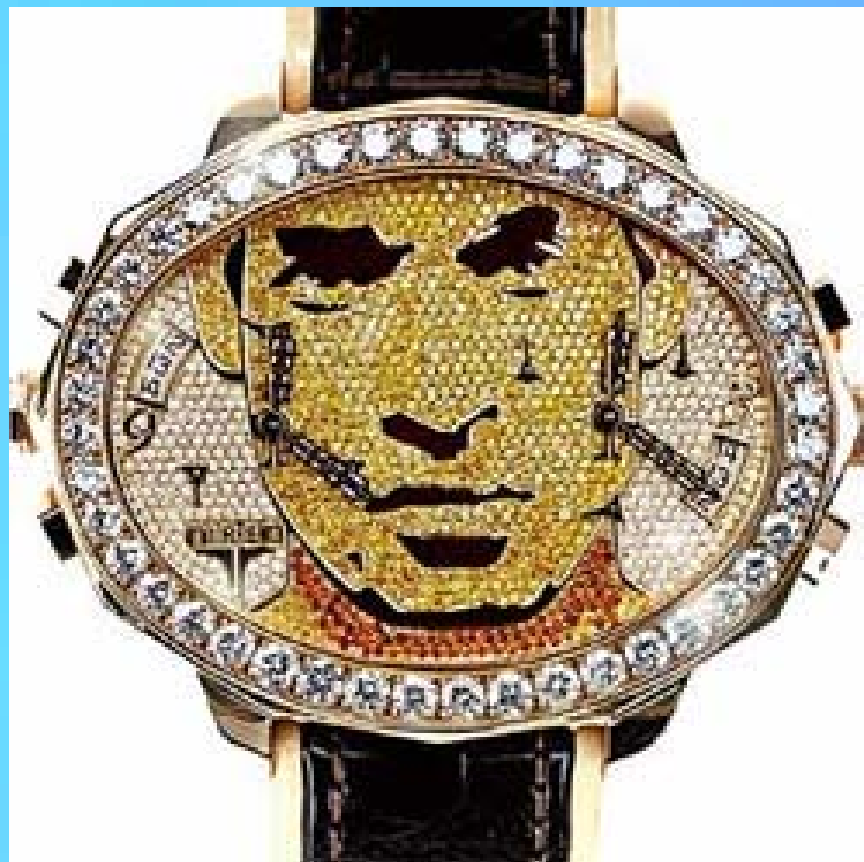
Everett

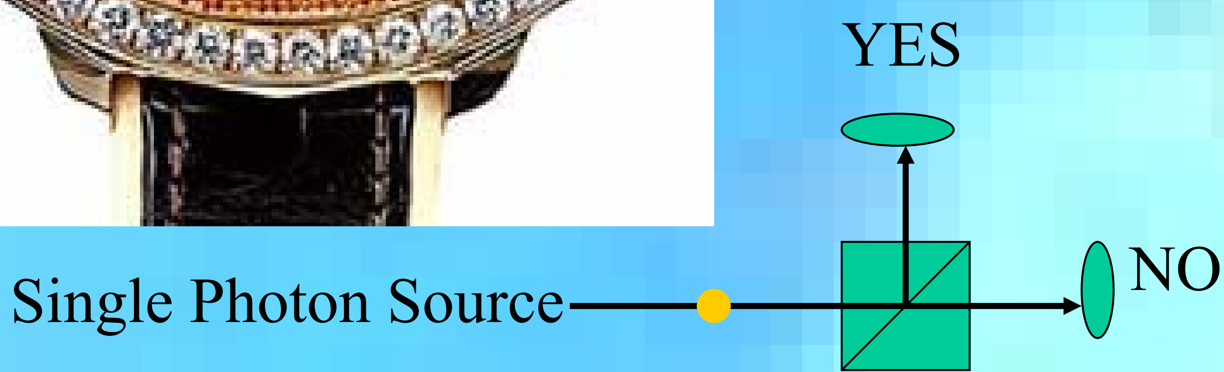
Many Worlds Interpretation

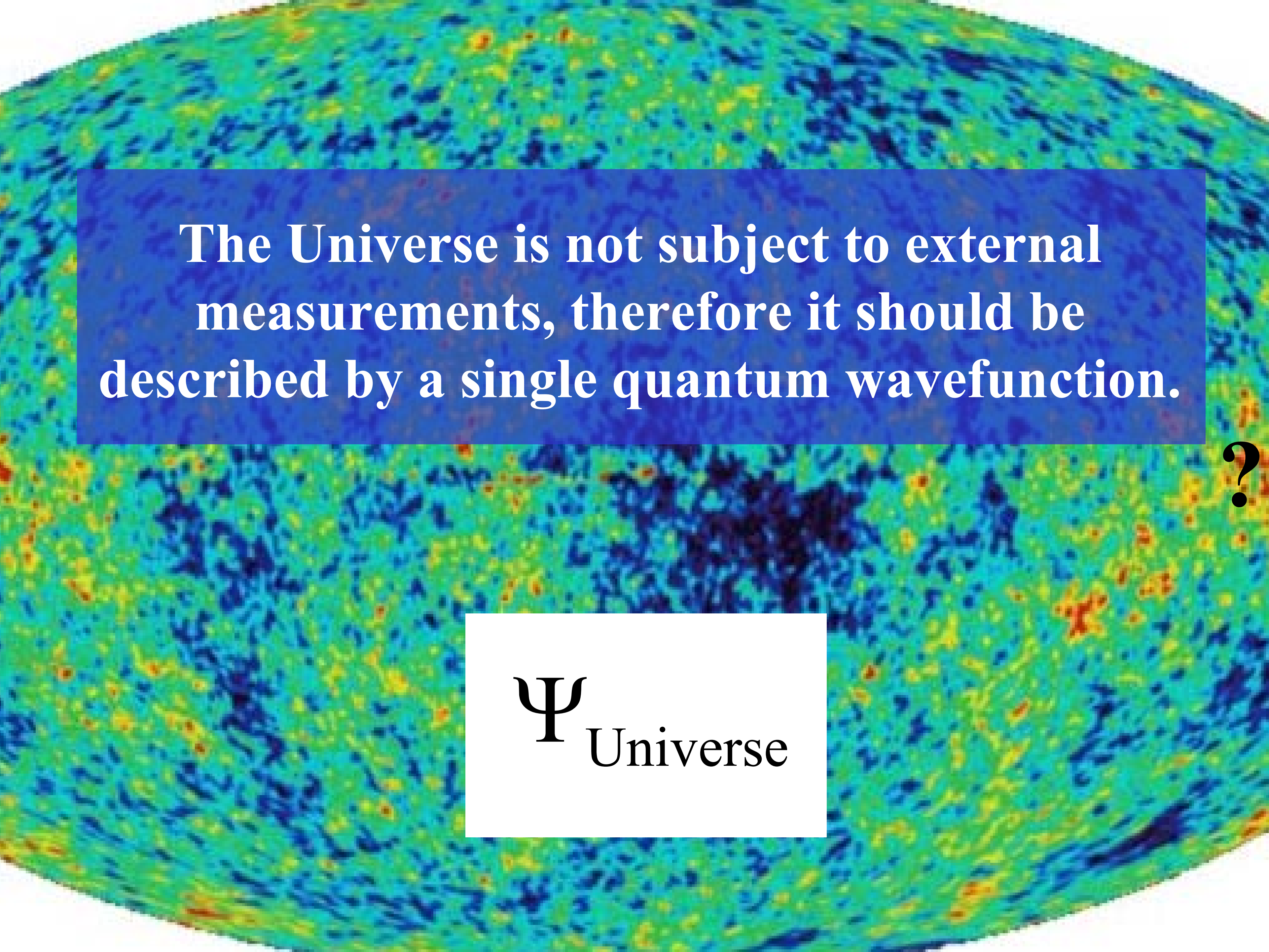
Deutsch



Vaidman's watch







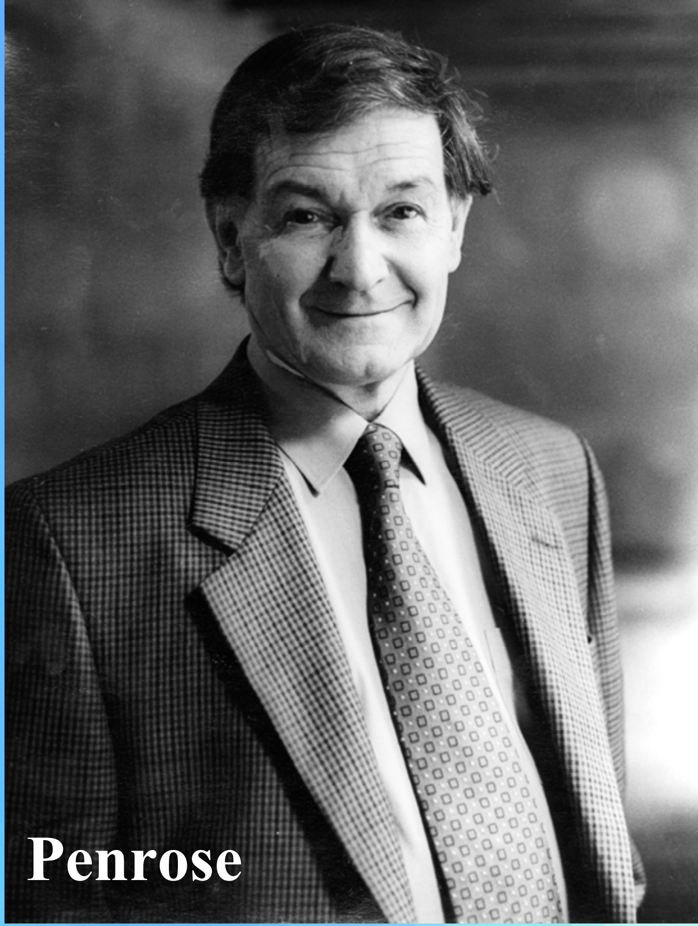
The Universe is not subject to external measurements, therefore it should be described by a single quantum wavefunction.

?

$$\Psi_{\text{Universe}}$$

Penrose:

There is a conflict between Einstein’s general covariance principle and the quantum superposition principle.



Two alternative locations of a massive object will each have stationary states, and have wavefunctions $|\Psi\rangle$ and $|\Phi\rangle$, that are eigenstates of the $\frac{\partial}{\partial t}$ operator with eigenvalues related to the energy.

$$\frac{\partial}{\partial t}|\Psi\rangle = -i\hbar E_{\Psi}|\Psi\rangle$$

$$\frac{\partial}{\partial t'}|\Phi\rangle = -i\hbar E_{\Phi}|\Phi\rangle$$

But how to deal with superpositions

$$\left.\frac{\partial}{\partial t}\right)\left(\alpha|\Psi\rangle + \beta|\Phi\rangle\right) = ???$$

Consider an equal superposition $\frac{1}{\sqrt{2}}\left(|\Psi\rangle + |\Phi\rangle\right)$

f and **f'** are the acceleration 3-vectors of the free-fall motion in the two space-times (**f** and **f'** are gravitational forces per unit test mass).

Penrose postulate: at each point the scalar (|**f-f'**|)^2 is a measure of incompatibility of the identification. The total measure of incompatibility (or "uncertainty) Δ at time *t* is:

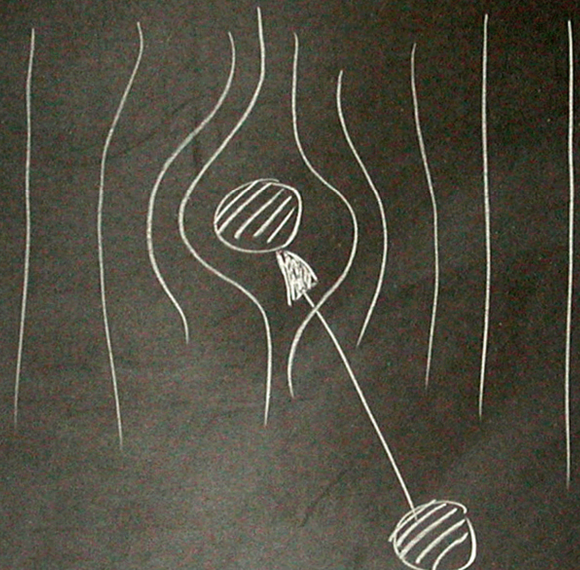
$$\Delta = \frac{1}{4\pi G}\int\left(\mathrm{f}\text{-}\mathrm{f}'\right)^2 \mathrm{d}^3x$$

$$\equiv E_G$$

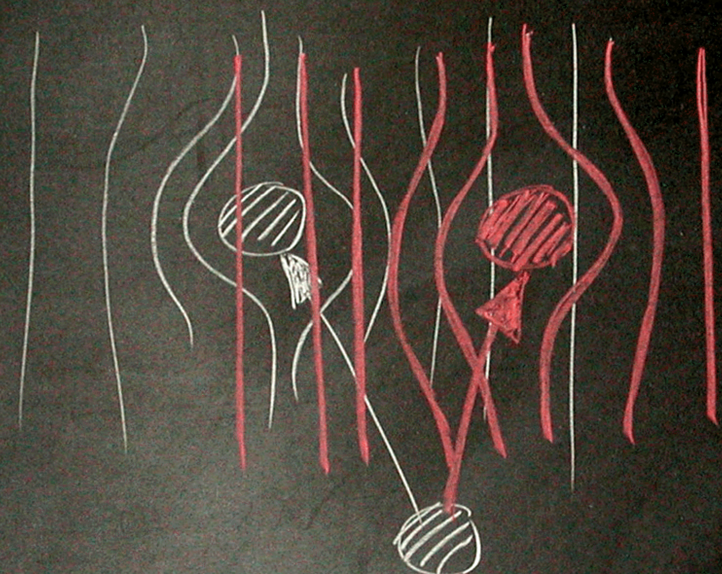
This is the gravitational self energy of the difference between the mass distributions of each of the two lump locations.

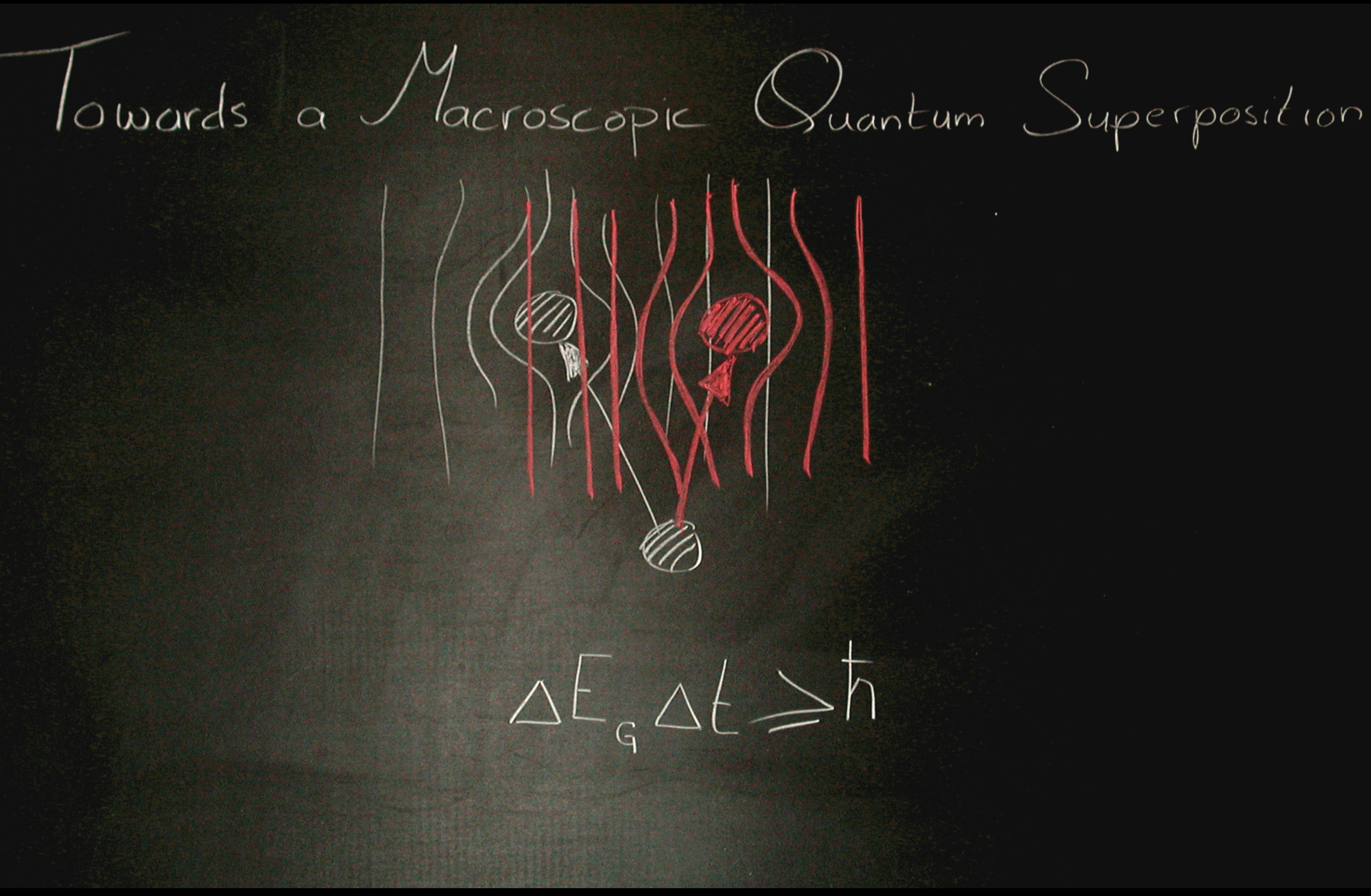
Prediction: The superposition state is unstable and has a lifetime of the order of $\frac{\hbar}{E_G}$

Towards a Macroscopic Quantum Superposition



Towards a Macroscopic Quantum Superposition




Towards a Macroscopic Quantum Superposition
$\Delta E_G \Delta t \geq \hbar$

$$E_{i,j} = -G \int\int d\vec{r}_1 d\vec{r}_2 \frac{\rho_i(\vec{r}_1)\rho_j(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|},$$

$$\Delta E = 2E_{1,2} - E_{1,1} - E_{2,2},$$

$$\Delta E = 2Gmm_1\left(\frac{6}{5a} - \frac{1}{\Delta x}\right), \qquad (\text{given}: \Delta x \geq 2a)$$

$m \sim 10^{-12}$kg,

$\omega_c \sim 1$-10kHz

$\kappa \sim 1$

$m_1 = 4.7 \times 10^{-26}$kg

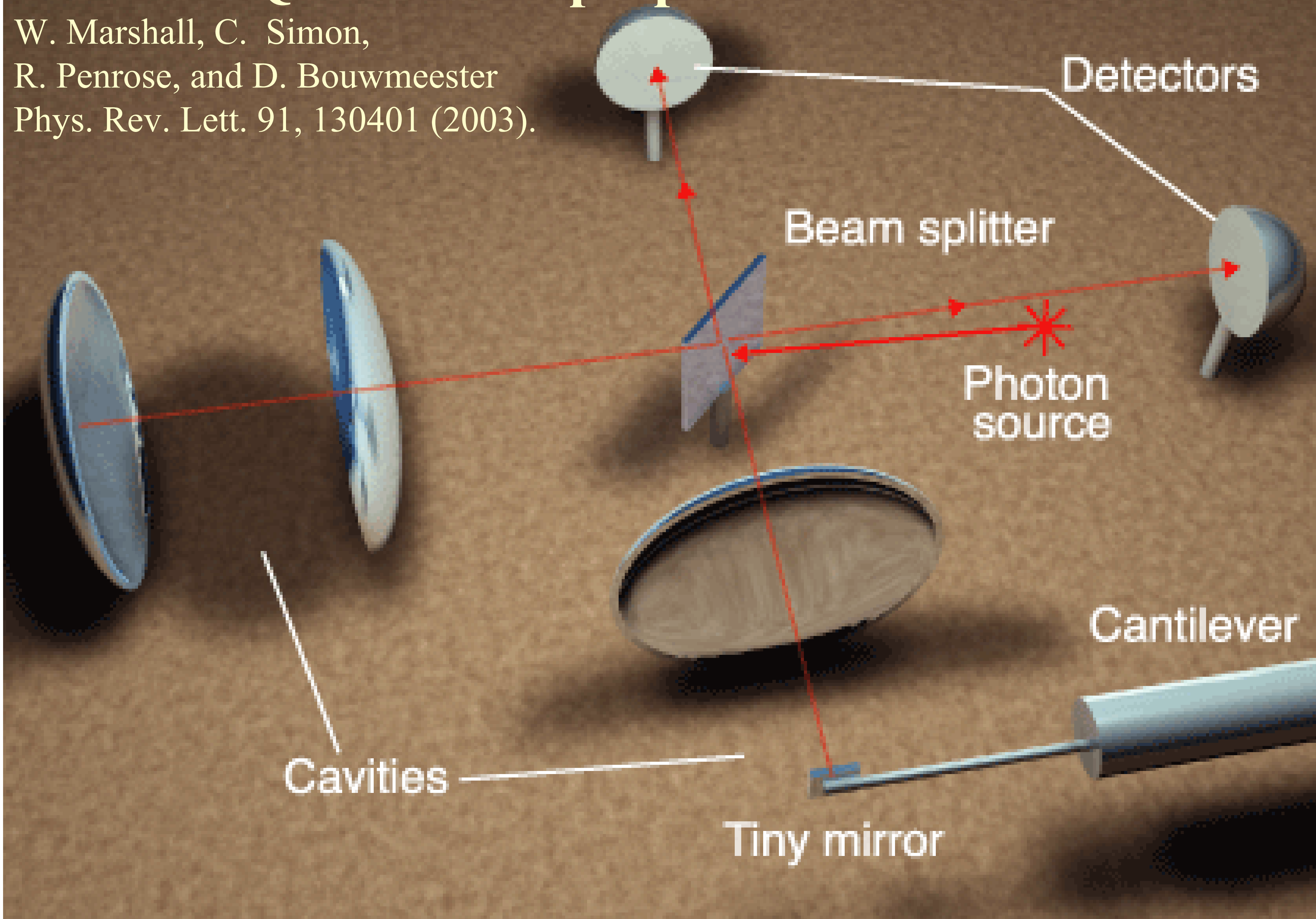
Take, $a = 10^{-15}$m size of nucleus, or size of ground-state wave function

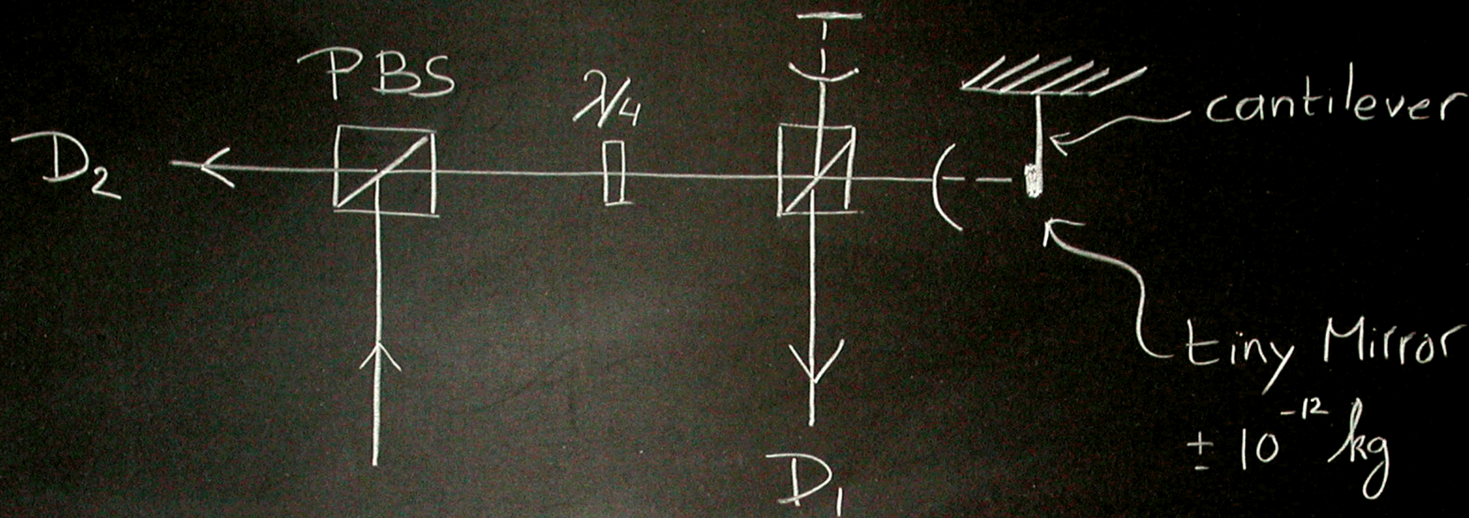
Decoherence time ~1 ms, or ~1s

Compare: For C_{60} experiments decoherence time is 10^{10}s

Towards Quantum Superpositions of a Mirror

W. Marshall, C. Simon,
R. Penrose, and D. Bouwmeester
Phys. Rev. Lett. 91, 130401 (2003).

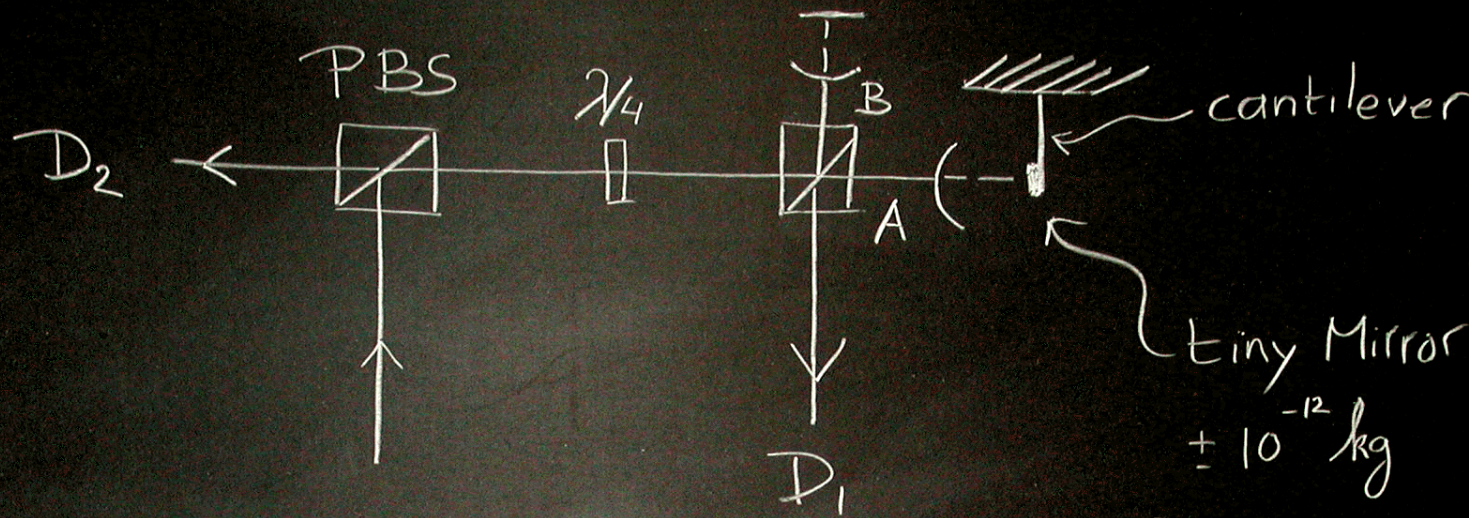




$$\mathcal{H} = \hbar\omega_c a^\dagger a + \hbar\omega_m b^\dagger b - \hbar g a^\dagger a (b + b^\dagger)$$

$$g = \frac{\omega_c}{L}\sqrt{\frac{\hbar}{2M\omega_m}}$$

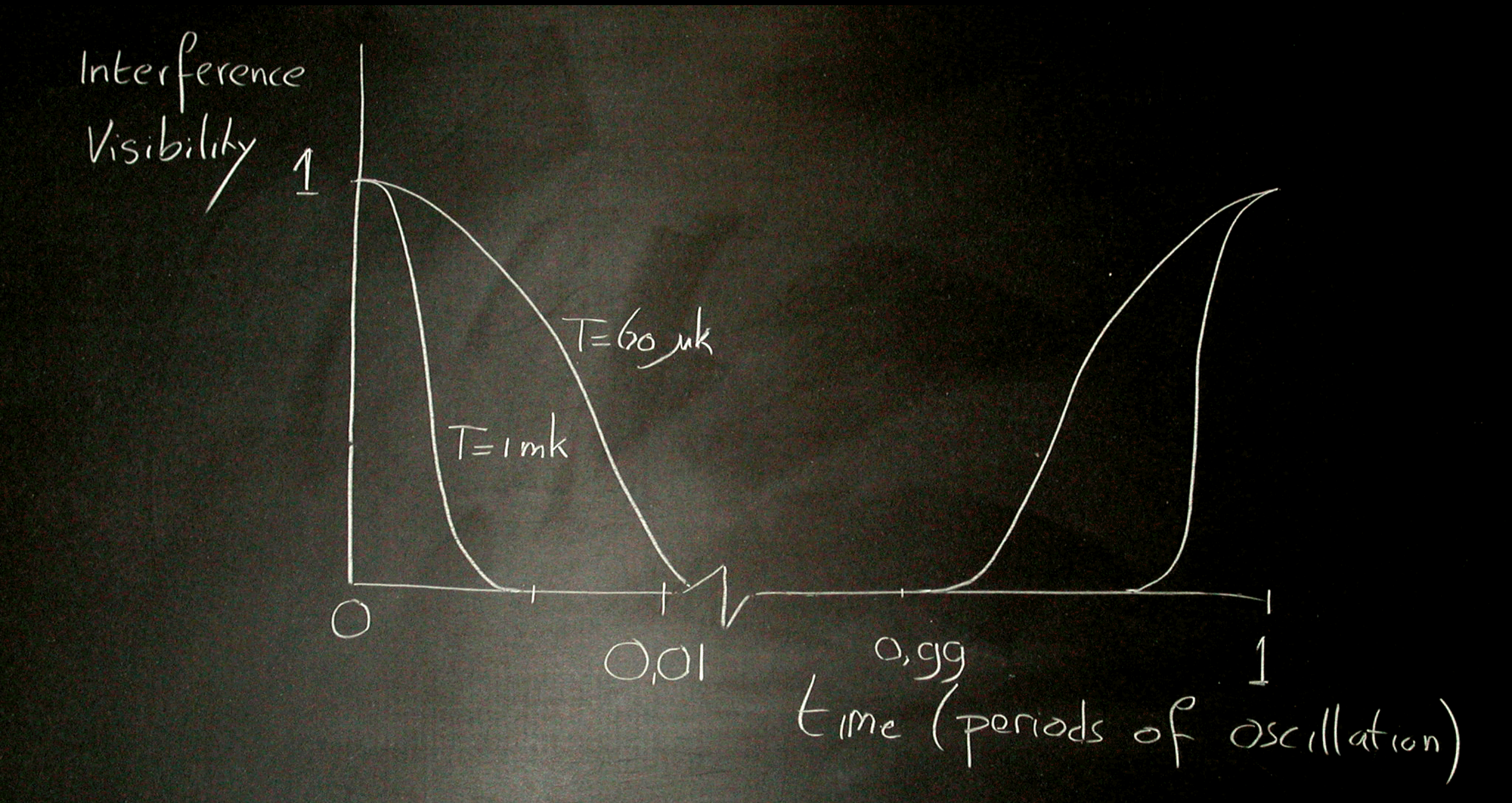
Law, PRA, **49**, 433 (1993)
Bose et al. PRA **59**, 3204 (1999)
Marshall et al. PRL **91**, 130401 (2003)



mirror in coherent state $|\beta\rangle = e^{-\frac{|\beta|^2}{2}} \sum_{n=0}^{\infty} \frac{\beta^n}{\sqrt{n!}} |n\rangle$

Initial state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B\right)|\beta\rangle$

$|\psi(t)\rangle$ = entangled state of mirror and photon
<u>except</u> after full period of oscillation



T is effective temperature of the fundamental resonance of cantilever

Experimental Requirements

1) momentum kick imparted by photon has to be larger than the initial quantum uncertainty of the mirror's momentum

$$\frac{2\hbar N^3 L}{\pi c M \lambda^2} \gtrsim 1$$

Optimum 700 nm $10 \times 10 \times 10\,\mu m$ SiO_2/Ta_2O_5 mirror

$N \sim 10^5 - 10^6$

$L \sim 1-5\,cm$

$\omega_m \sim 2\,kHz \longrightarrow \Delta X_{mirror} = 10^{-13}\,m$

Experimental Requirements

2) environmental decoherence time ~ 1 period

$$\gamma_D = \gamma_m k_B T M (\Delta x)^2 / \hbar^2$$ (Zurek et al)

↑ damping rate cantilever

$$\rightarrow Q = \omega_m / \gamma_m \gtrsim 10^5$$ (@3mK Rugar et al)

Q=150.000 leads to required T<8mK for bulk material

Experimental Requirements

3) Stability of order $\lambda/20N \sim 10^{-14}$ m

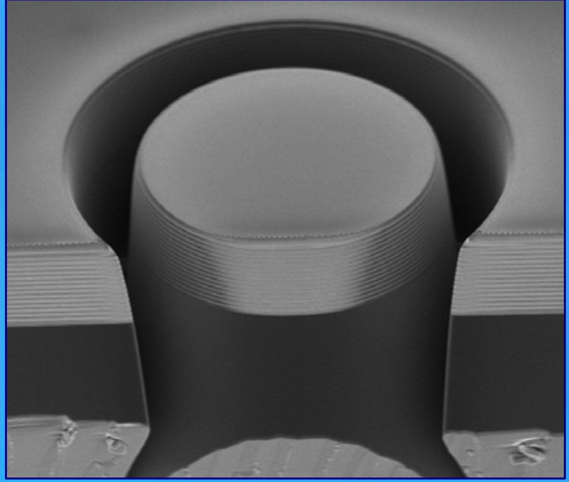
on timescale of experiment.

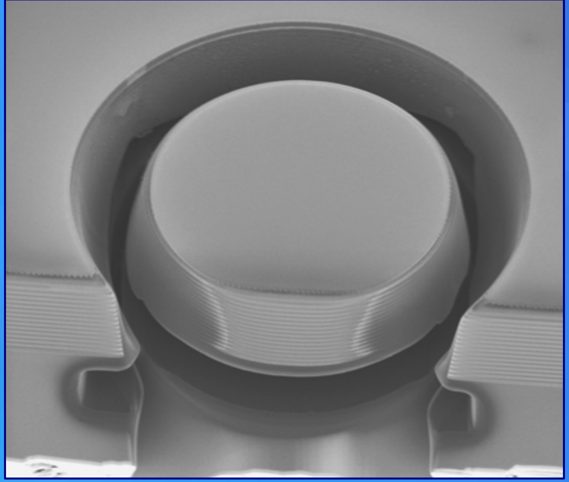
(
STM 10^{-13} m/min

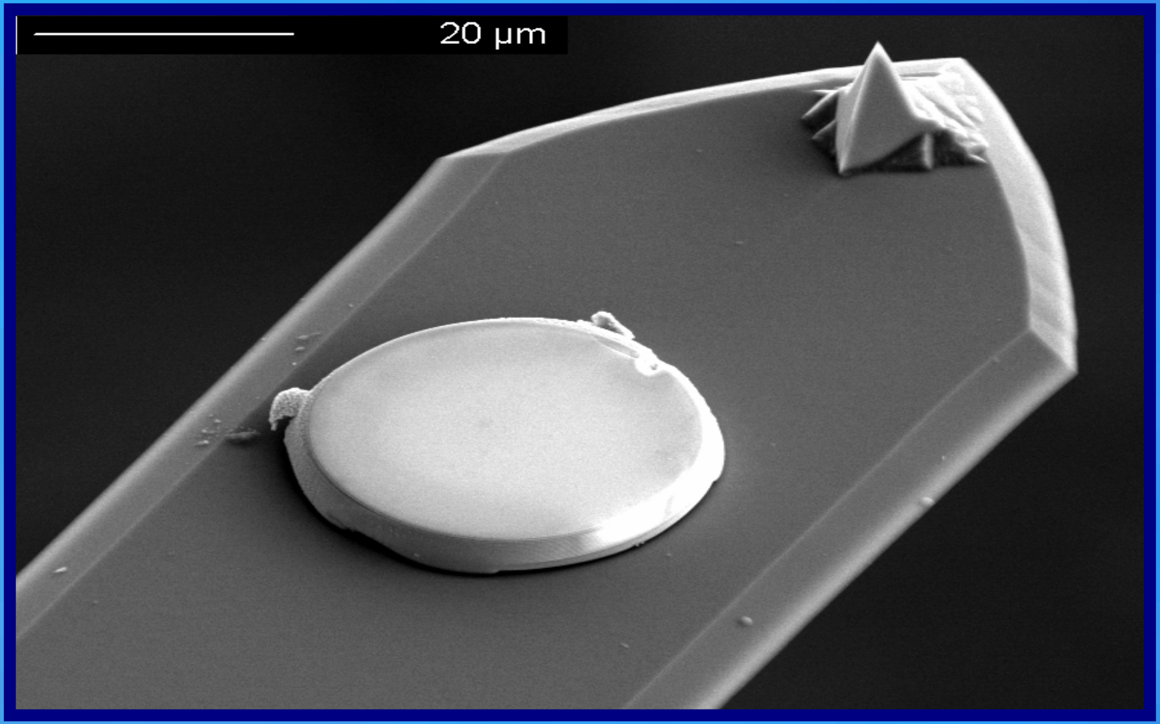
Gravitational wave detection 10^{-19} m/ms
)

Great help: Switchable mirrors

4) UUHV background density ~ 100 particles/cm³

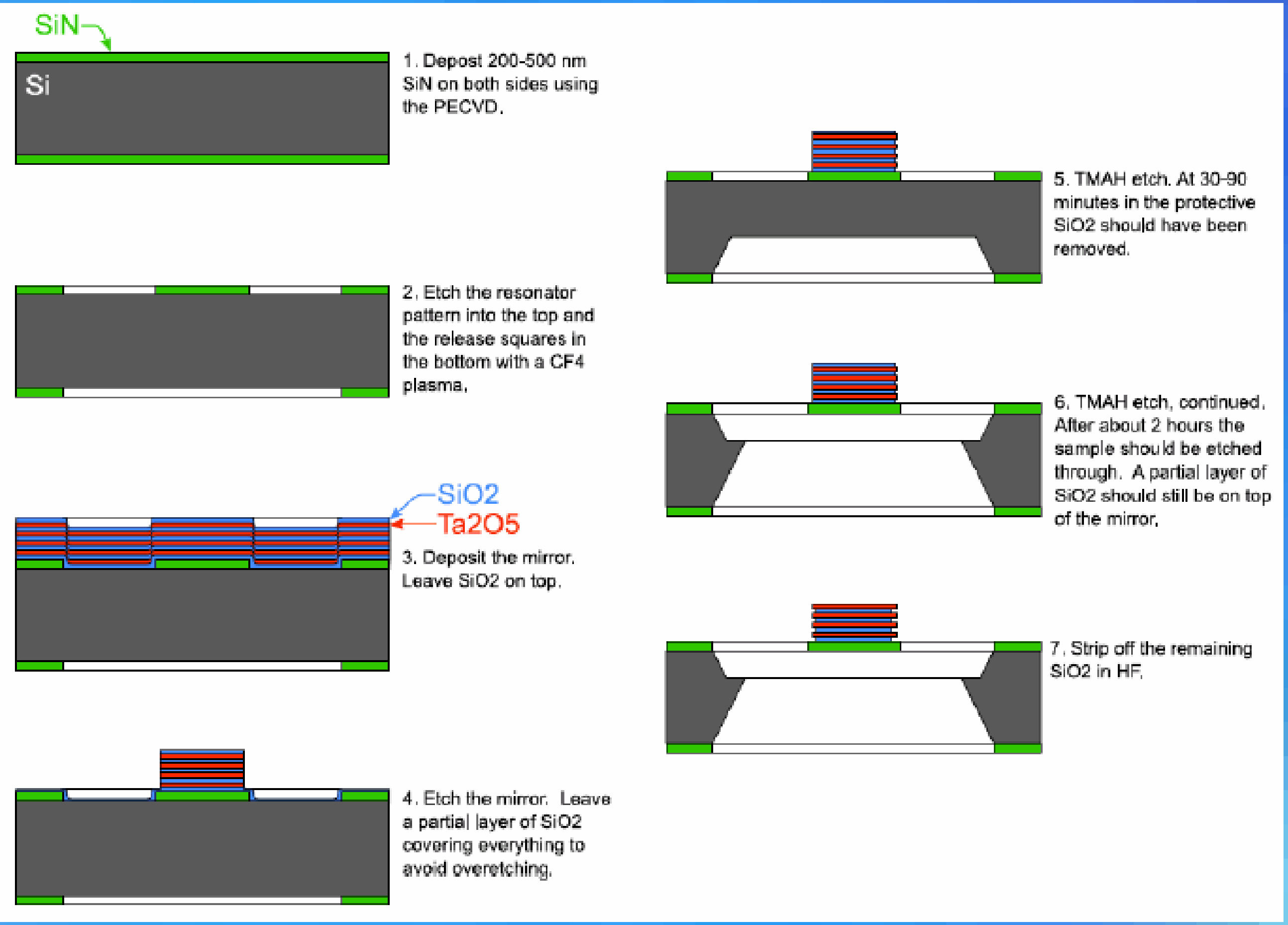


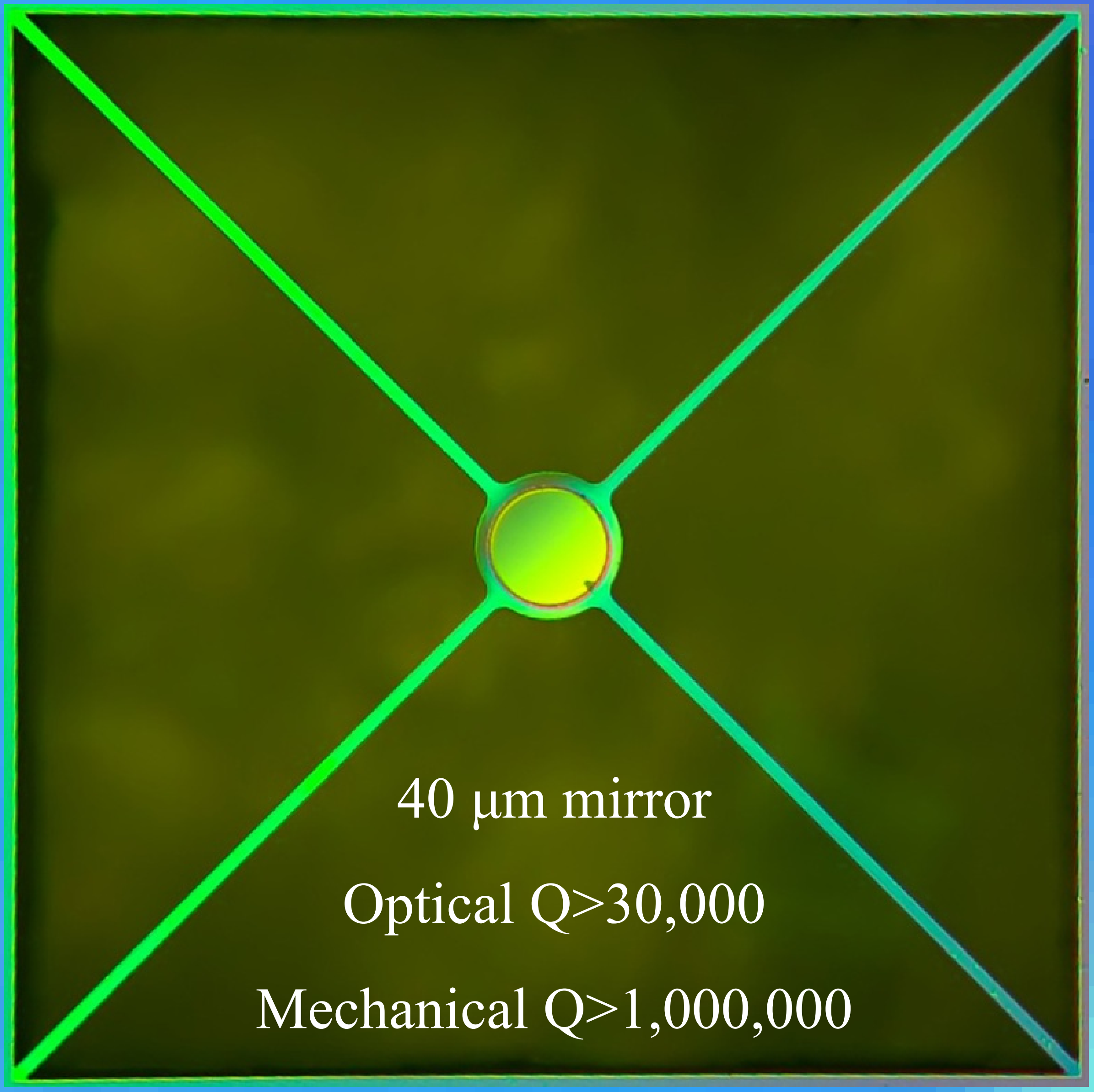




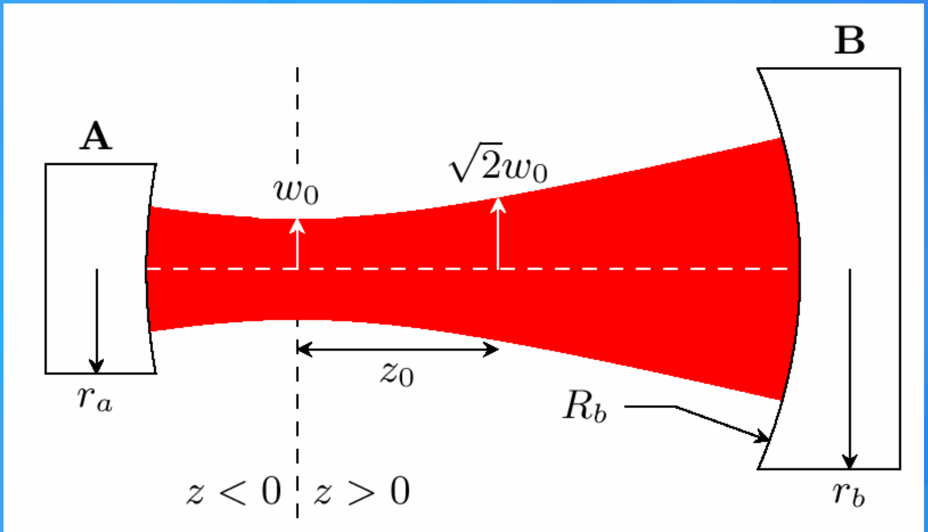
Optical Q=2100
Mechanical Q=137.000
PRL **96**, 173901 (2006)

Dustin Kleckner UCSB, Si_3N_4 based resonator with SiO_2 TaO_5 mirror




40 μm mirror
Optical Q>30,000
Mechanical Q>1,000,000

Simulate diffraction limited finesse

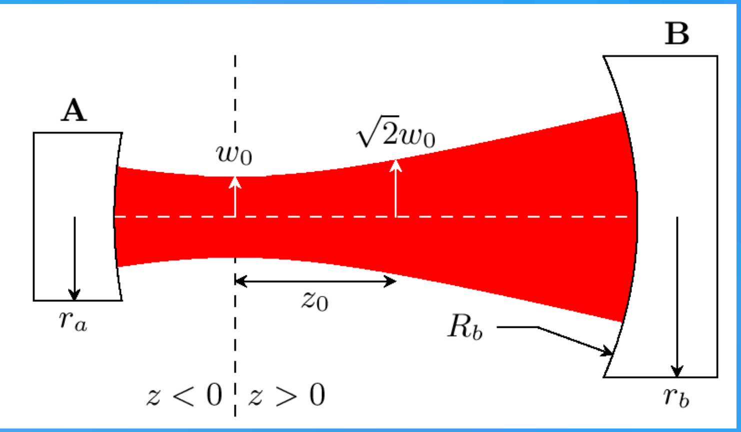


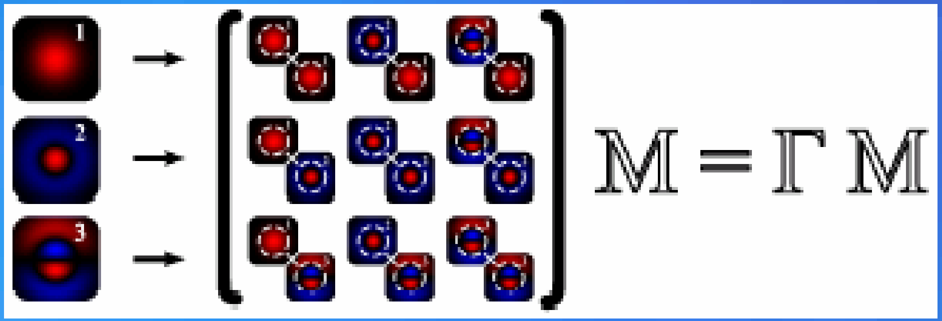
Laguerre Gaussian mode decomposition

$$
\begin{aligned}
E^{\pm}_{n,m}(r,\phi,z) &\propto \left[\frac{r^{|m|}}{w(z)^{|m|+1}}\right] L_n^{|m|}\left[\frac{2r^2}{w(z)^2}\right] \exp\left[-\left(\frac{r}{w(z)}\right)^2 - im\phi \pm i\Theta(r,z)\right] \\
\Theta(r,z) &= (2n+|m|+1)\tan^{-1}\left(\frac{z}{z_0}\right) - k\left(z + \frac{r^2}{2R(z)}\right) \\
z_0 &= \frac{kw_0^2}{2} \\
w(z) &= w_0\sqrt{1+\left(\frac{z}{z_0}\right)^2} \\
R(z) &= z\left[1+\left(\frac{z_0}{z}\right)^2\right]
\end{aligned}
$$

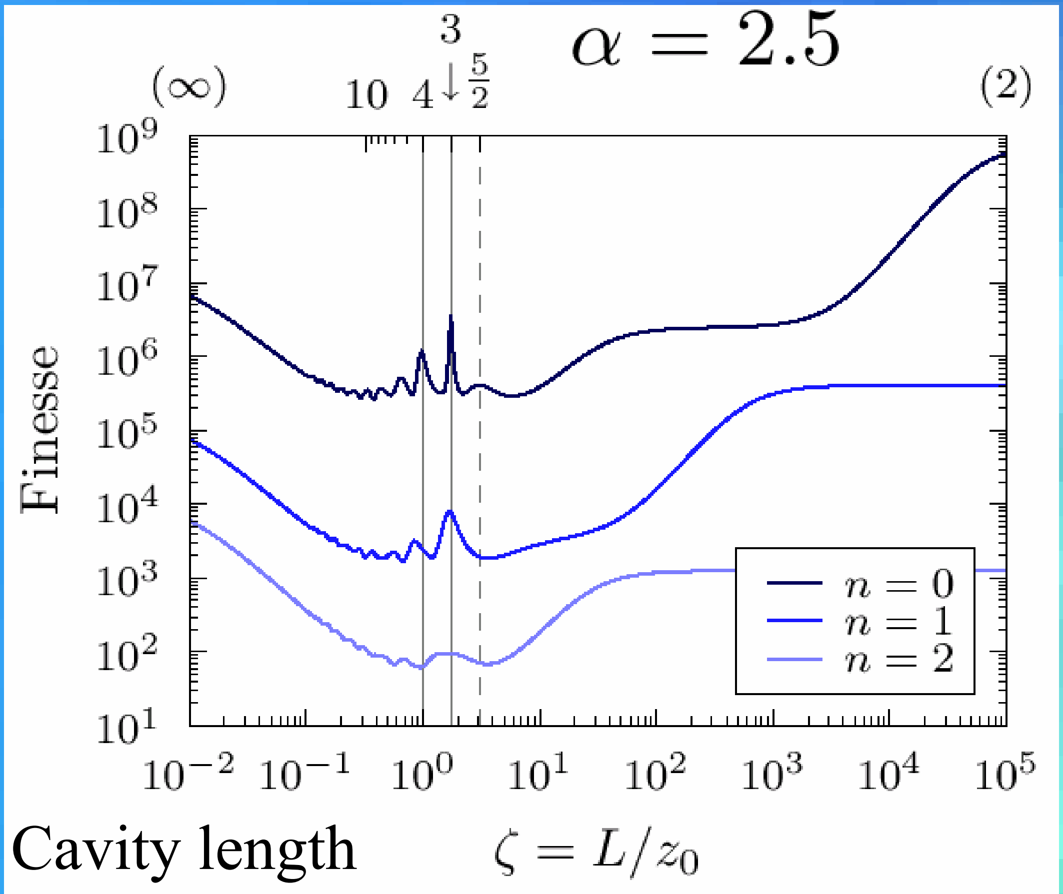
Gouy shift

Simulate diffraction limited finesse

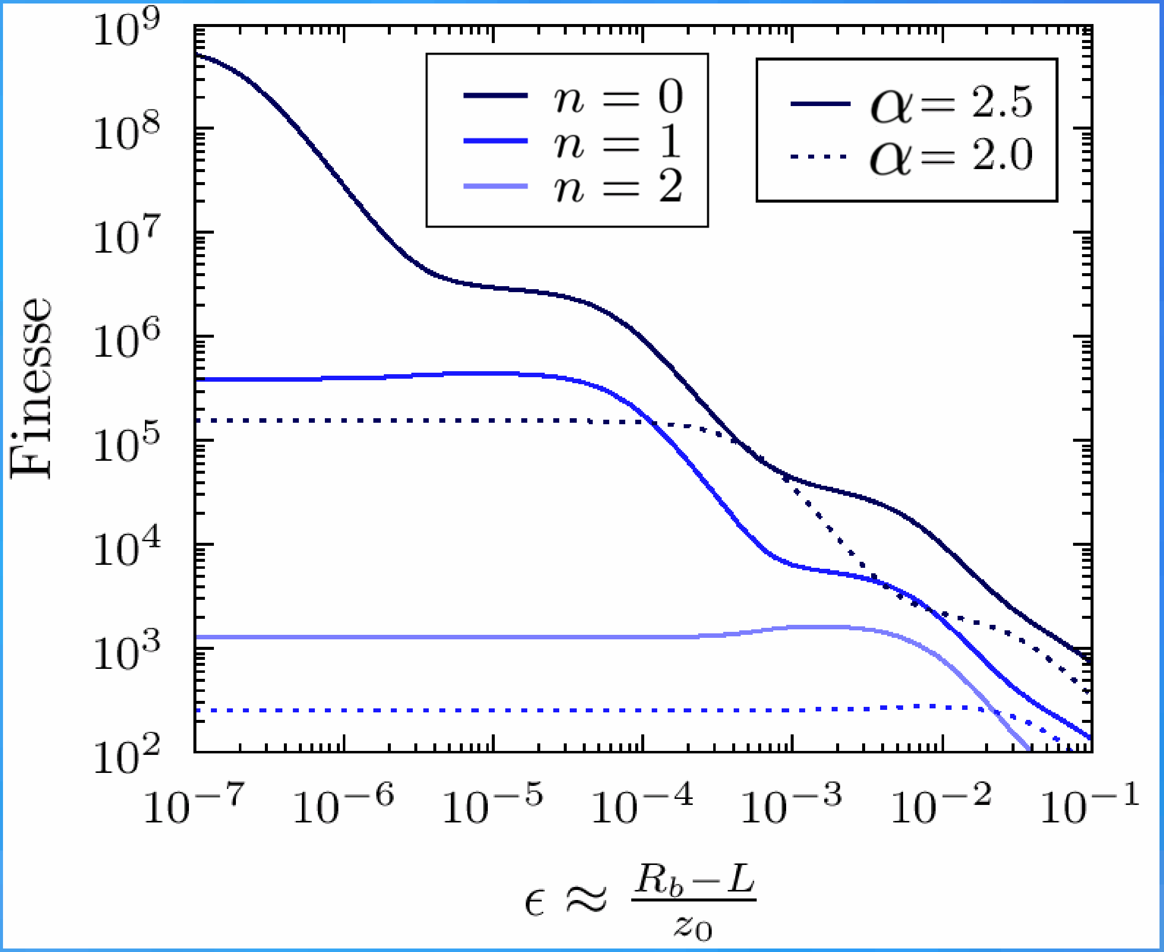




$$
\begin{aligned}
r_a &= \alpha w_0 \\
R_a &= \infty \\
z_a &= 0 \\
r_b &= \alpha w(z_b) \\
R_b &= R(z_b) \\
z_b &= L = \zeta z_0
\end{aligned}
$$



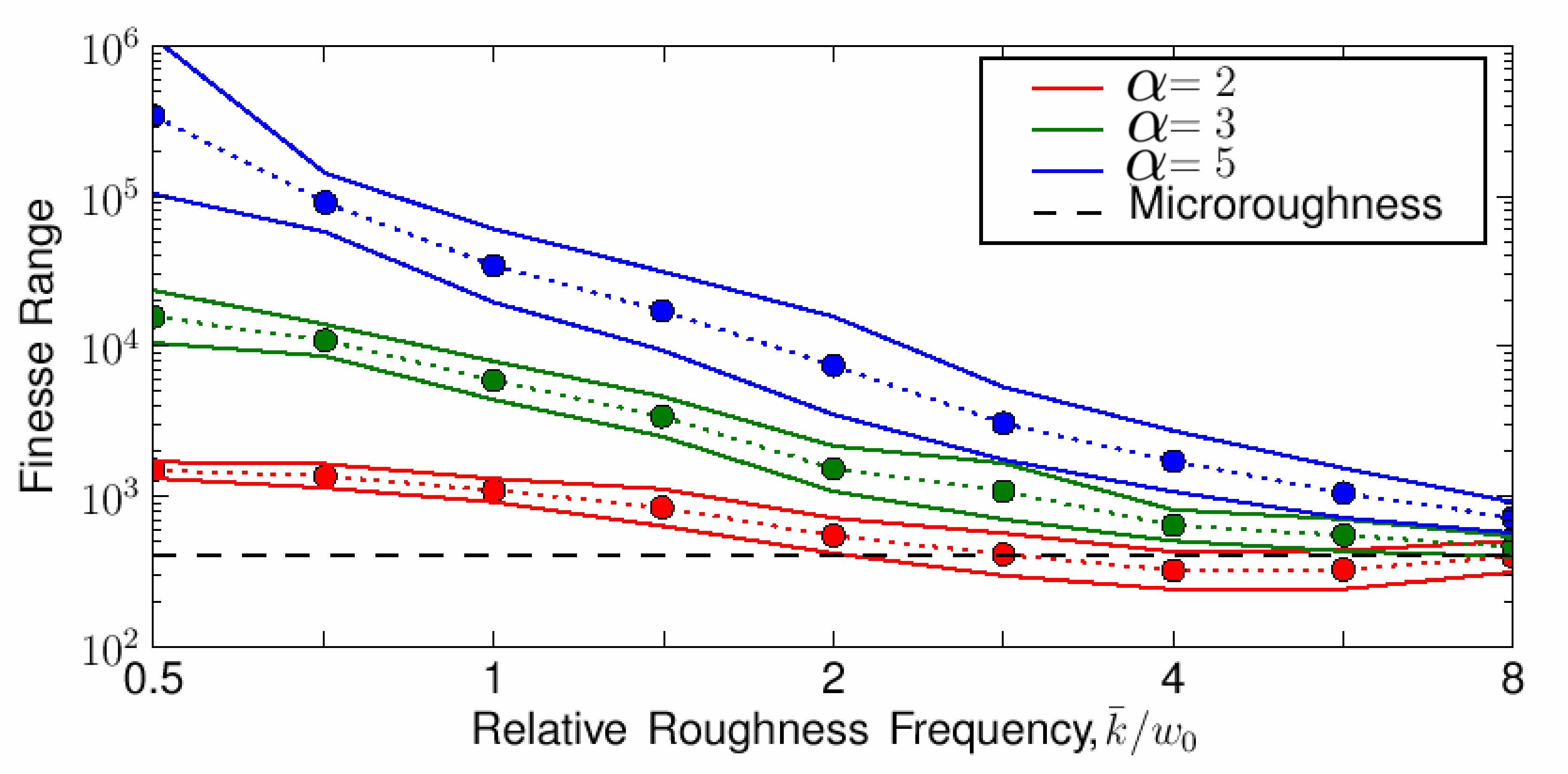
Effect of defocusing: radial phase shift $\exp[-2i\epsilon\rho^2]$



For Finesse 10^6 and z_0=10µm alignment accuracy 1nm required!

Effect of mirror roughness

$\sigma = 10^{-2}\lambda$



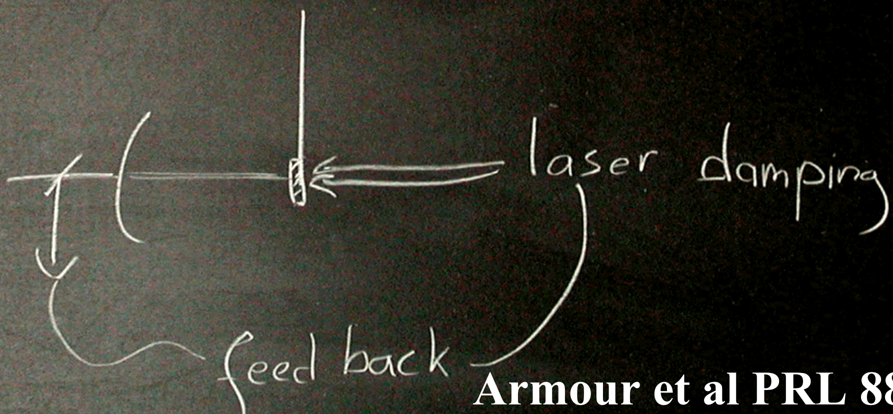
Super polished mirrors might not be good enough

D. Kleckner et al, PRA 2010

Experimental Requirements

2) Cooling

- standard 50 mK
- nuclear demagnetization 50 μK
- optical cooling



⟹ groundstate

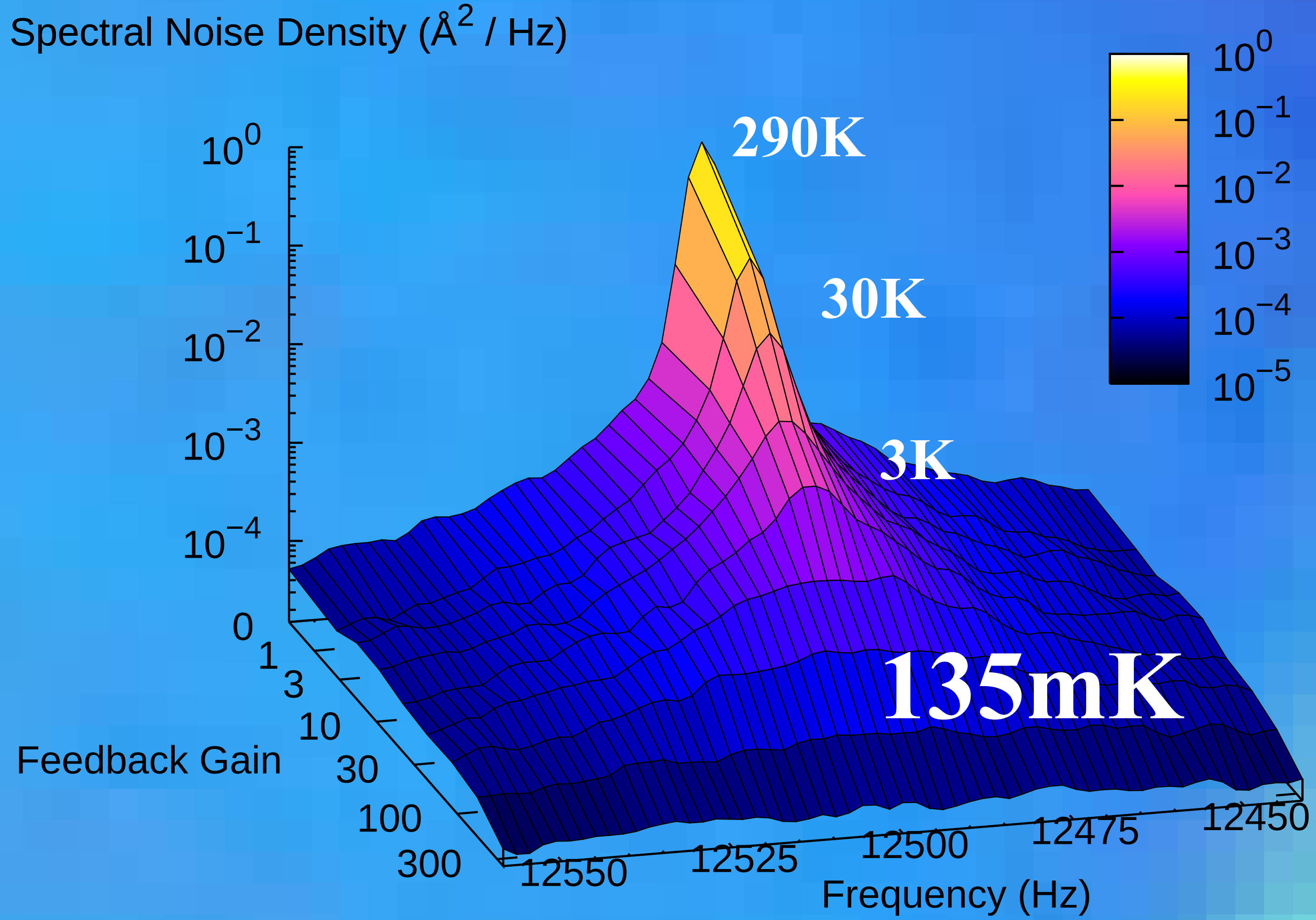
Armour et al PRL 88, 1483010 (02)

Mancini et al PRL 80, 688 (98)

Cohadon et al PRL 83, 3174 (99)

Optical Cooling

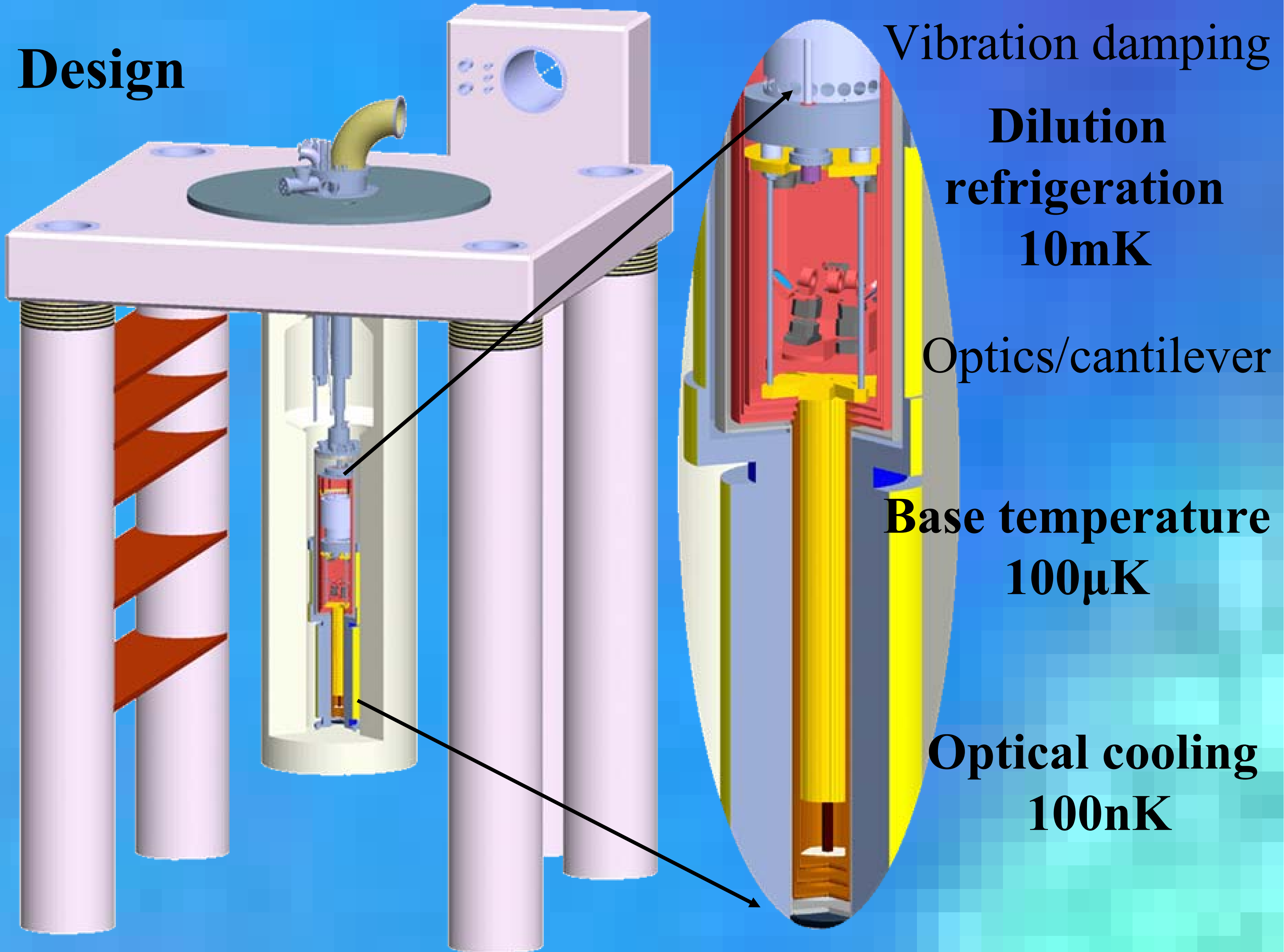
Gain factor 2500

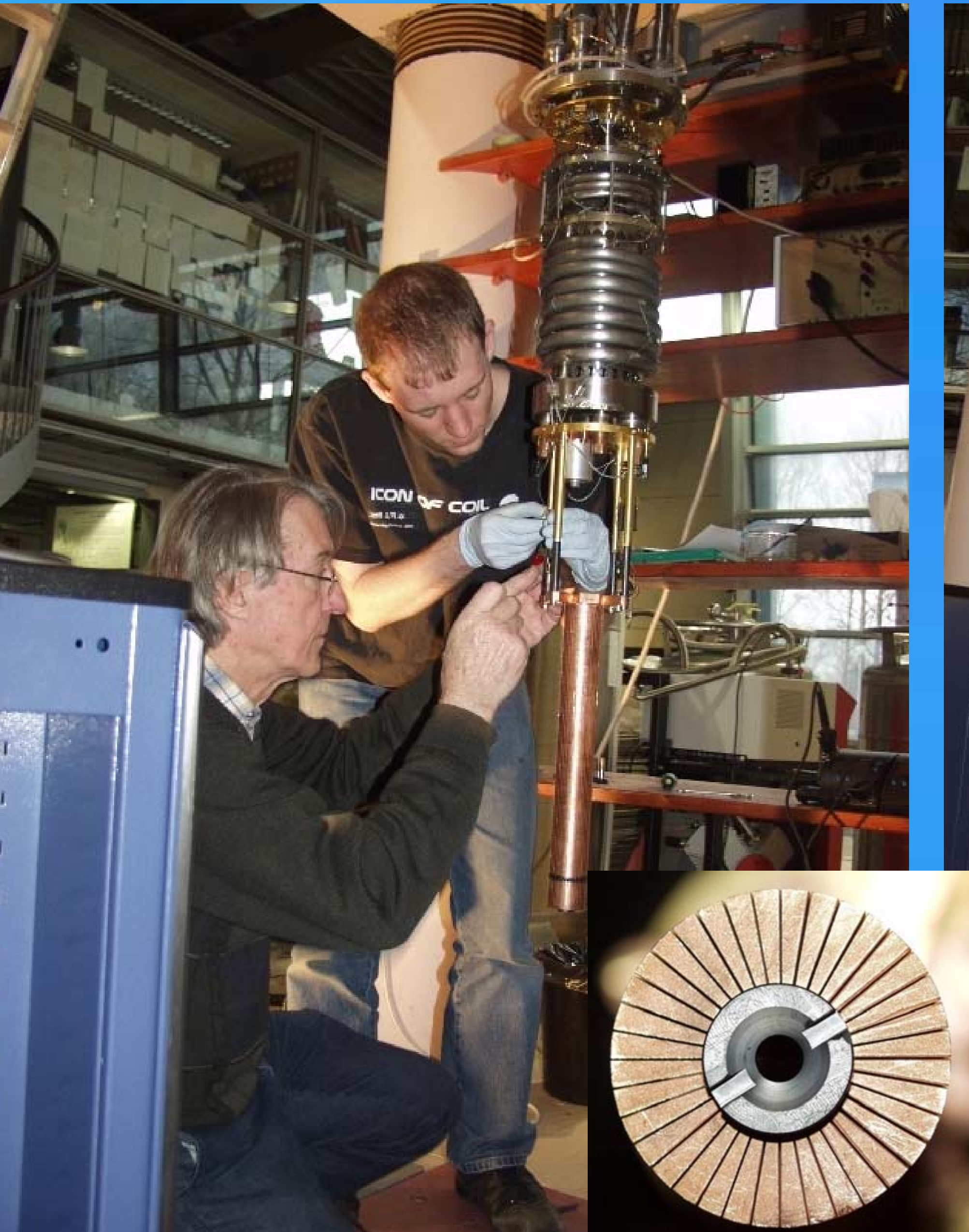


D. Kleckner and D.B. Nature **444**, 75 (2006).

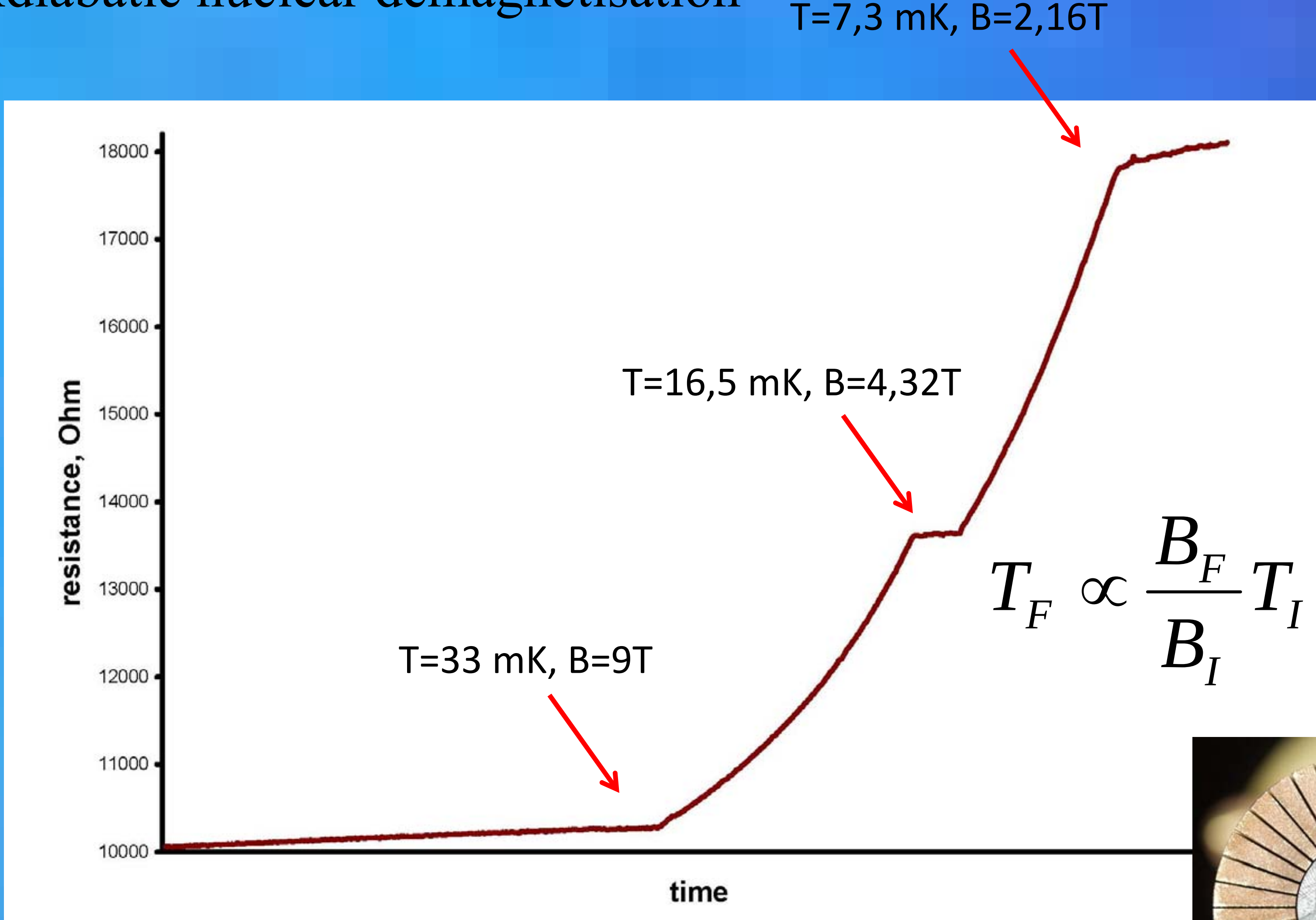
Leiden, the Netherlands

Design


Vibration damping
Dilution refrigeration 10mK
Optics/cantilever
Base temperature 100μK
Optical cooling 100nK


ICON OF COIL

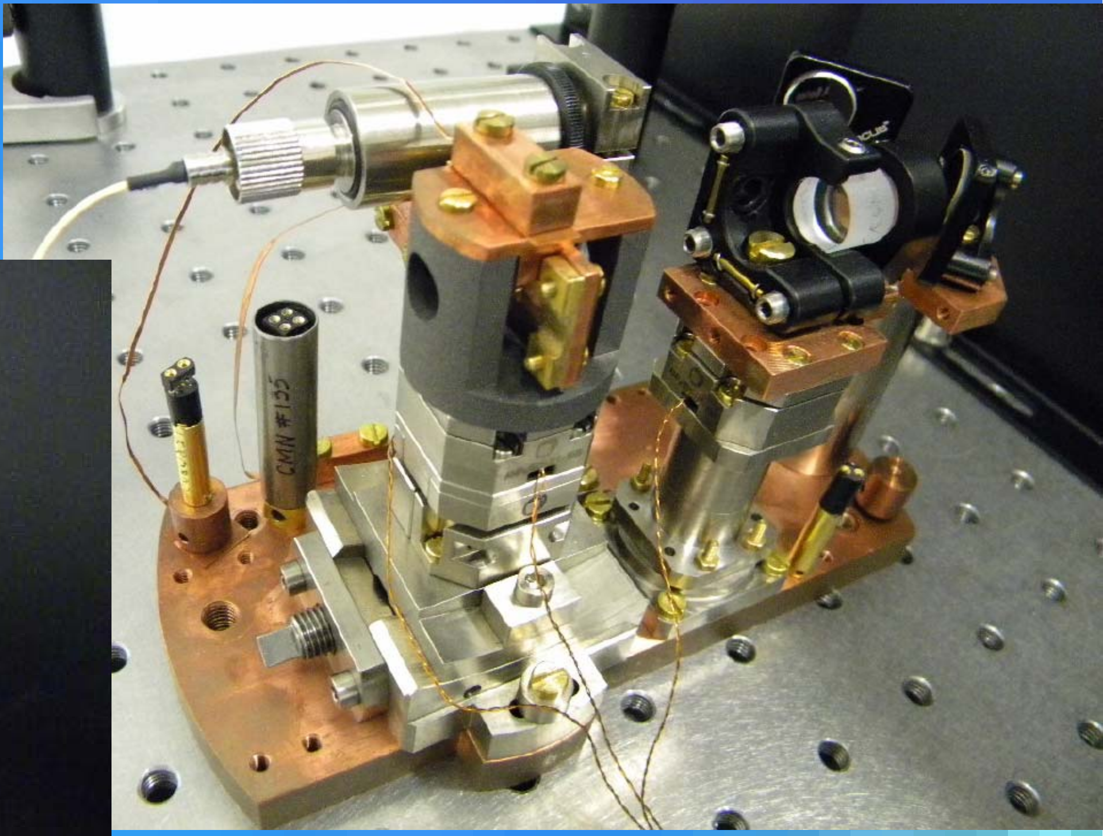
Adiabatic nuclear demagnetisation

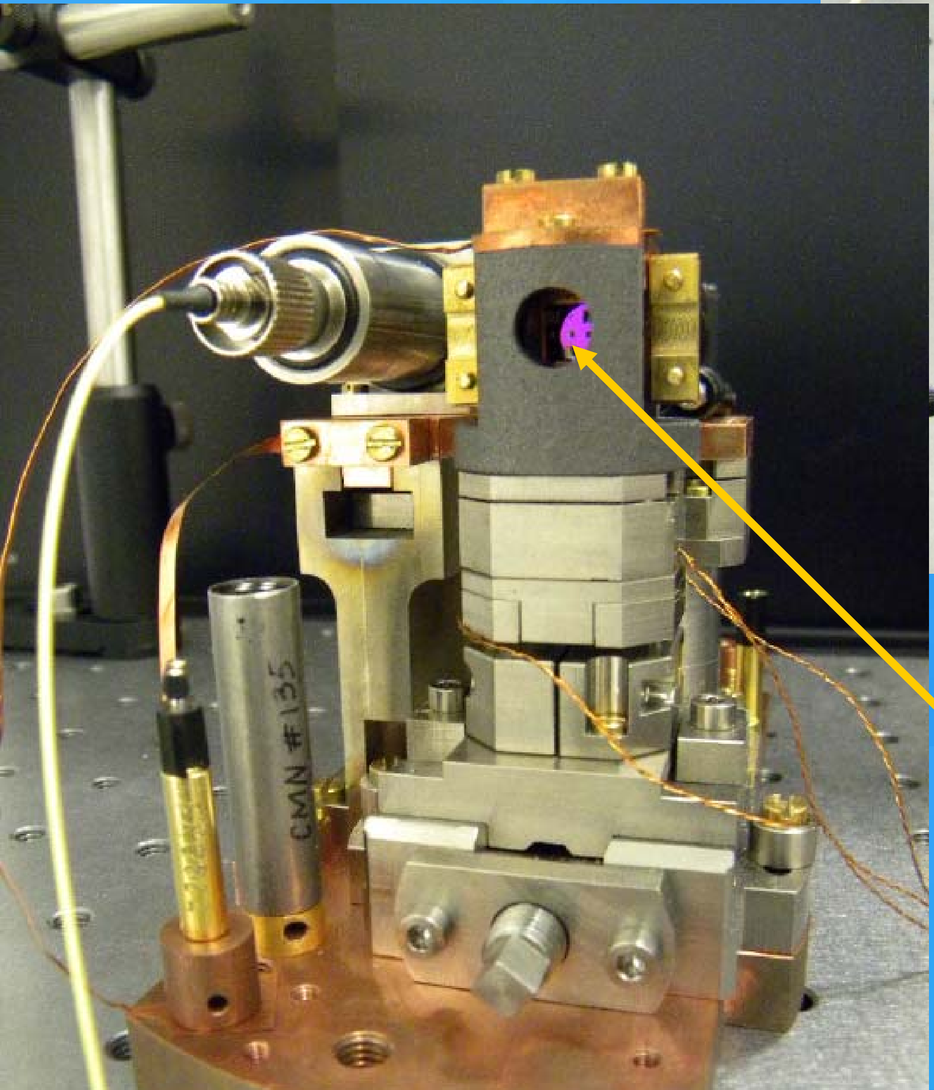


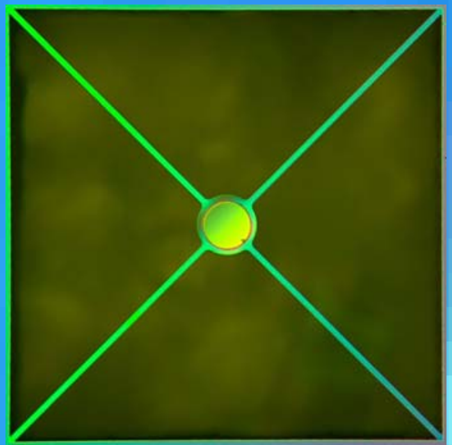
$$T_F \propto \frac{B_F}{B_I} T_I$$

Test run indicates cooling to <100microK possible with system

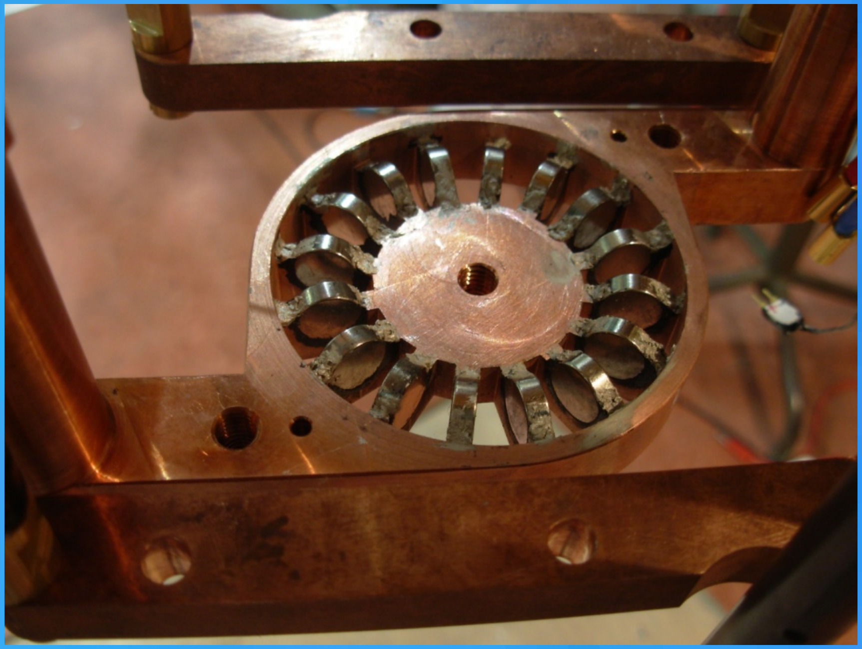
Optical Setup

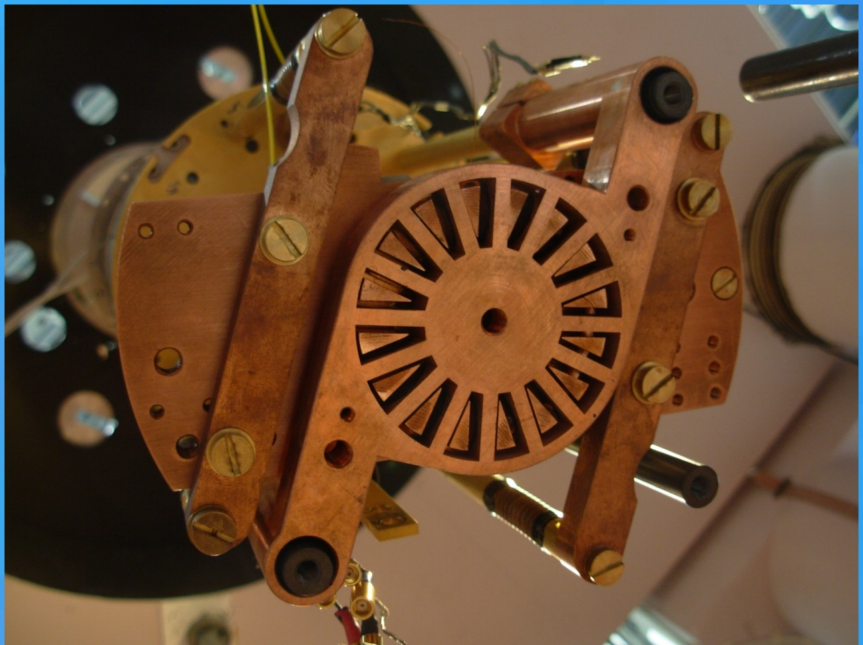


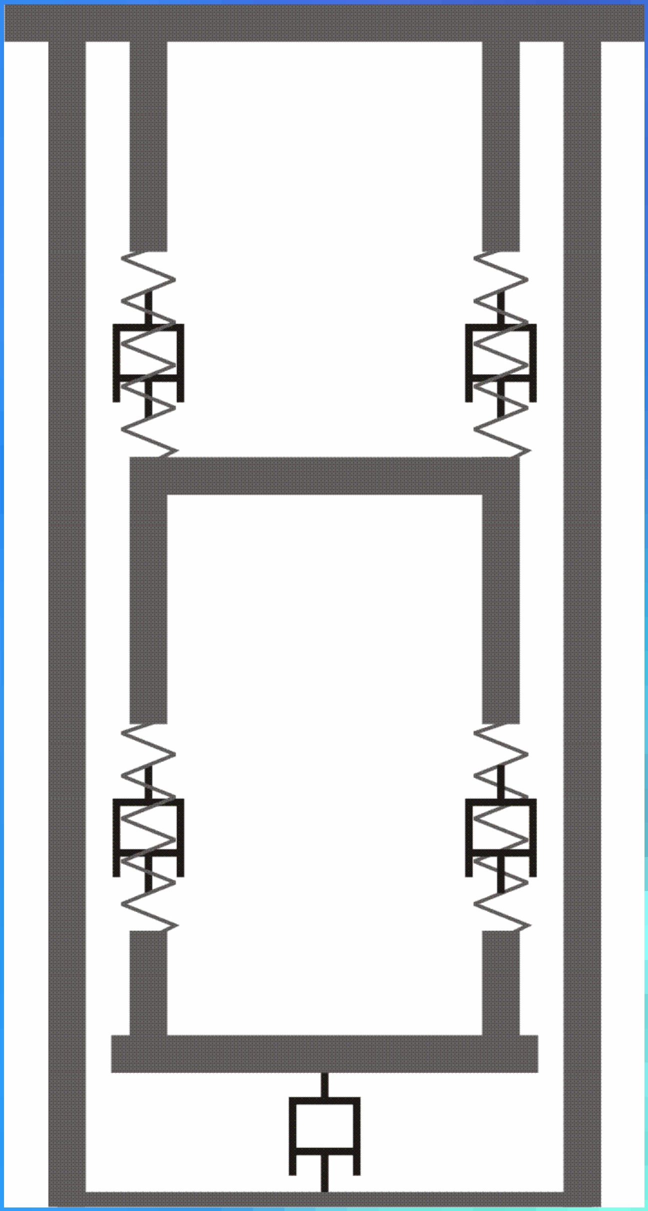


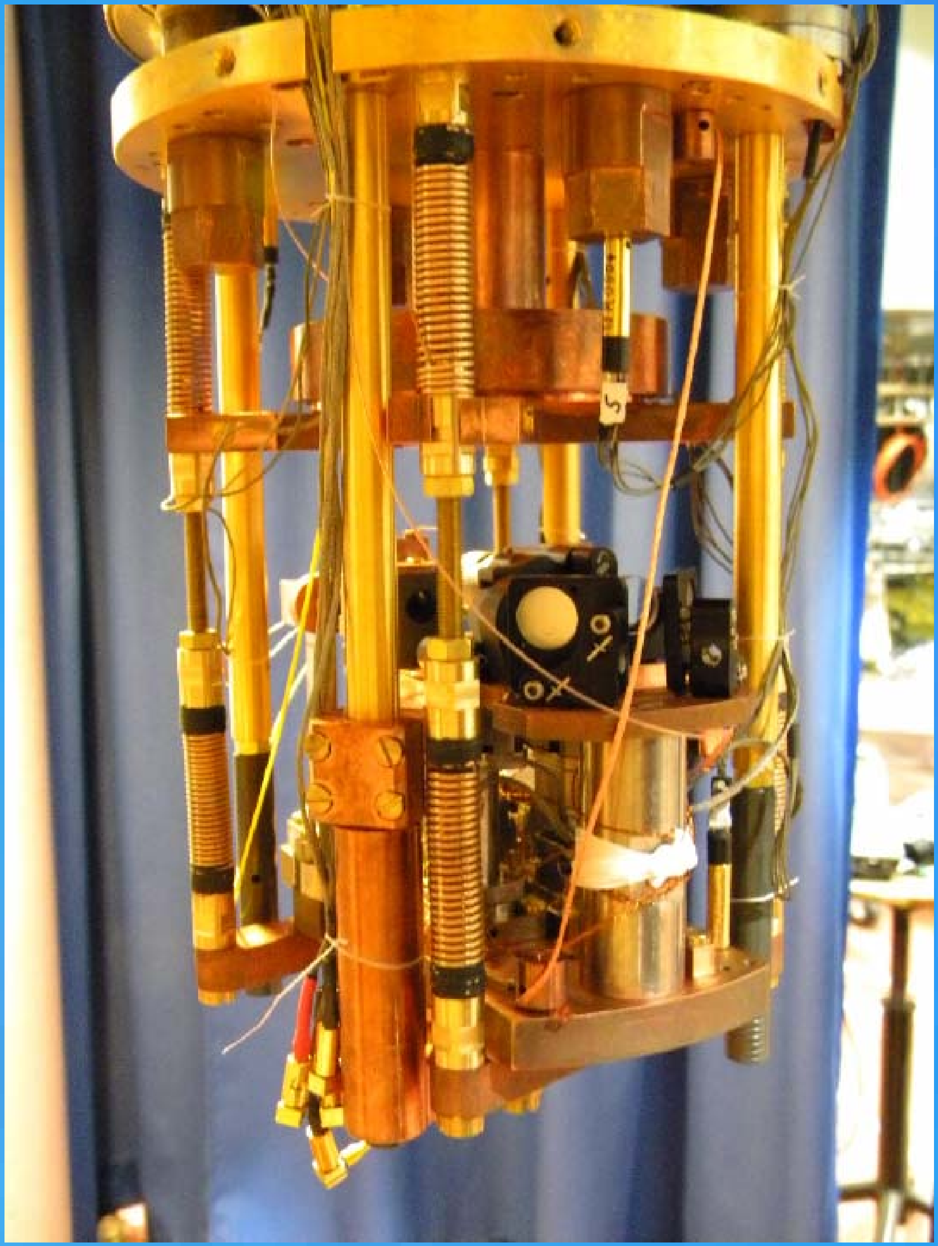


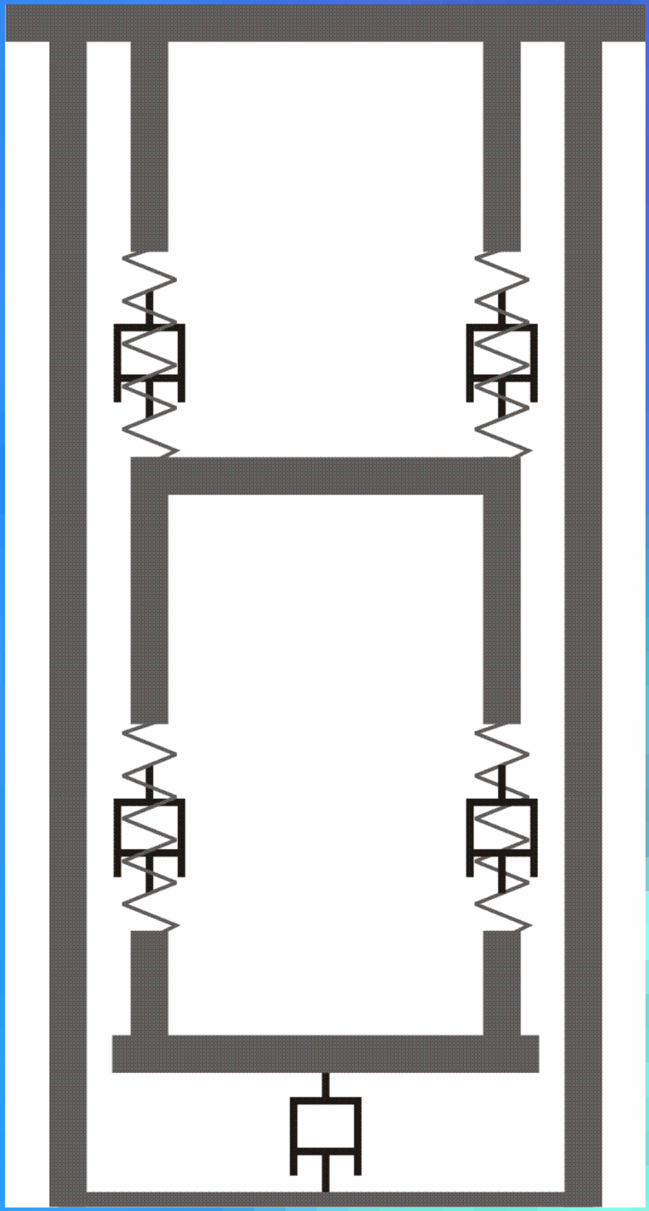
Vibration damping: Multi stage Eddy current damping











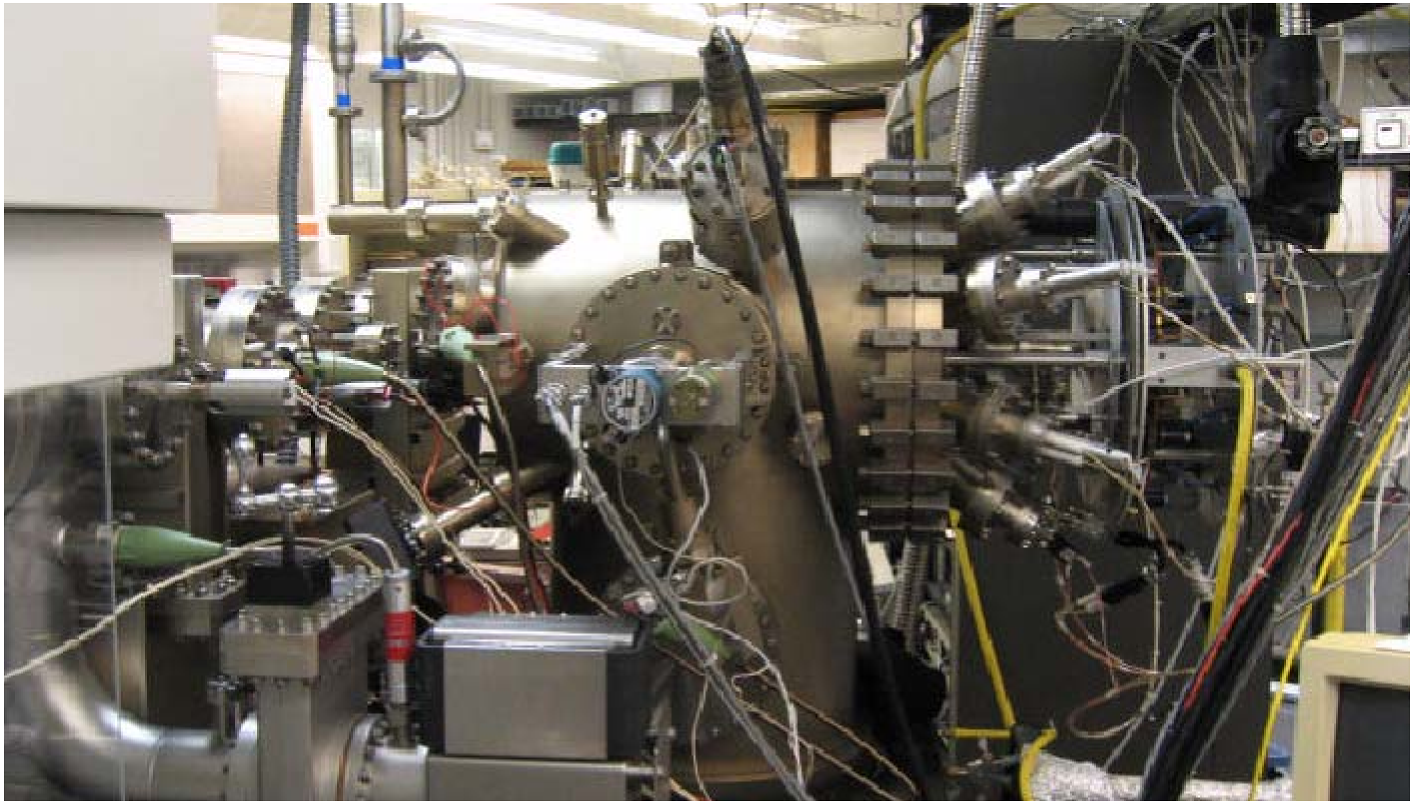
Work in progress

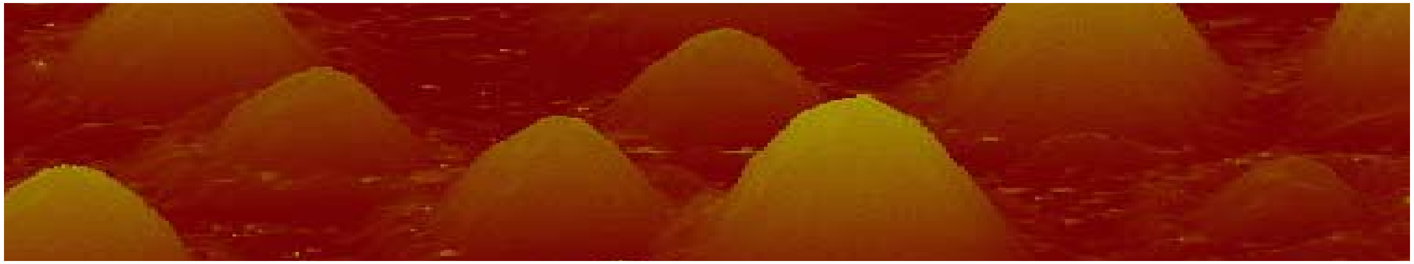
Dustin Kleckner
Brian Pepper

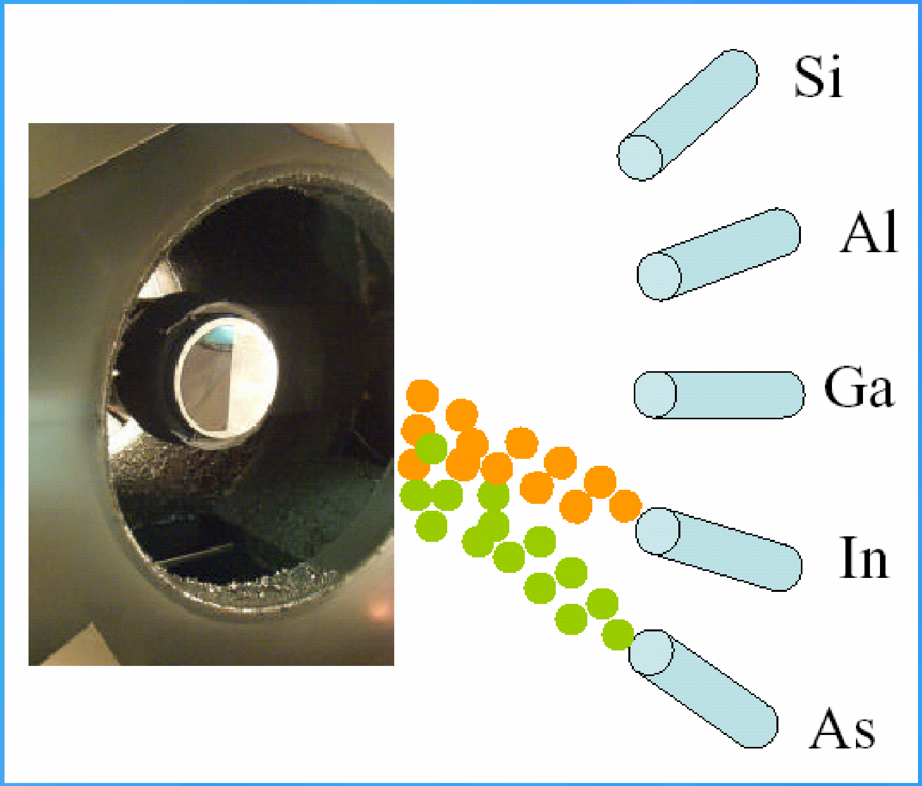
Evan Jeffrey
Petro Sonin

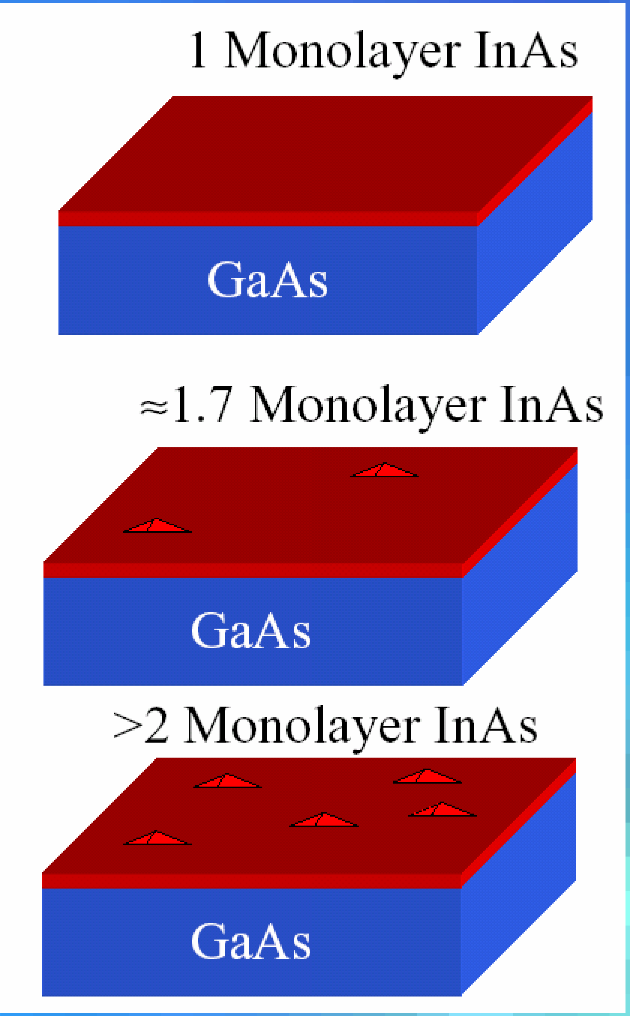
Harmen van der Meer

Molecular Beam Epitaxy (MBE) grown quantum dots

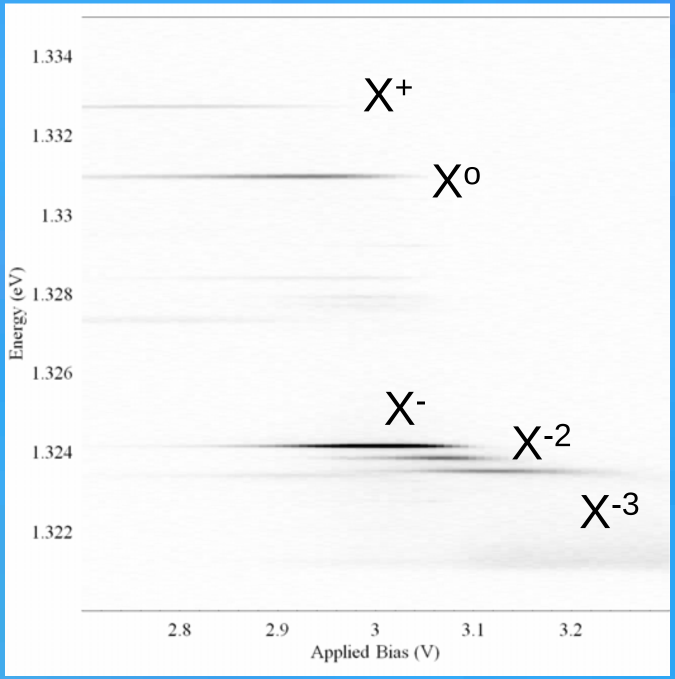





Si
Al
Ga
In
As

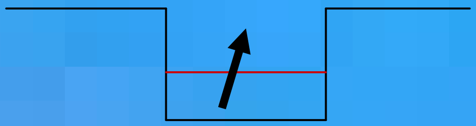

1 Monolayer InAs
GaAs
≈1.7 Monolayer InAs
GaAs
>2 Monolayer InAs
GaAs

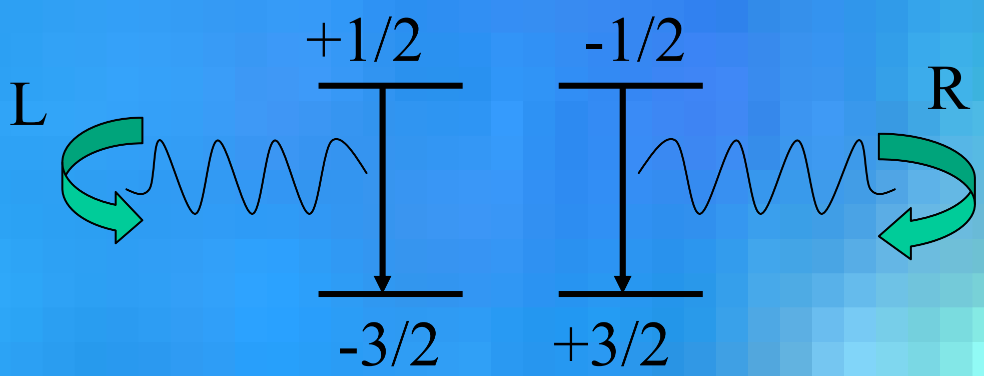
Self-assembled GaAs/InGaAs QUANTUM DOTS

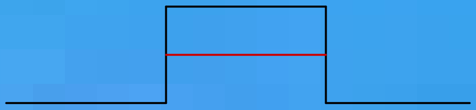


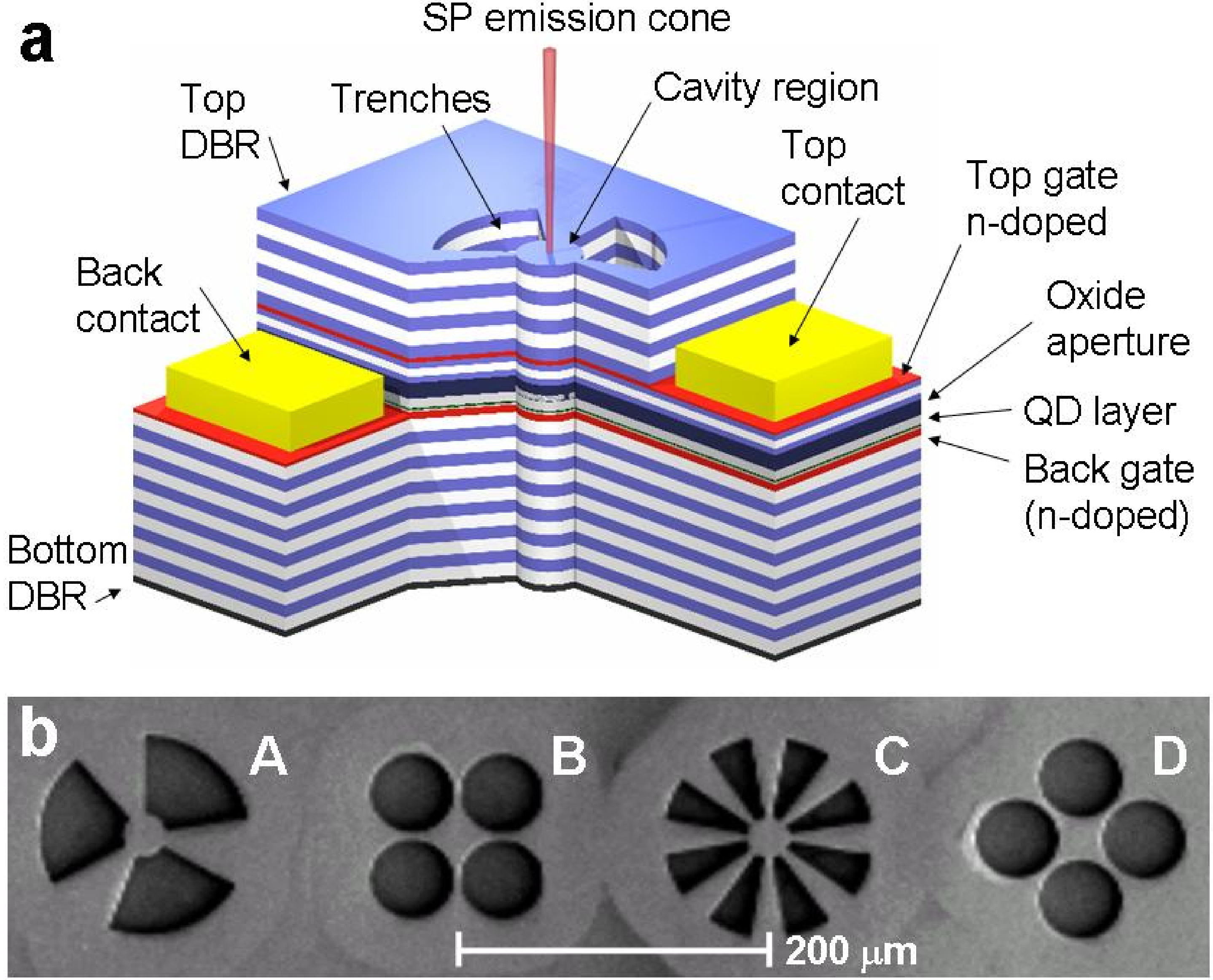
add extra electron to QD

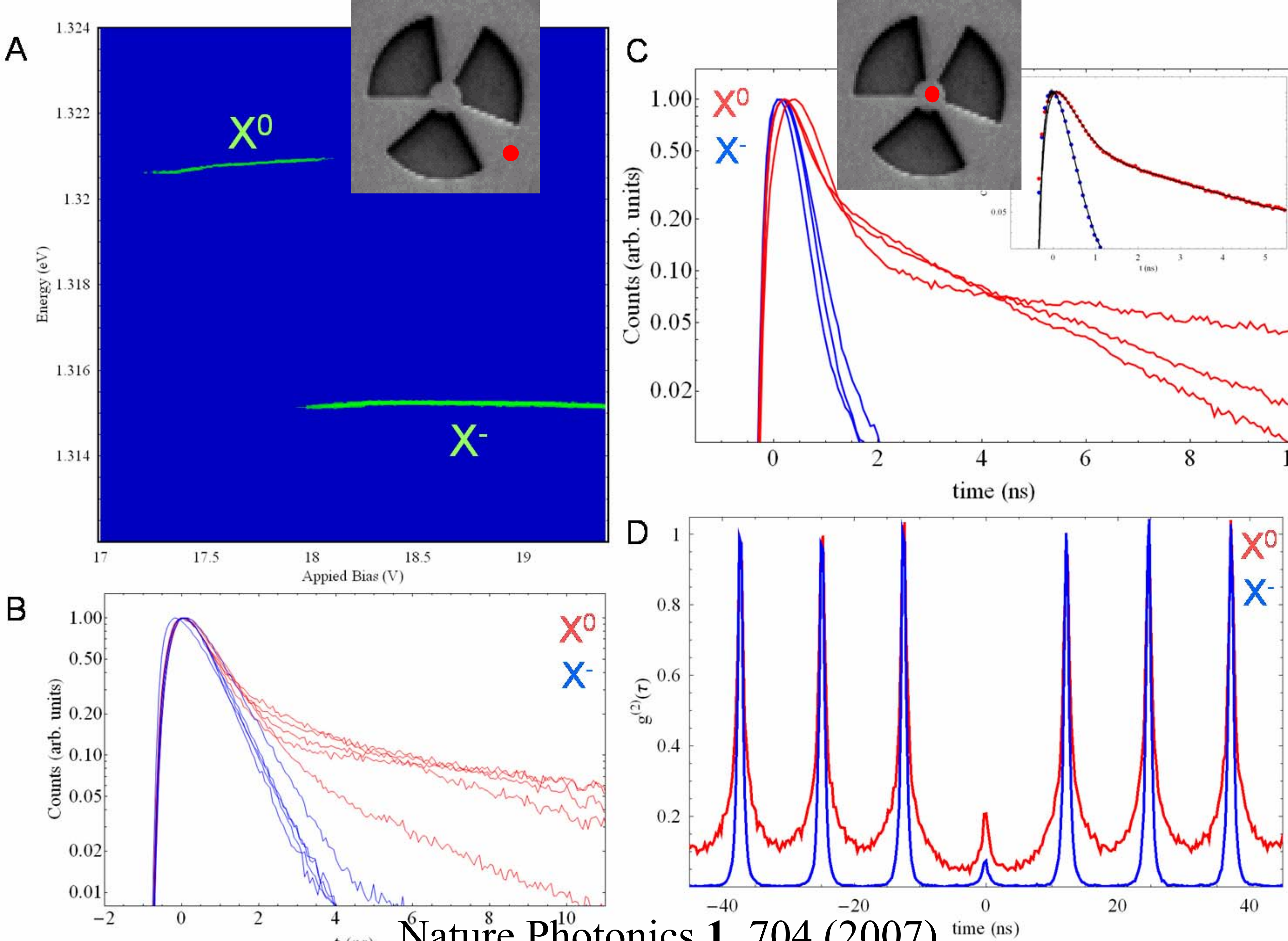
Spin of extra electron is qubit (0.1ms?) coupled to excitons (gates ns)



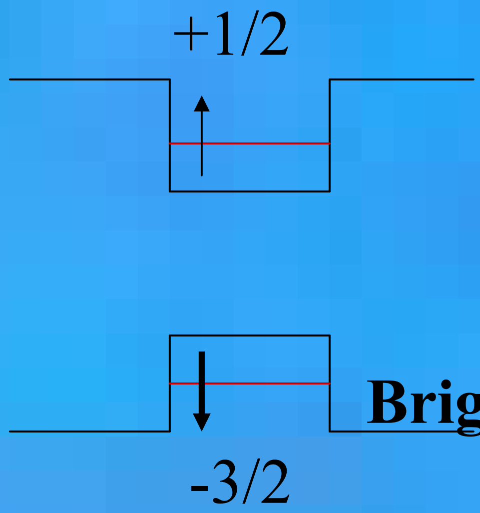


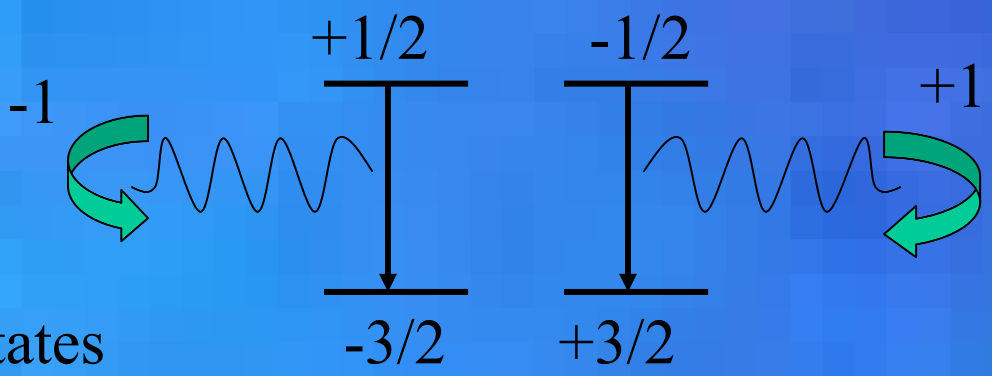



a
SP emission cone
Top DBR
Trenches
Cavity region
Top contact
Top gate n-doped
Back contact
Oxide aperture
QD layer
Back gate (n-doped)
Bottom DBR
b
A
B
C
D
200 μm

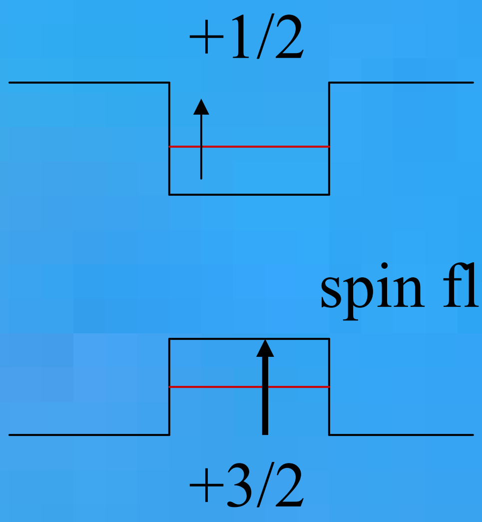


Nature Photonics **1**, 704 (2007)

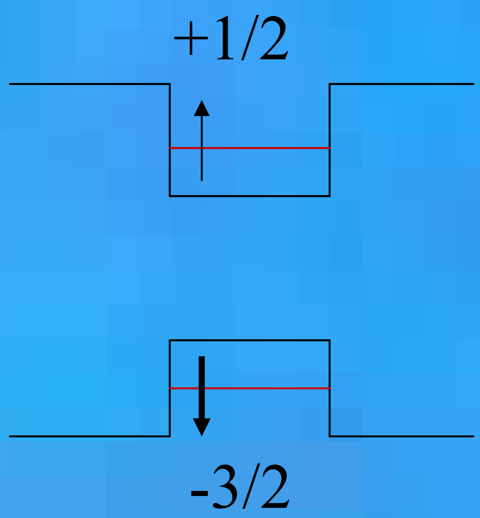


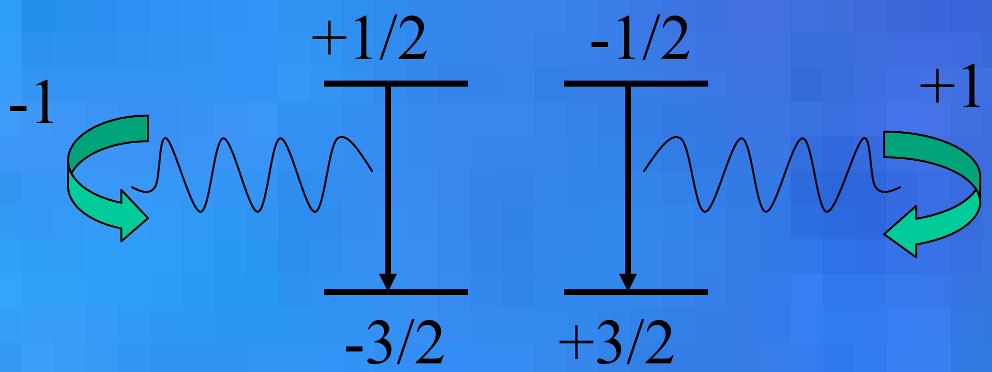


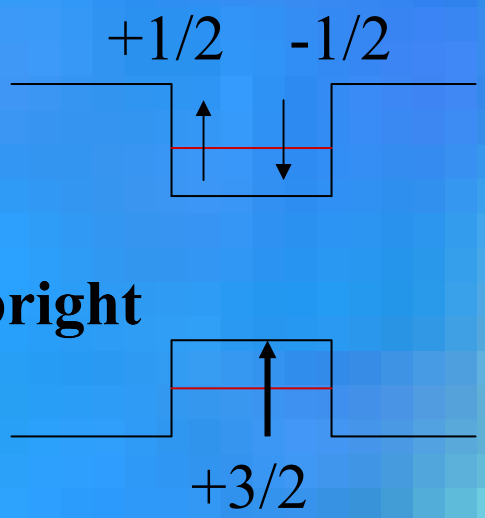
Bright states



spin flip gives **Dark** state

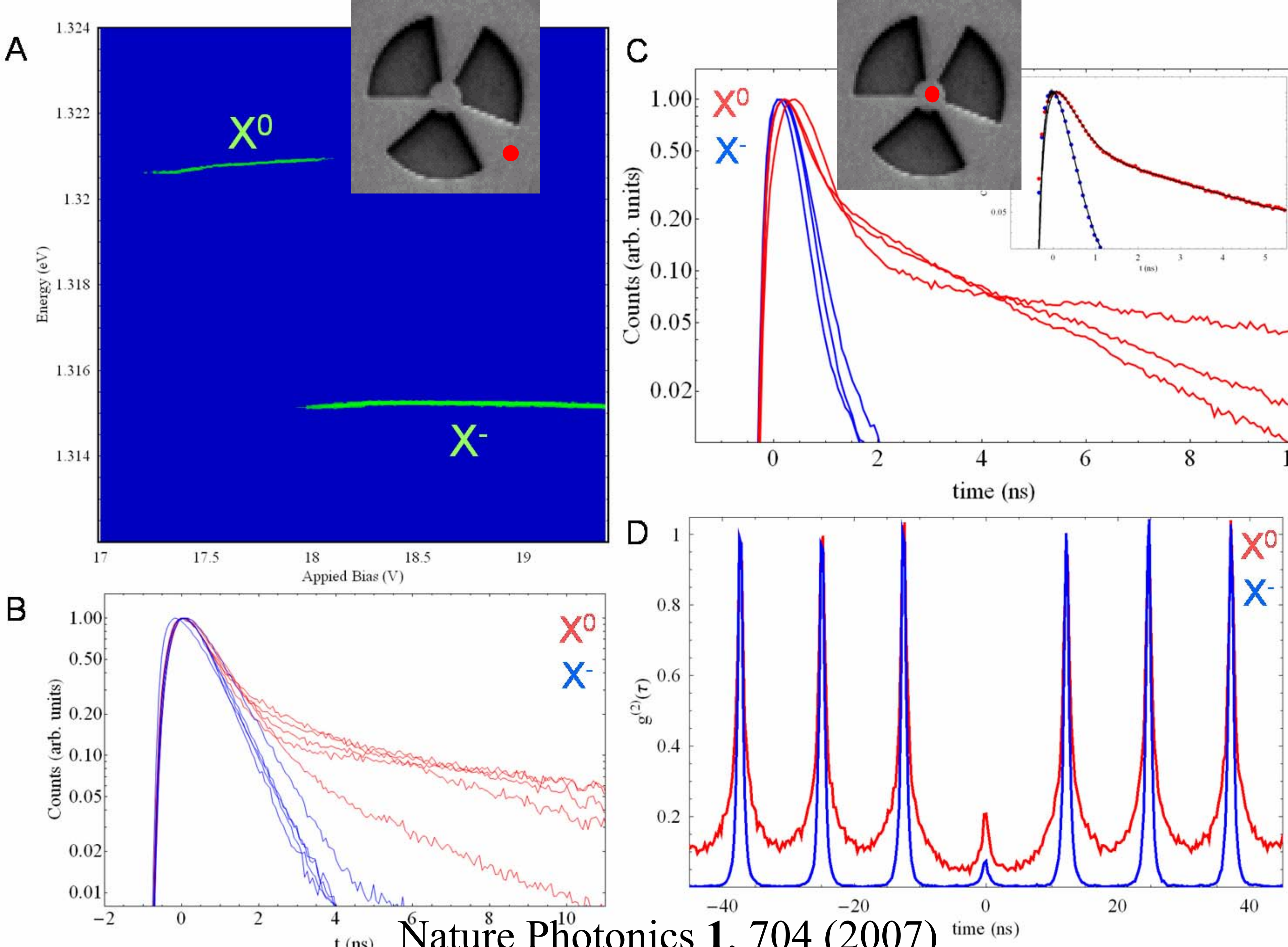




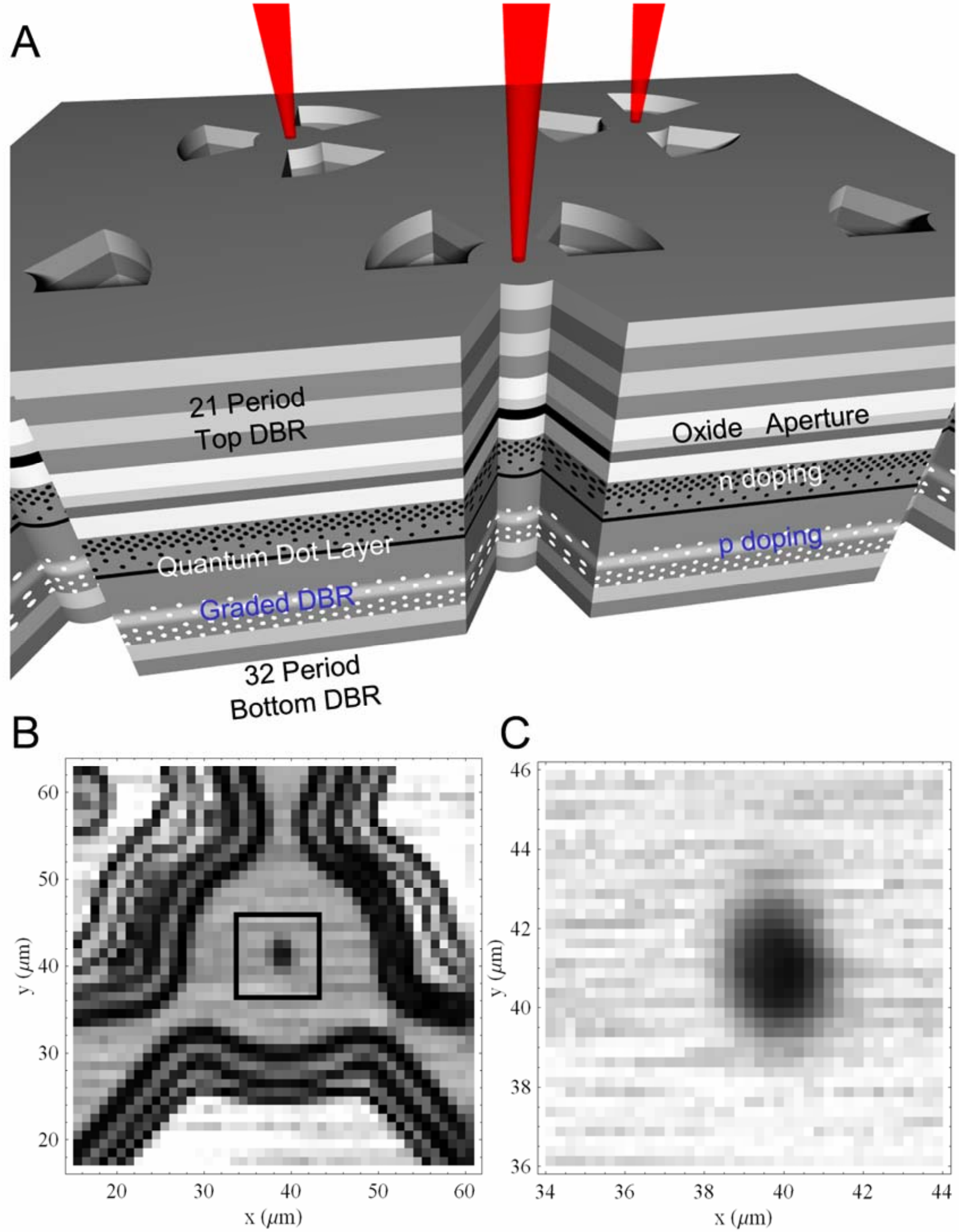


Add single electron

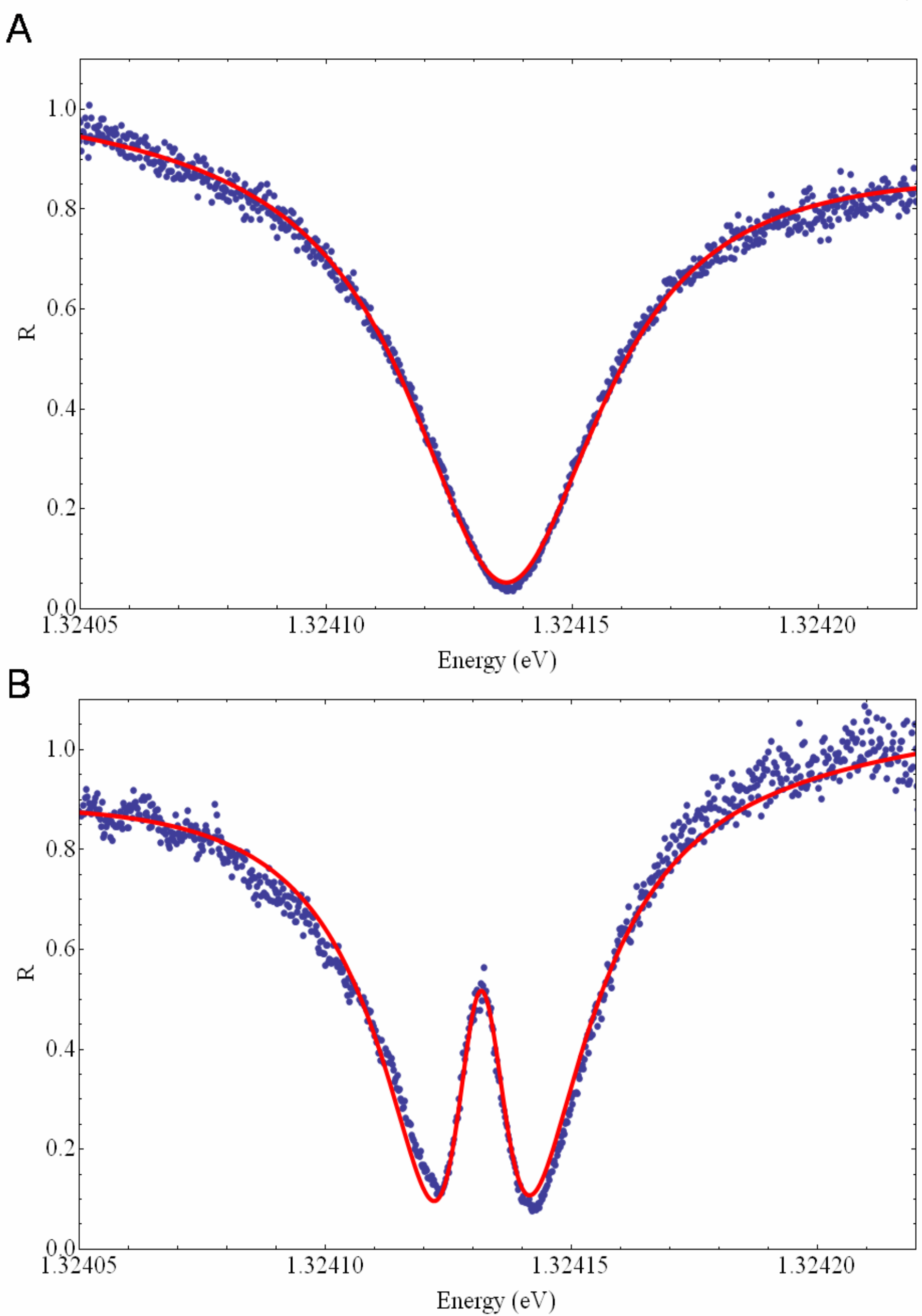
Trion state, always **bright**



Nature Photonics **1**, 704 (2007)


A
21 Period
Top DBR
Oxide Aperture
n doping
Quantum Dot Layer
p doping
Graded DBR
32 Period
Bottom DBR
B
y (μm)
x (μm)
C
y (μm)
x (μm)

Reflection Spectroscopy



Jaynes-Cummings model

$$R(\omega)=\left|1-\frac{\kappa(\gamma-i(\mu\omega-\omega_{QD}))}{(\gamma-i(\omega-\omega_{QD})\kappa)-i(\omega-\omega_c)+g^2}\right|^2$$

κ is cavity field decay rate:

κ = 24.1μeV, corresponding to Q =27,000,

g is emitter-cavity coupling

g = 9.7μ eV,

γ is emitter decay rate:

γ = 1.9μ eV,

$\frac{g}{\kappa}$ = 0.40, deep in Purcell (weak-coupling) regime,

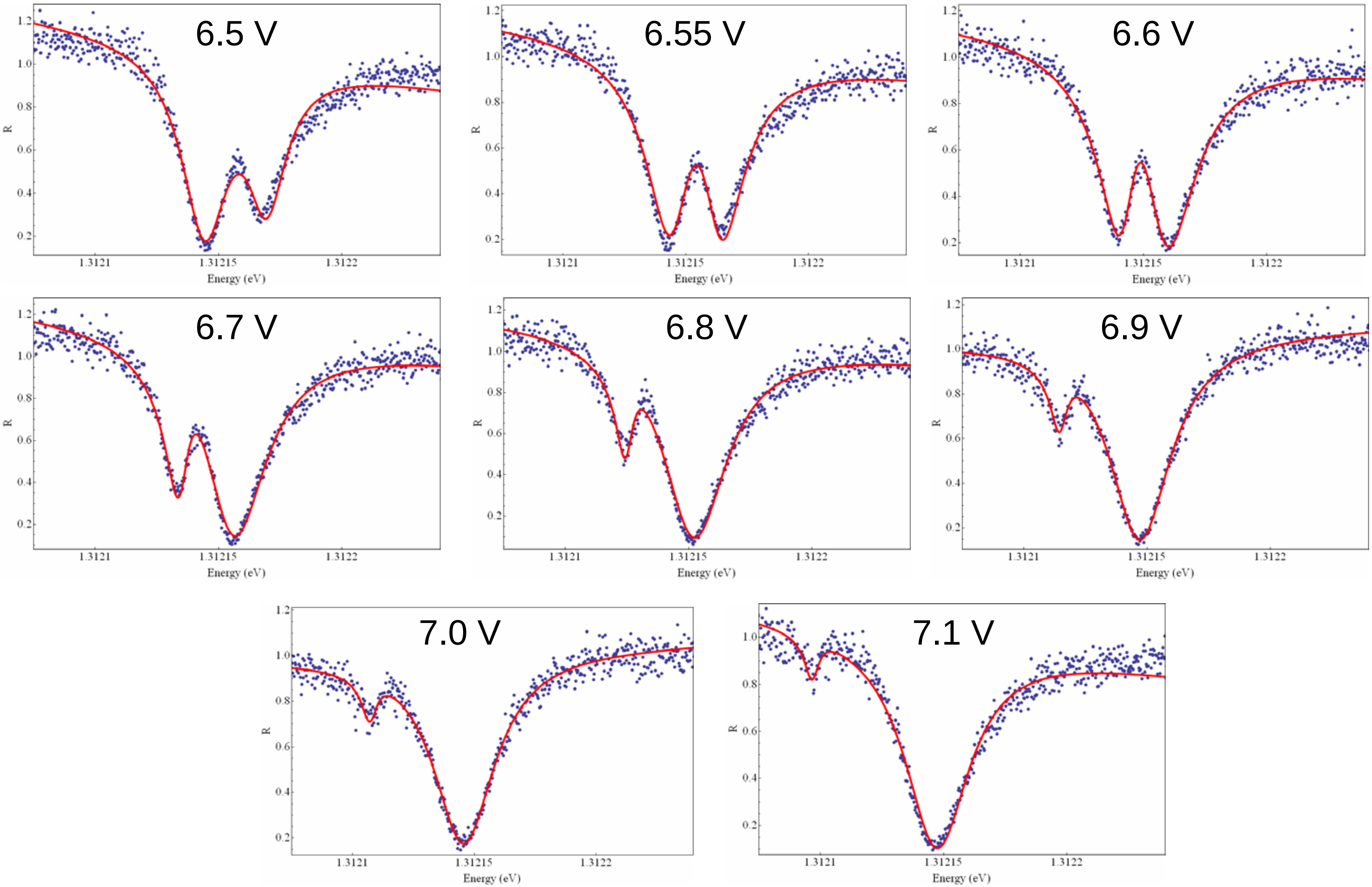
$\frac{g}{\kappa}$ > 0.5 is strong coupling

96% mode matched!!!

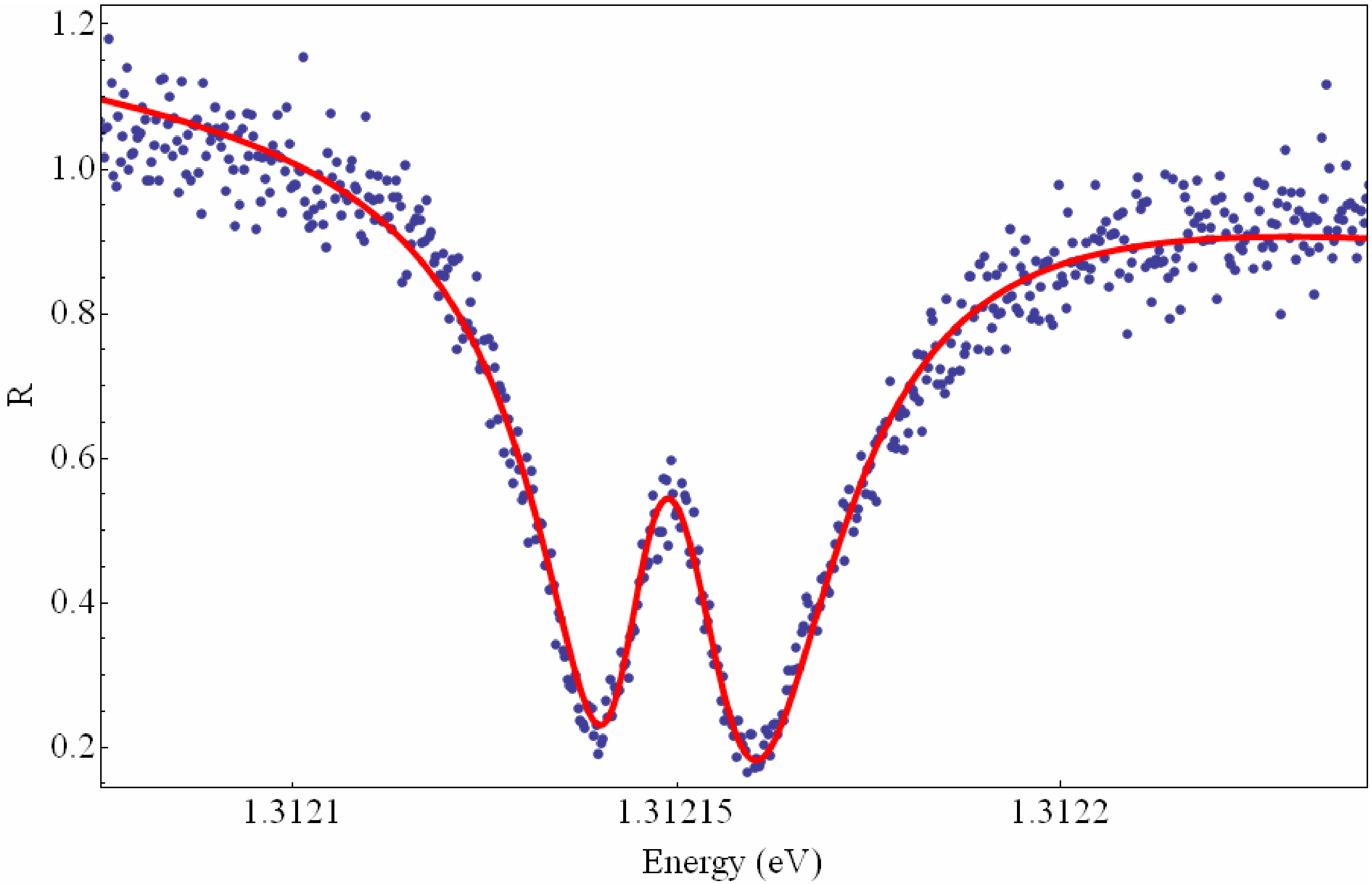
Ideal for hybrid QIP schemes

PRL Rakher et al. '09

Reflection Spectroscopy



Fit Parameters: Strong Coupling



$$g = 9.96\mu eV$$

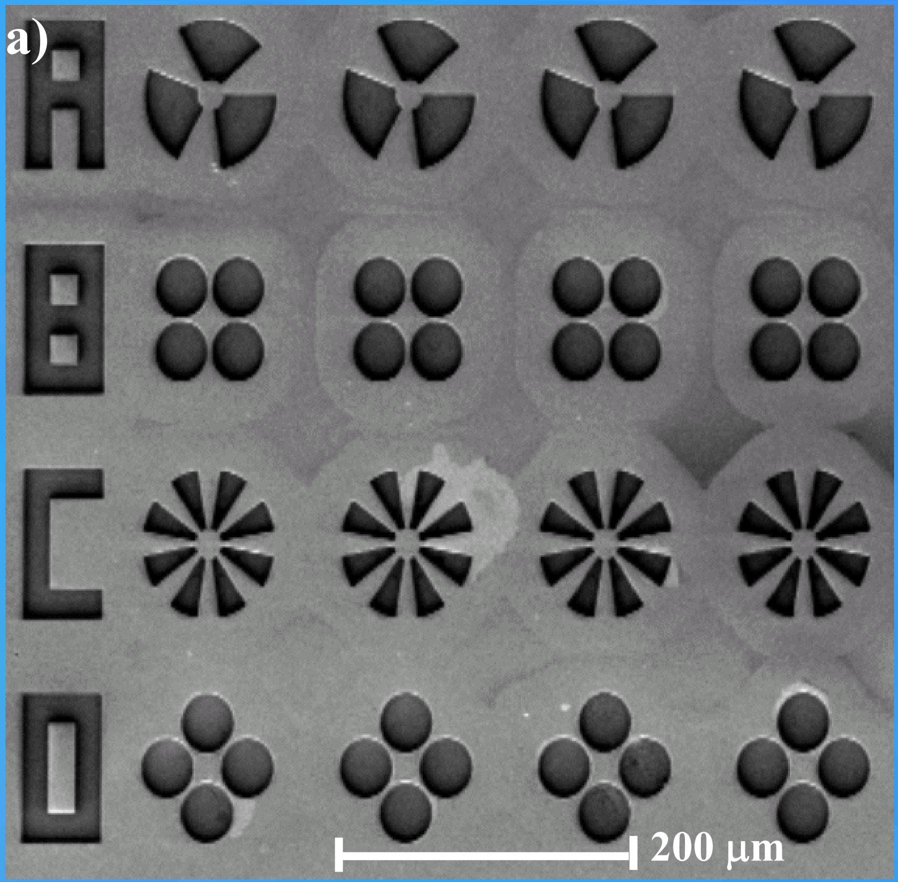
$$\kappa = 16.4\mu eV$$

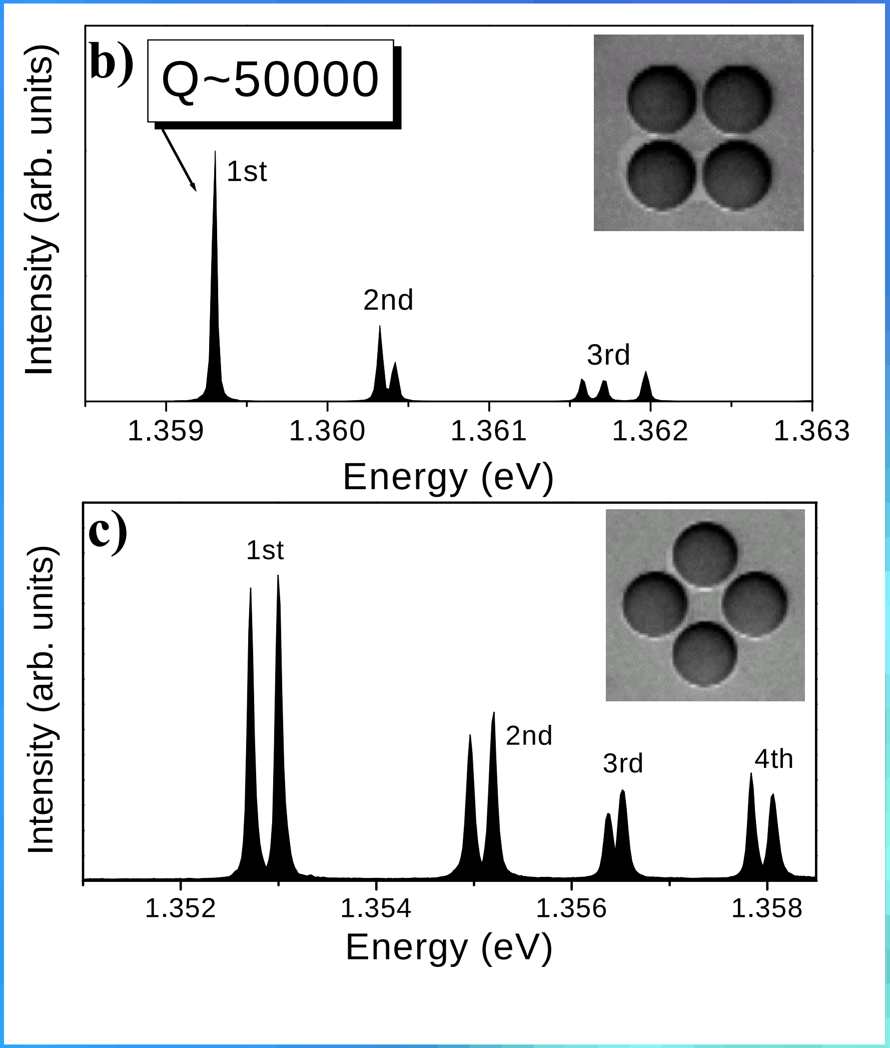
$$Q = 39800$$

$$\frac{g}{\kappa} = 0.607 > 0.5$$

$$\gamma = 3.1\mu eV$$

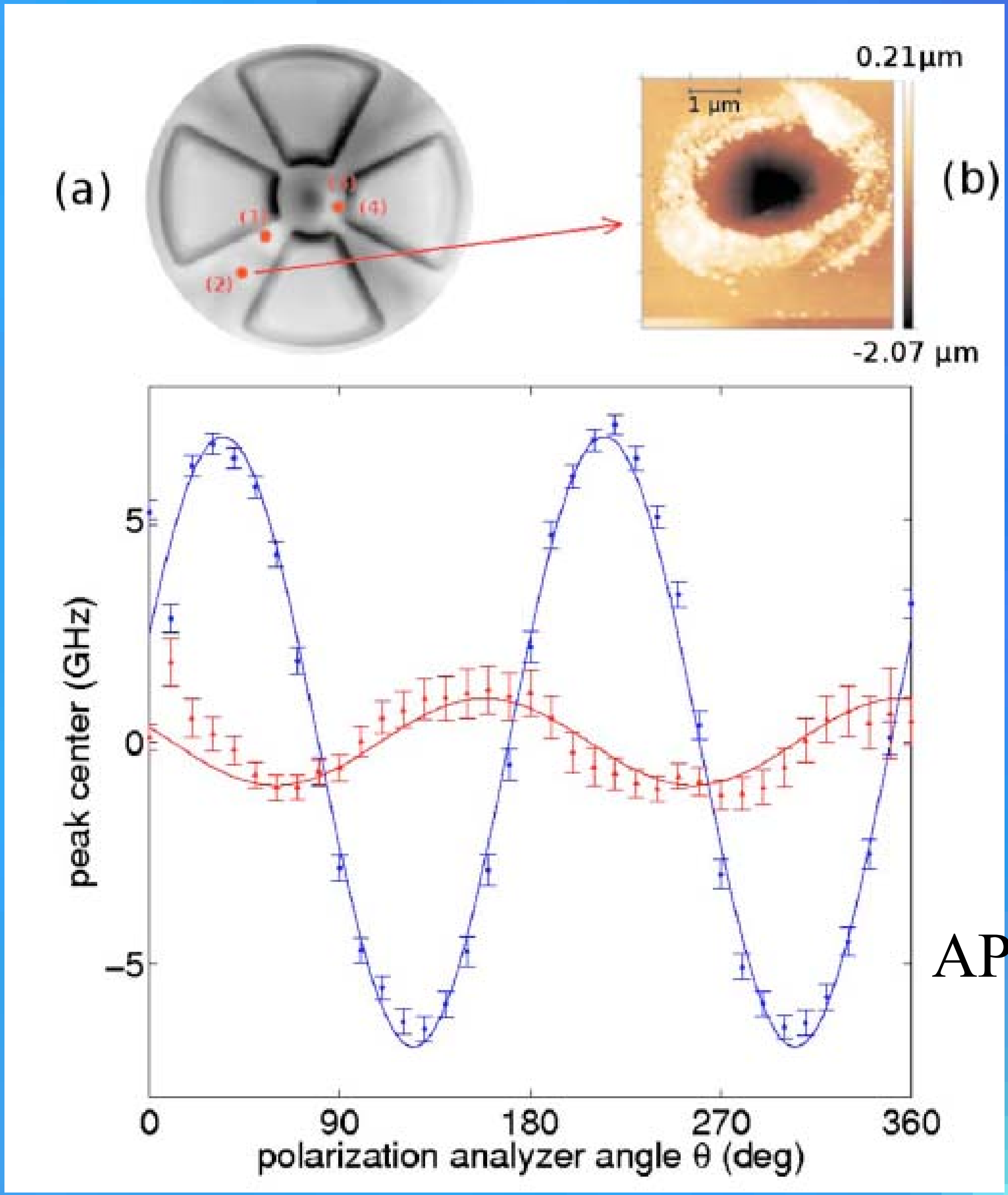
Mode polarization tuning



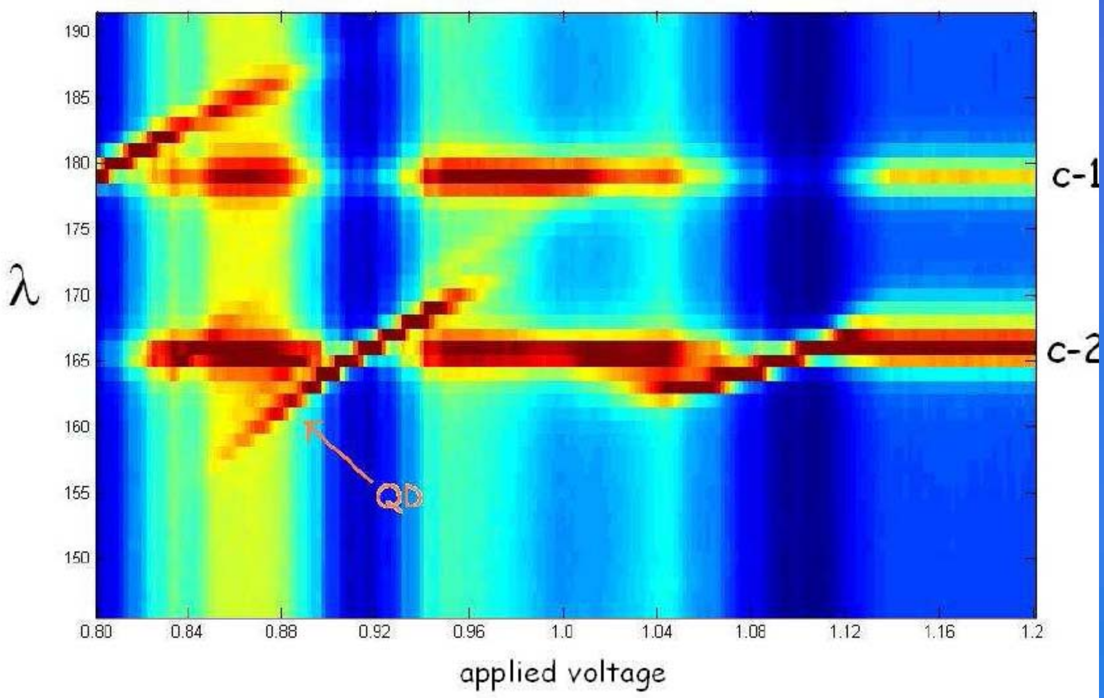


- Fine tuning by hole burning
- Fibre coupling (two sided)

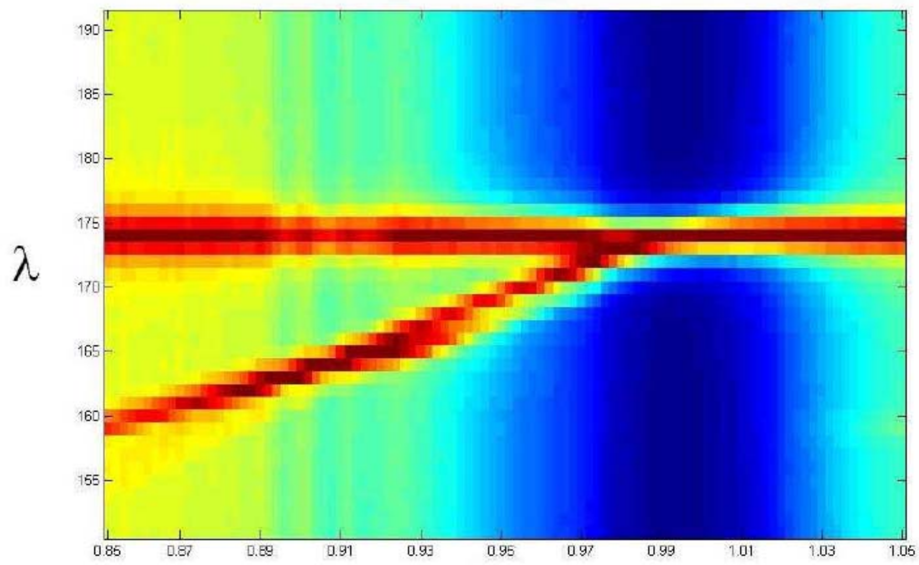
Birefringence tuning by hole burning



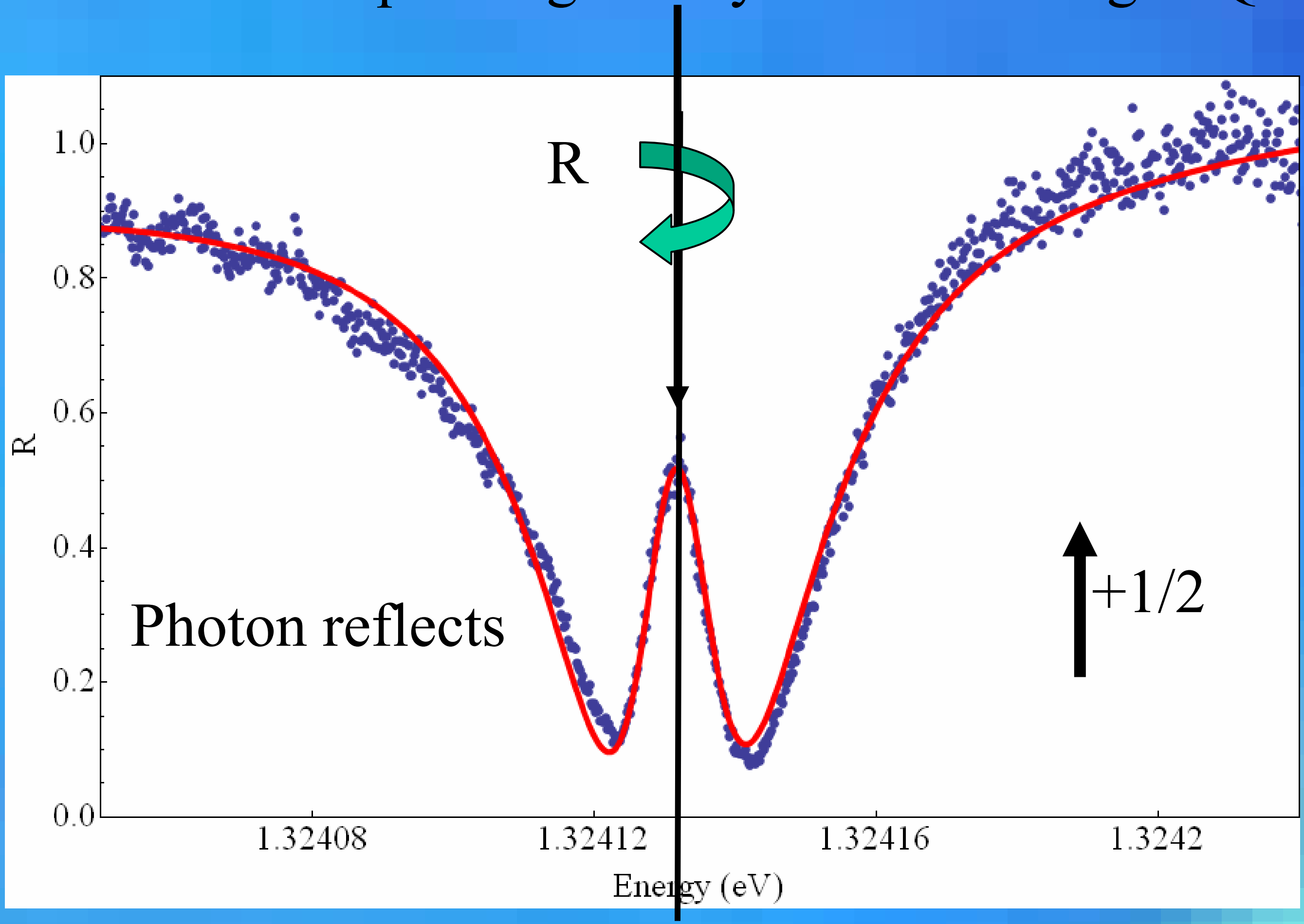
APL Bonato et al 09



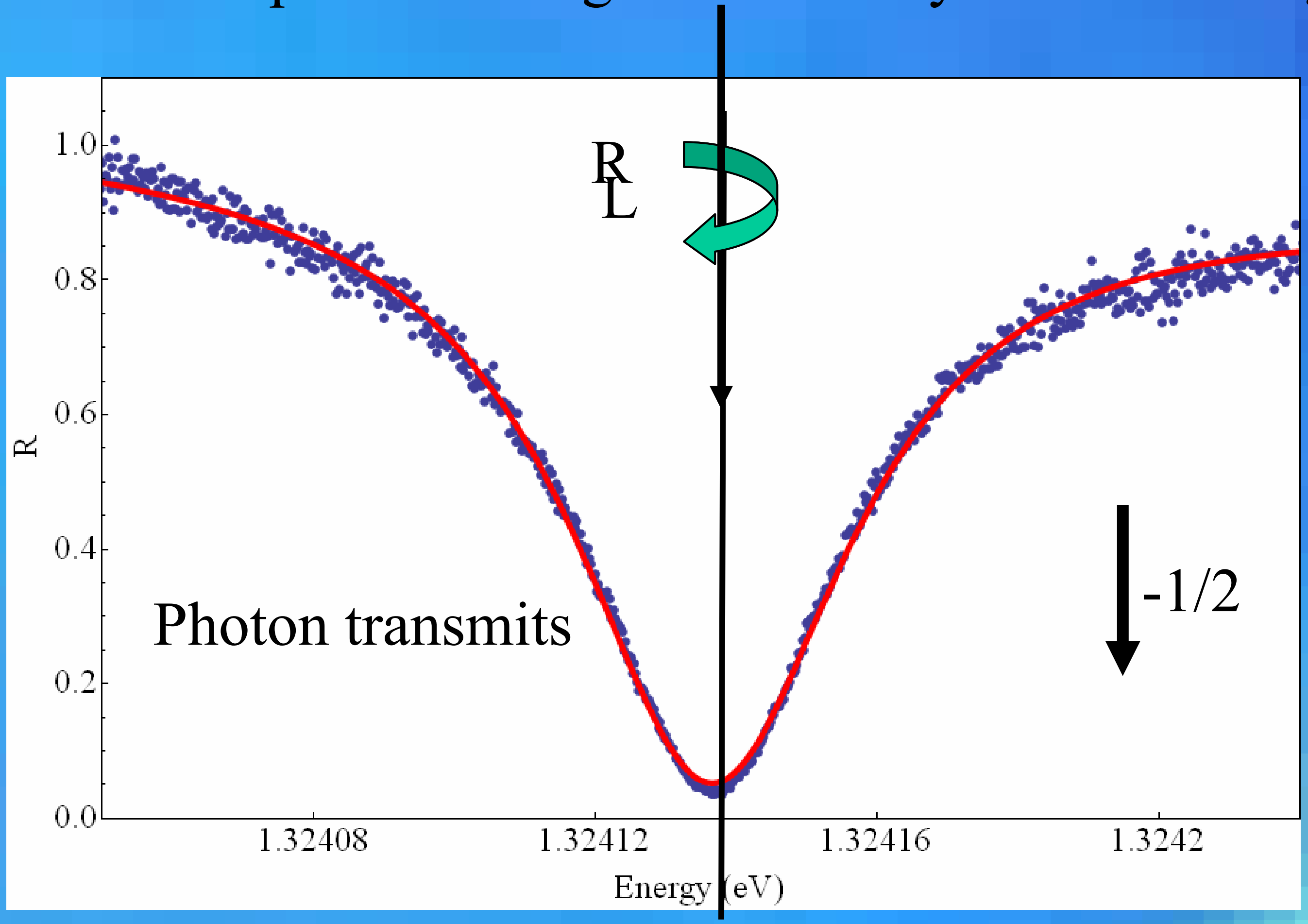
After hole burning:

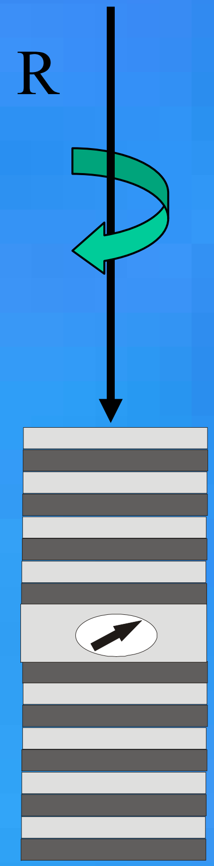


Prediction: For pol. deg. cavity and a X^- charged QD

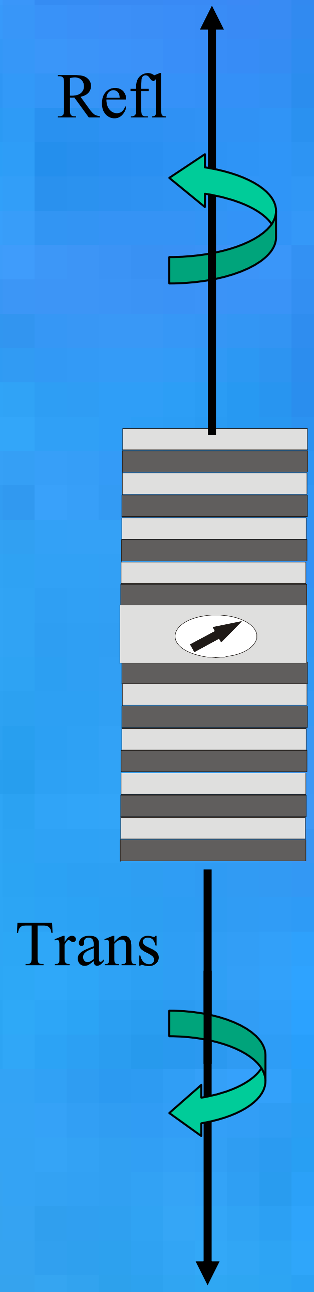


Prediction: For polarization generate cavity and a X^- charged QD



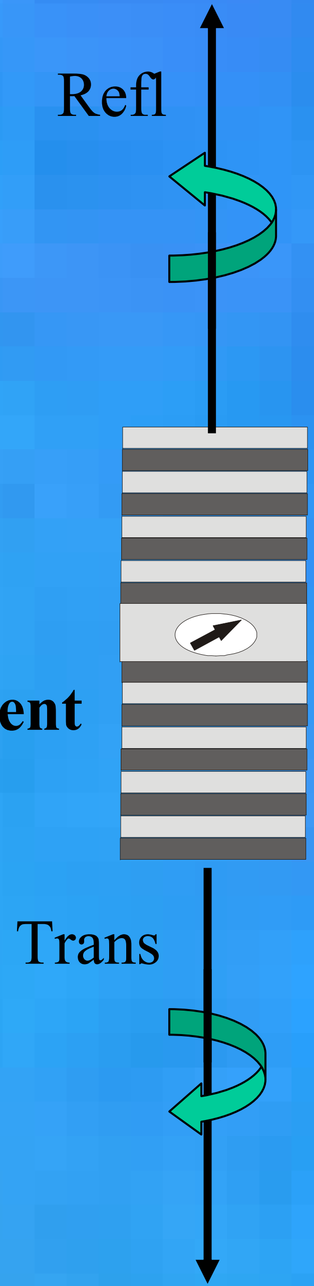


$$|\Psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$$



$$|\Psi\rangle = \alpha|\uparrow;\mathrm{Refl}\rangle + \beta|\downarrow;\mathrm{Trans}\rangle$$

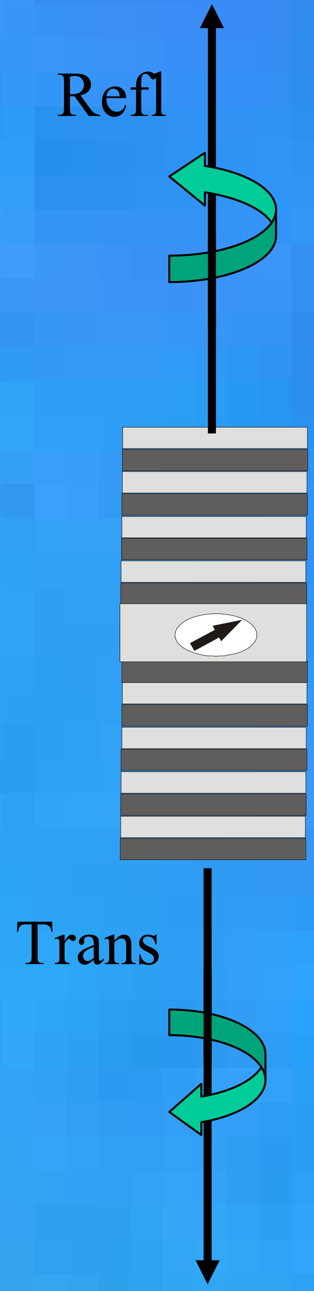
Single photon
“interaction free”
single electron spin
entanglement/measurement



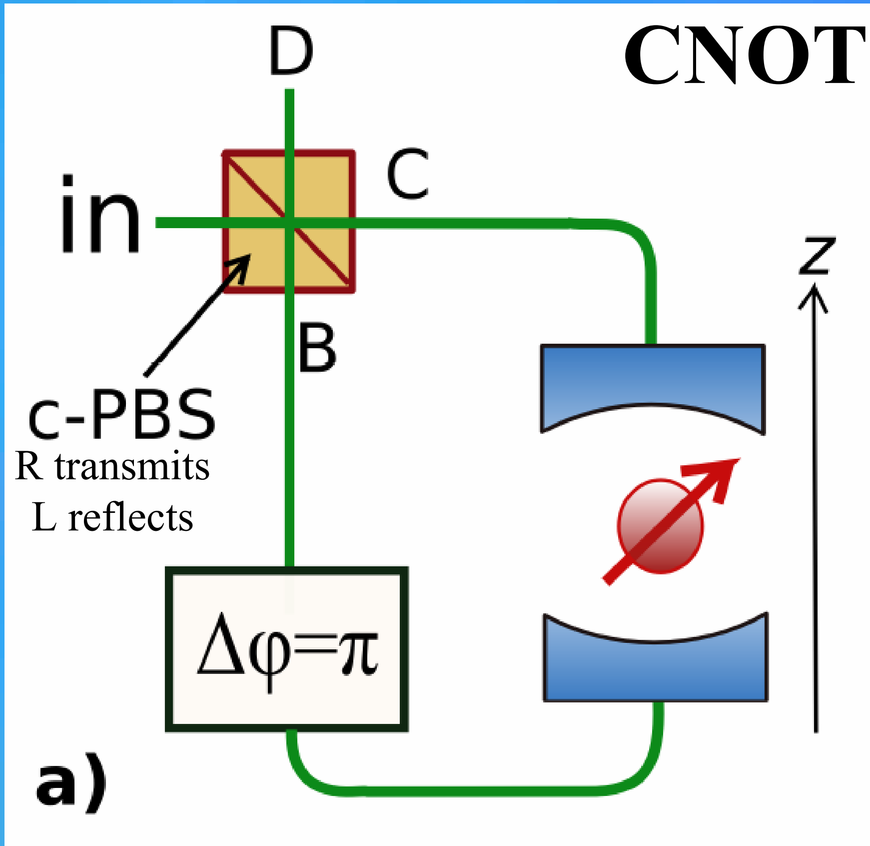
$$|\Psi\rangle = \alpha|\uparrow;\mathrm{Refl}\rangle + \beta|\downarrow;\mathrm{Trans}\rangle$$

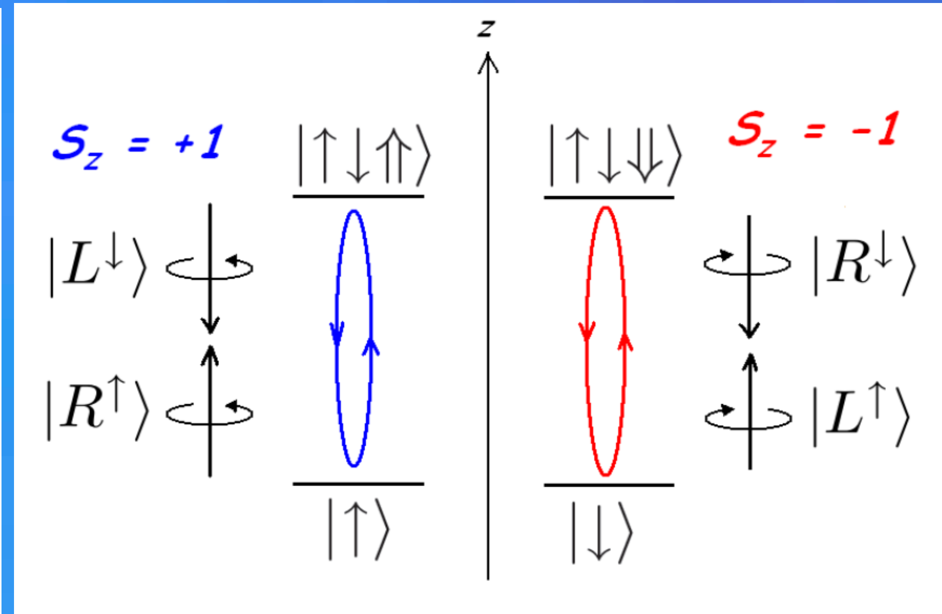
Repeated projection measurments
Quantum zeno effect

PRA **80**, 023812 (09)



$$\left|\Psi\right\rangle = \alpha\left|\uparrow;\mathrm{Refl}\right\rangle + \beta\left|\downarrow;\mathrm{Trans}\right\rangle$$





Transmission π mod2π
with respect to reflection

$$\left|\psi_{ph}\right\rangle = \alpha\left|R\right\rangle + \beta\left|L\right\rangle, \quad \left|\psi_{el}\right\rangle = \gamma\left|\uparrow\right\rangle + \delta\left|\downarrow\right\rangle$$

$$\left|\psi\right\rangle_{in} = \left|\psi_{ph}\right\rangle \otimes \left|\psi_{el}\right\rangle$$

$$\left|\psi\right\rangle_{out} = \gamma\left|\uparrow\right\rangle\left[\alpha\left|R\right\rangle + \beta\left|L\right\rangle\right] + \delta\left|\downarrow\right\rangle\left[\alpha\left|L\right\rangle + \beta\left|R\right\rangle\right]$$