Decoherence & Entanglement

A Philosopher's View

Amit Hagar

Department of History & Philosophy of Science
College of Arts & Sciences
Indiana University, Bloomington

hagara@indiana.edu

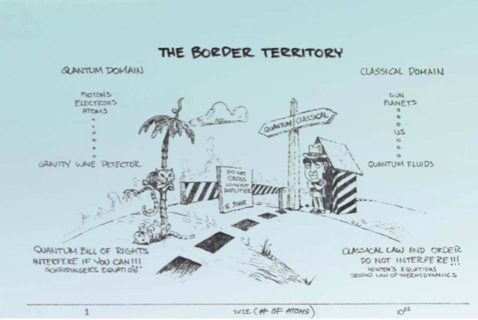
XIV Seven Pines Symposium
May 2010

When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled.

Schrödinger 1935

When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled

Schrödinger 1935



The phenomenon of Decoherence, or the delocalization of phase relations, is known since 1929, but only recently (1970 onwards) has been recognized as the “new orthodoxy”

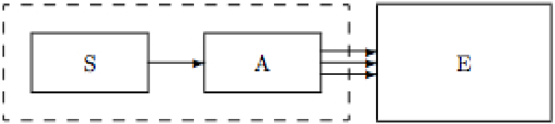
The Plan

1. Basic concepts
2. the role of decoherence in the foundations of QT
3. The “emergence of the classical world”
4. Decoherence and the complexity of noise

The Open System Approach — Classical

- All objects in the universe interact with each other to a greater or lesser degree
- In classical physics, $H_{Sys}(q, p, t)$ which determines the evolution of a local system described by canonical co-ordinates depends on the actual state $\{q_{env}(t), p_{env}(t)\}$ of the environment
- In a statistical description, ensembles of Hamiltonians may be more appropriate since the effect of the environment is treated as "uncontrolable perturbation" (Borel 1914)
- Nevertheless, a unique state of the system is still supposed to exist even though we are dealing with an ensemble of Hamiltonians

The Open System Approach — Quantum



- In QM the situation is completely different
  - There are no definite states of the subsystems
  - Interactions generally lead to a non–separating global state for the whole system.
- The only possible description of the system in the standard formalism is by means of a *reduced* density matrix.

Pure Decoherence

- Phase relations are dislocalized (extremely fast) into the total system (S+A) – finally the entire universe (S+A+E):

$$\left(\sum_n c_n|n\rangle|\Phi_n\rangle\right)|E_0\rangle \quad \xrightarrow{t} \quad \sum_n c_n|n\rangle|\Phi_n\rangle|E_n\rangle \tag{1}$$

- The behavior of $S + A$ is then described by the density matrix

$$\rho_{SA} \approx \sum_n |c_n|^2 |n\rangle\langle n| \otimes |\Phi_n\rangle\langle\Phi_n| \qquad \text{if} \qquad \langle E_n|E_m\rangle \approx \delta_{nm} \tag{2}$$

- Furry (1936): the reduced density matrix yields the same statistics for observables of S, A, or $S + A$, and these are indistinguishable from the statistics given by a classical mixture

Proper vs. Improper Mixture

d'Espagnat (1966):

- However, we should bear in mind that the formal ensemble of states characterizing the reduced density matrix is not a 'real' ensemble in the sense of classical physics.
- Even if a complete set of density matrices for all subsystems were given, such a description would remain incomplete in an essential way, in contrast to classical physics (where a specification of the state of each degree of freedom implies a complete characterization of the global state).

Proper vs. Improper Mixture

- To distinguish between proper and improper mixtures we need specific observables of the *composite* system $S_n + A + E$ (of which the entangled composite state is an eigenstate)
- As the number of subsystems S_n increases, this distinction becomes more and more difficult, and FAPP impossible
- Yet even if this distinction is practically hard to attain, QM itself tells us that this is not impossible in principle

Speakable or Unspeakable?

Decoherence is claimed to be relevant to a variety of questions:

- The Quantum Measurement Problem
- The problem of the Arrow of Time
- CM (e.g., chaotic trajectories) from QM formalism

The Measurement Problem

- Decoherence is nothing but standard (linear) Schrödinger equation; we are still left with the problem of interpreting the state of the system, ψ
- One might be tempted to
  - regard ψ as a catalogue of *knowledge*
  - view the measurement process as primitive
  - dismiss the problem as “a problem about people”
- But this leads to a rather radical view of QT
- And besides, there *are* observables that can distinguish true from false collapse...

The Arrow of Time

Because of the apparent collapse and the time–reversal non–invariance of the Master Eq., some (e.g., Zeh) claim decoherence is relevant to the problem of the arrow of time

However

- Since the evolution of the $S + A + E$ is still unitary, then the issue here is no different than the standard problem of accounting for the thermodynamic arrow in time with classical mechanics
- In other words, one would have to point at initial conditions that ensure the occurrence and persistence of decoherence
- In the two rigorous derivations of the Master eq. (Davies, Lindblad), one assumes a TD arrow in time in the first place (by assuming a heat bath to which the open system relaxes)

CM from QM

There are several results (e.g., Zurek & Paz 1994) where Newtonian chaotic trajectories are derived within the open system approach (QM + decoherence)

However

- Wigner's function (the quantum analogue to the density function in Liouville's theorem) can sometimes take negative values hence cannot be interpreted straightforwardly as a probability distribution
- But even when positive, the Wigner function alone would not suffice, as one would have to interpret the reduced state as an effective collapse, and assign single–time and multi–time probabilities to such states, as in the no–collapse interpretations

What is Classicality?

- Recall Schrödinger: I would not call [entanglement] one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought
- So let's define classicality as absence of entanglement
- And let's see whether decoherence is relevant to that

Disclosure: not only there are several precise measures of entanglement, but also additional "classicality measures", e.g., macroscopic, irreversible, chaotic. . . but I only have 30min :)

Explaining the Unobserved

Why don't we see macroscopically distinguished entangled states?

- Dynamical: Collapse theories (GRW, CSL) give a dynamical answer (these states become unstable as the system's mass density increases)
- Combinatorial: Assume a uniform measure on all possible experimental set–ups. Then the chance of detecting entanglement with an entanglement witness decreases exponentially with the system's size (Pitowsky 2004)

Things Are Different Than What They Seem

Why don't we see macroscopically distinguished entangled states?

- Apologetic: well, (almost) all states are entangled but to see them we need to be able to achieve control of so many degrees of freedom, and the project becomes FAPP impossible

Note

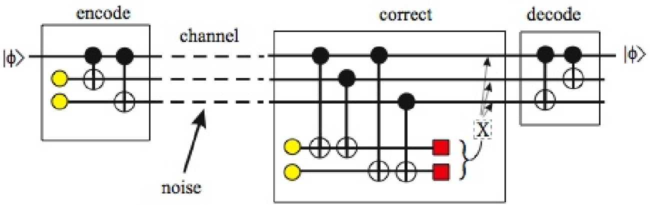
1. Classicality *is* a lie
2. But this lie can never be detected as such
3. So Decoherence can only give rise to an *appearance* of classicality
4. And while the prior two are *explanations*, decoherence is only a consistency proof

Those who cannot remember the past are condemned to repeat it

The timeline

- 1994: Shor's Algorithm
- 1995: Unruh and Landauer: Decoherence constrains scalability
- 1995: QEC
- 1996–7: FTQEC and the threshold theorems (TT) for Markovian noise
- 2005: First published criticism on the physical significance of TT with Markovian noise
- 2007–?: FTQEC for non Markovian noise

Error Correction



A single qubit is encoded into three, and these are sent down the noisy channel. The receiver introduces further qubits (the ancilla) and uses them to extract a syndrome by applying controlled-not gates and then measuring the ancilla bits. The received state can then be corrected. Finally, the single qubit of information may be re-extracted (decoded) from the three (Steane 2003)

Surpassing the No–Cloning Theorem

- QEC is depicted as a successful quantum application of the classical error correction method of the repetition code
- Its expected outcome is the reduce of the error probability when recovering noisy information (for the above example, by a factor of $\frac{1}{3}$
- And do so in such a way that overcomes the major constraints imposed by the unitarity of quantum mechanics, namely the no–cloning theorem

The Threshold Theorems

- Originally conceived (1996) for Markovian noise
- Assumptions:
  1. Error correlations decay exponentially in time and space
  2. Gates can be executed in time τ_g such that $\tau_g \omega = O(\pi)$, where ω is the Bohr or the Rabi frequency
  3. A constant supply of 'fresh', nearly pure, ancilla qubits is available

The Threshold Theorems

Aharonov, Ben Or, Knill, Laflamme, Zurek:

- The error rate decreases faster than the growth in the size of the circuit
- An error threshold exists such that if each gate in a physical implementation of a quantum network has error less than this threshold, it is possible to perform an arbitrary long quantum computation with arbitrary accuracy

The Threshold Theorems

- Unfortunately, these assumptions were shown to be inconsistent:
  - (1) and (2) are incompatible with WCL, and so require SCL, which means that the reservoir (the source for the ancillas) must posses a high temperature, which then contradicts (3)
  - (1) and (3) are incompatible with SCL, and so require WCL, which means that the gate velocity must be slow, which then contradicts (2)

Alicki, Lidar & Zanardi 2006

- This double standard (system – correlated; environment – uncorrelated) appears already in interventionist models of SM

The Threshold Theorems

For non–Markovian noise the issue is still open:

- Length of the quantum computation? requires a delicate analysis of different time scales.
- Error–rates? TT must now deal with *amplitudes* and not with *probabilities*. Thresholds are now worse than the previous (uncorrelated) case
- TT for correlated (non–Markovian) noise explicitly rely on the norm of the interaction Hamiltonian. Low error–rate $\equiv$ very–high–frequency component of the noise is particularly weak
- Physically ill motivated: in some decoherence models it even implies that the system and the environment are practically *de*coupled

Decoherence Free Subspaces

Instead of actively correcting errors, one looks for subspaces which are noise–resilient:

- First determine the commutant of the errors, which is the set of operators that commute with all errors. Then find a subset of the commutant that is algebraically equivalent to the operators characterizing a qubit
- To keep a qubit within this subspace, one uses the quantum Zeno effect
- Open question: how do the resources for doing so (e.g., the measurement frequency) scale?