The development of quantum field theory in condensed matter physics

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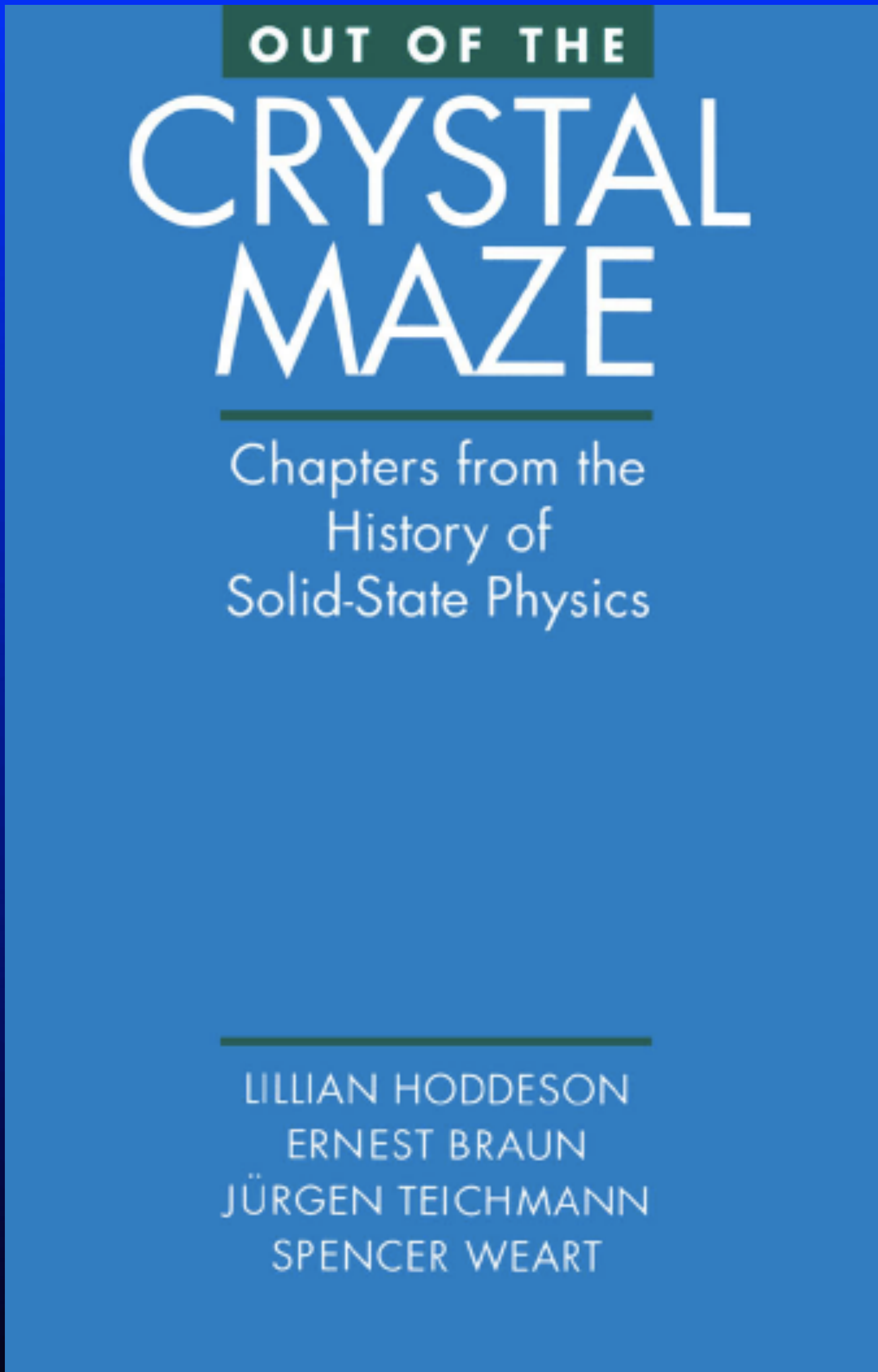
University of Illinois

Seven Pines Symposium
May 18, 2013



The International Project in the History of Solid State Physics

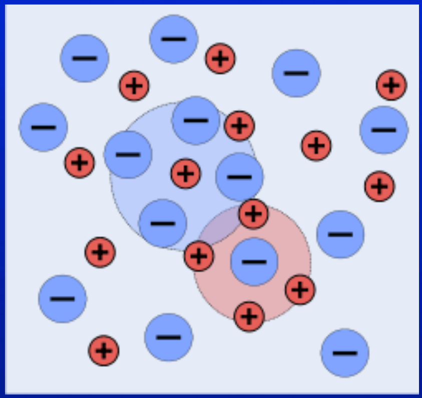
Especially, Chapter 9:
Collective Phenomena,

L. Hoddeson, H. Schubert, S. J. Heims, & GB

Oxford Univ. Press 1992

Pre-World War II roots

Screening of charges: Debye and Hückel, Zürich 1922



short ranged interactions between charges

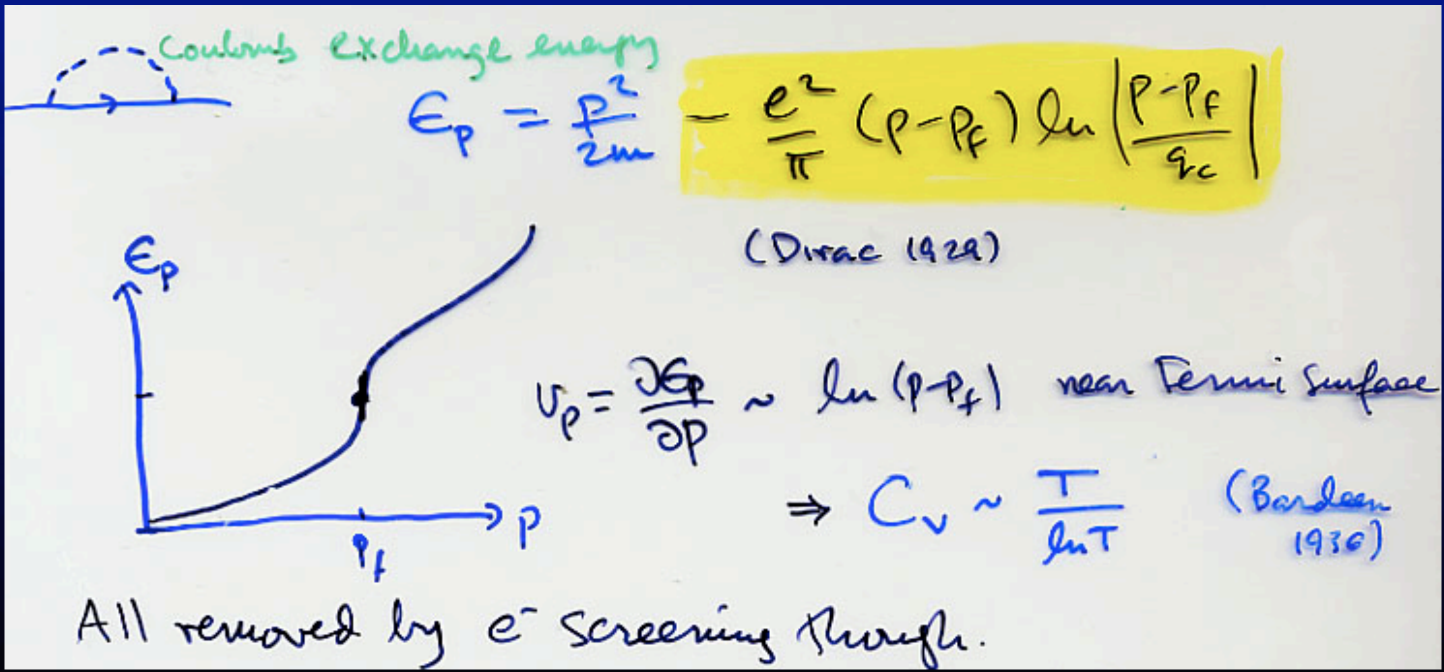
Classical plasmas: Langmuir (GE) 1925-29

understanding vacuum tubes =>
theory of classical plasmas

Correlation energy of electrons in metals:
Wigner, 1934, 1938

how to go beyond simple Fermi gas of electrons to understand binding of real metals?

Specific heat of electron gas and screening:
Bardeen 1936-38



Post-World War II routes to the field-theoretic approach to condensed matter physics

Quantum electron gas

Nuclear matter

Theory of Fermi liquids

BCS theory of superconductivity

Superfluid ^{4}He

Quantum electron gas:

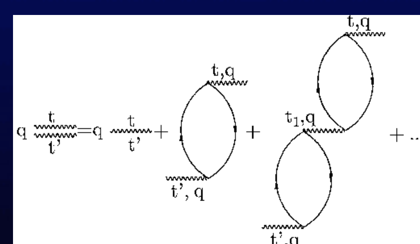
D. Bohm (Berkeley, WWII) works on plasmas, synchrotron radiation; continues at Princeton with D. Pines and E. Gross.


1947 Shelter Island Conference on QED :
Bohm–Pines theory of electron gas ⇔ QED vacuum
Schwinger's canonical transformations

1948 Pocono meeting
Bohm realizes from Schwinger that dynamical screening of electrons in plasmas ⇔ renormalization in quantum electrodynamics.

Bohm-Pines: plasmas oscillations, single particle modes, equations of motion, random phase approximation (RPA).


Gell-mann--Brueckner 1956 electron gas correlation energy
sum bubble diagrams to
remove Coulomb divergence.
Matsubara 1955!

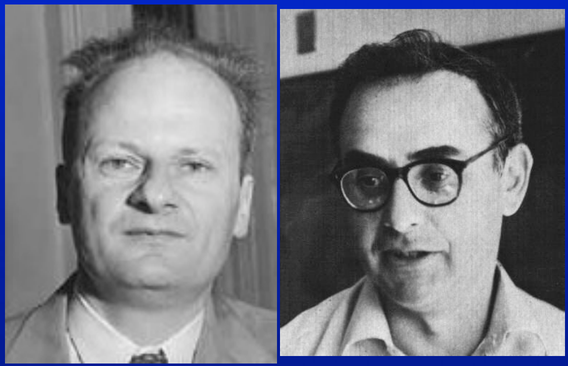


Connection of two approaches: Nozières and Pines, Brueckner and Sawada

Theory of nuclear matter

Understand binding and saturation of extended nuclear matter
(neutrons and protons) in terms of nuclear forces.
Early application of Feynman diagrams to many-body problem

Brueckner 1955
Bethe-Goldstone 1955

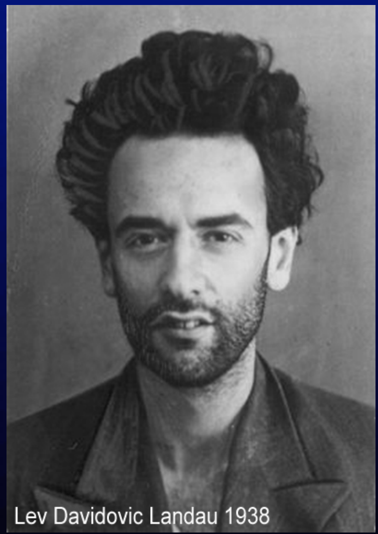


Theory of Fermi liquids

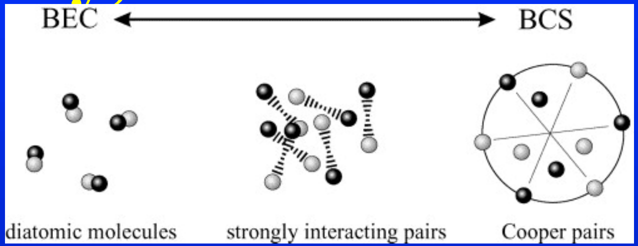
Landau theory of Fermi liquids, 1957
exact description of strongly interacting system
in terms of quasipartilces -- application to ^{3}He liquid

Derivation by Luttinger, Nozières, 1962

Galitskii- Migdal, microscopic calculations, 1957



BCS Theory of Superconductivity, 1956-7



Bohm-Pines => Bardeen at Princeton 1950.
Pines to Urbana 1952.

Strong coupling polaron problem. Lee, Low & Pines, 1953

Quantum field theory connections:
  Lee's intermediate coupling theory in polaron
  Tomonaga – 1D strongly coupled electron gas.

Bardeen: *"it was becoming clear that [quantum] field theory might be useful in solving the many-body problem of a Fermi gas with attractive interactions between the particles."*

Cooper (nuclear and field theory at Columbia) brought field theory ideas to Urbana .

Bardeen-Pines effective interaction, 1955 – phonons + Coulomb = attractive

L. Gor'kov 1958: formulation of BCS theory diagramatically

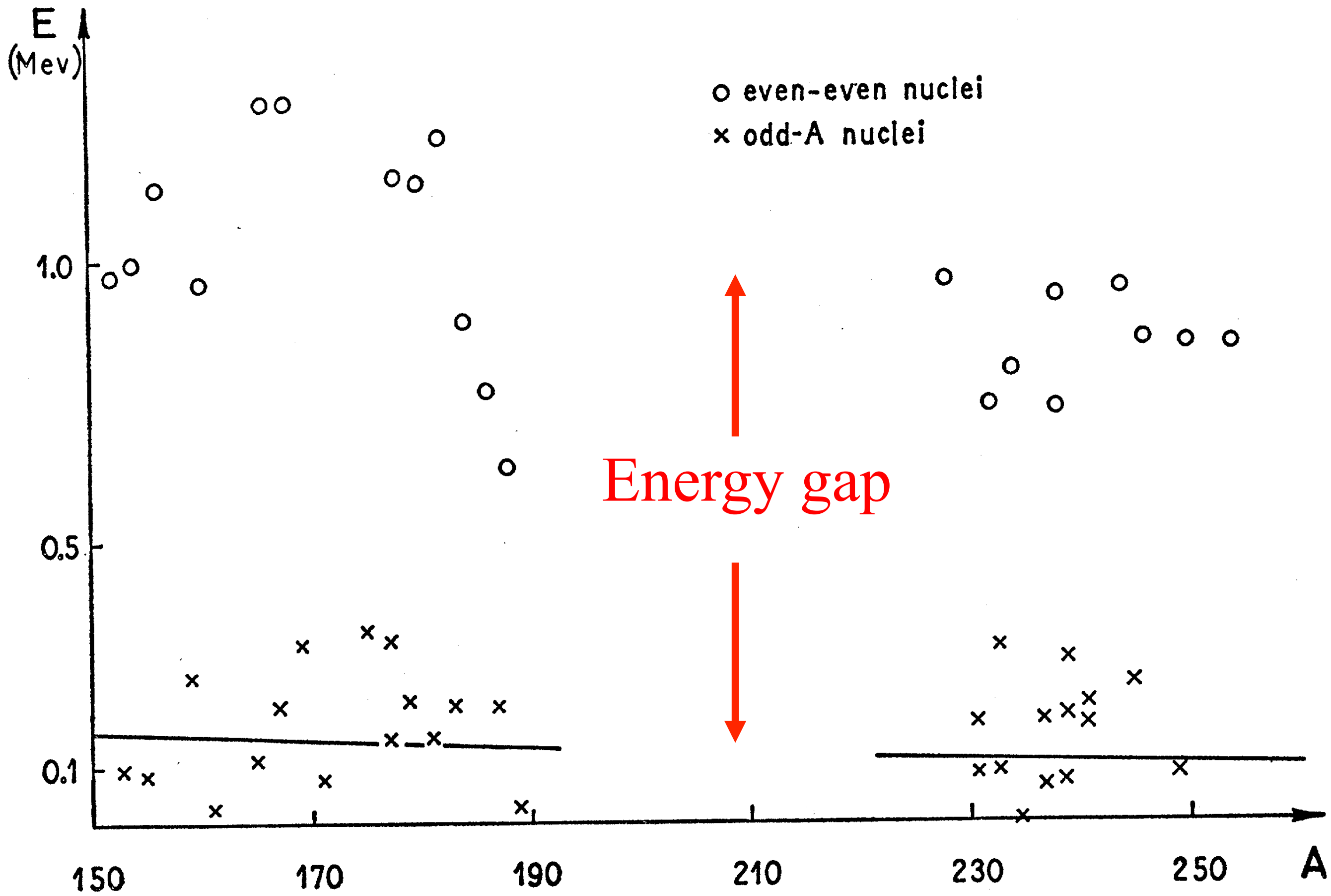
Immediate application to nuclei, A. Bohr, B. Mottelson & D. Pines, 1958; and neutron stars, Migdal, 1959.

Energies of first excited states: even-even (BCS paired) vs. odd A (unpaired) nuclei

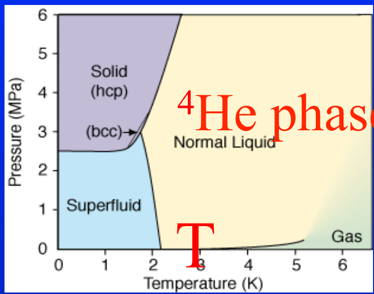
FIG. 1. Energies of first excited intrinsic states in deformed nuclei, as a function of the mass number. The experimental data may be found in *Nuclear Data Cards* [National Research Council, Washington, D. C.] and detailed references will be contained in reference 1 above. The solid line gives the energy $\delta/2$ given by Eq. (1), and represents the average distance between intrinsic levels in the odd-A nuclei (see reference 1).

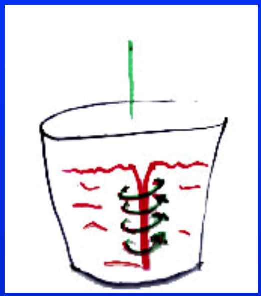
The figure contains all the available data for nuclei with $150<A<190$ and $228<A$. In these regions the nuclei are known to possess nonspherical equilibrium shapes, as evidenced especially by the occurrence of rotational spectra (see, e.g., reference 2). One other such region has also been identified around $A=25$; in this latter region the available data on odd-A nuclei is still represented by Eq. (1), while the intrinsic excitations in the even-even nuclei in this region do not occur below 4 Mev.

We have not included in the figure the low lying $K=0$ states found in even-even nuclei around Ra and Th. These states appear to represent a collective odd-parity oscillation.



Superfluid ^{4}He





quantized vortices

Brueckner and Sawada (U. Penn) 1956 -- diagrammatic approach

S. Beliaev 1956 -- generalization of electron gas calculations to formulate Bogolioubov canonical transformation diagrammatically

E. Gross and L. Pitaevskii 1961 -- independently introduce spatially dependent order parameter – earlier treated as a numerical parameter in the theory. cf. Ginzburg-Landau

Superfluid ^{3}He?

Brueckner & Soda 1960, Anderson & Morel 1961, Balian & Werthamer 1963

Putting it all together – Green's function formalism

Basic tool is propagators or Green's functions as in QED – Feynman and Schwinger

Work on many-body problem at zero temperature – how to generalize to do statistical mechanics at non-zero temperature?

Diagrammatics for partition function: Matsubara, A New Approach to Quantum-Statistical Mechanics, 1955

"the grand partition function, which is a trace of the density matrix expressed in terms of field operators, can be evaluated in a way almost parallel with the evaluation of the vacuum expectation values of the S-matrix in quantum field theory, ..."

Introduction of imaginary time: $t \subset [0,-i\beta]$ where $\beta = 1/KT$.

Statistical mechanics in terms of imaginary time:

Rudolf Peierls, Werner Heisenberg; *(rear) G.* Gentile, George Placzek, Giancarlio Wick, Felix Bloch, Victor Weisskopf, F. Sauter: Leipzig, 1931

Statistical density matrix $\mathbf{e^{-H/T}} \rightarrow$

time-evolution operator $\mathbf{e^{-iHt/\hbar}}$ for $\mathbf{t = -i\hbar/T}$

No deep underlying physics!

F. Bloch, Habilitationsschrift*:*
Zur Theorie des Austauschproblems und der Remanenzerscheinung der Ferromagnetika, *Zs. Phys.* 74 (1932): 295-335.

Matsubara focusses on calculating partition function $\mathbf{Z} = \mathrm{Tr}\,\mathbf{e}^{-\beta\mathbf{H}}$
of interacting system, $H = H_0 + H_1$,
cf. that of non-interacting system

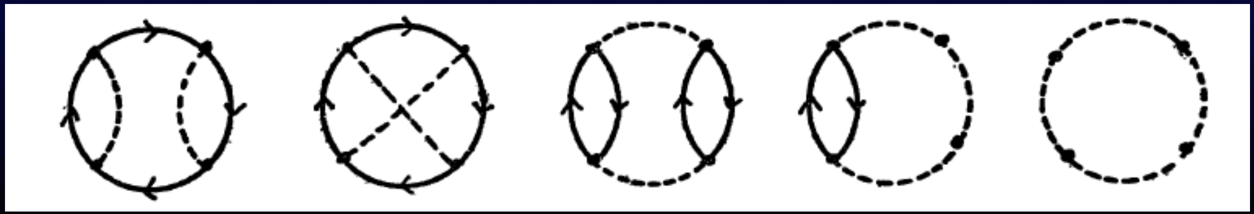
$$\exp(-\beta H) = \exp(-\beta H_0) \cdot S(\beta)$$

Expands in terms of perturbation H_1

$$S(\beta) = \sum_{n=0}^{\infty} (-1)^n \int_0^\beta dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} \cdots \int_0^{t_{n-1}} dt_n H_1(t_1) H_1(t_2) \cdots H_1(t_n)$$
$$= \sum_{n=0}^{\infty} (-1)^n / n! \int \cdots \int P[H_1(t_1) \cdots H_1(t_n)] dt_1 \cdots dt_n,$$

P = time ordering from 0 to β

Diagrammatic expansion of partition function



JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN Vol. 12, No. 6, JUNE, 1957

Statistical-Mechanical Theory of Irreversible Processes. I.

General Theory and Simple Applications to Magnetic and Conduction Problems

By Ryogo KUBO

Department of Physics, University of Tokyo

(Received March 2, 1957)

Understanding boundary conditions in imaginary time.

Derivation of Onsager relations in terms of thermal correlation functions

Kubo relations for transport coefficients, viscosity and conductivity in terms of correlation functions

A few papers from U.S. East Coast on development of quantum field theory for the many-body problems, c. 1960

P.C. Martin & J. Schwinger, *Theory of many-particle systems*,
Phys. Rev. 115, 1342 (1959)

J.M. Luttinger and J.C. Ward, *Ground-state energy of a many-fermion system*. II, Phys. Rev. 118, 1417 (1960)

J. Schwinger, *Brownian motion of a quantum oscillator*
J. Math. Phys. 2, 407 (1961) -- the round trip contour

GB and L.P. Kadanoff, *Conservation laws and correlation functions*,
Phys. Rev. 124, 287 (1961)

GB, *Self- consistent approximations in many body systems*,
Phys. Rev. 127, 1391 (1962)

L.P. Kadanoff and GB, *Quantum statistical mechanics* (1962)

PHYSICAL REVIEW VOLUME 115, NUMBER 6 SEPTEMBER 15, 1959

Theory of Many-Particle Systems. I*†

PAUL C. MARTIN AND JULIAN SCHWINGER
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts

(Received March 20, 1959)

[1] The many-body problem has been studied with the aid of perturbation theory by many authors. This paper will not draw on any results of these works but has various points of contact with them. We mention the work of T. Matsubara, Progr. Theoret. Phys. (Kyoto) **14**, 351 (1955); K. M. Watson, Phys. Rev. **103**, 489 (1956); W. Riesenfeld and K. M. Watson, Phys. Rev.

explicit boundary conditions on single particle Green's fcns

$$G_<^{i\lambda,\, i\tau}(\mathbf{r}t;\mathbf{r}'t') = \pm e^{-W}\,\mathrm{Tr}[e^{-i\lambda}e^{-iN\lambda - iH\tau}(1/i) \times \psi(\mathbf{r}, t+\tau)\psi^\dagger(\mathbf{r}'t')] = \pm e^{-i\lambda}G_>(\mathbf{r}t+\tau;\mathbf{r}'t').$$

implementation in terms of Fourier sums over discrete "Matsubara frequencies"

$$\exp(-i\pi\nu t/\tau). \quad \begin{cases} \nu \text{ even} & (\text{B.E.}) \\ \nu \text{ odd} & (\text{F.D.}) \end{cases}$$

$$\omega_\nu = \frac{\pi\nu}{-\mathbf{i}\beta}$$

Green's function look-up sheet

Greater and lesser correlation functions:

$$G^{>}(1,2) = \langle \psi(1)\psi^{\dagger}(2)\rangle, \quad 1 = r_1, t_1, \ldots$$
$$G^{<}(1.2) = \langle \psi^{\dagger}(2)\psi(1)\rangle$$

Fourier transforms in terms of spectral weight A:

$$G^{<}(p,\omega) = A(p.\omega) f(\omega)$$
$$G^{>}(p,\omega) = A(p,\omega)(1 \pm f(\omega))$$

$$f(\omega) = \frac{1}{e^{\beta\omega} \mp 1}$$

Green's function in complex frequency plane:

$$G(p,z) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{A(p,\omega)}{z-\omega}$$

Self-energy:

$$G^{-1}(p,z) = z - p^2/2m - \Sigma(p,z)$$

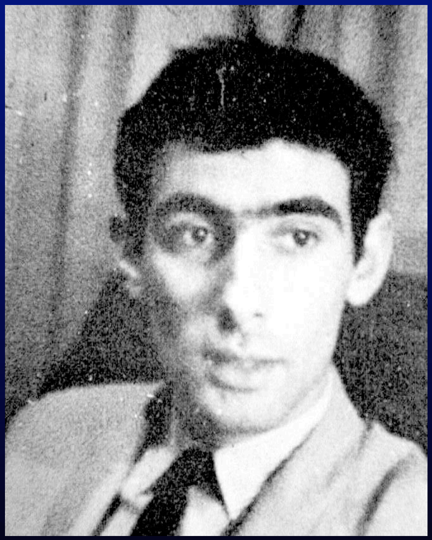
Retarded and advanced Green's functions for real frequency

$$G_{ret}(p,\omega_1) = \lim_{\eta \to 0+} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{A(p,\omega)}{\omega_1 + i\eta - \omega}$$

$$G_{adv}(p,\omega_1) = \lim_{\eta \to 0+} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{A(p,\omega)}{\omega_1 - i\eta - \omega}$$

1960-2

Niels Bohr Institute, Copenhagen



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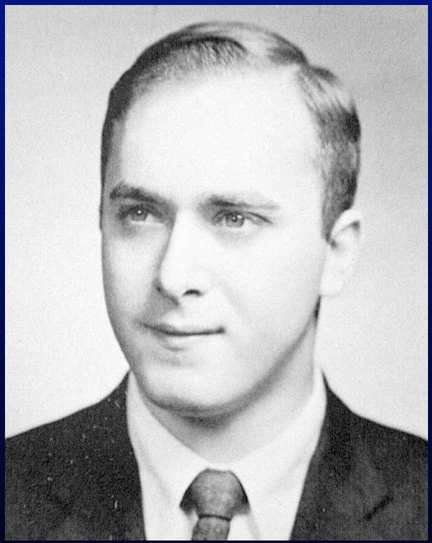
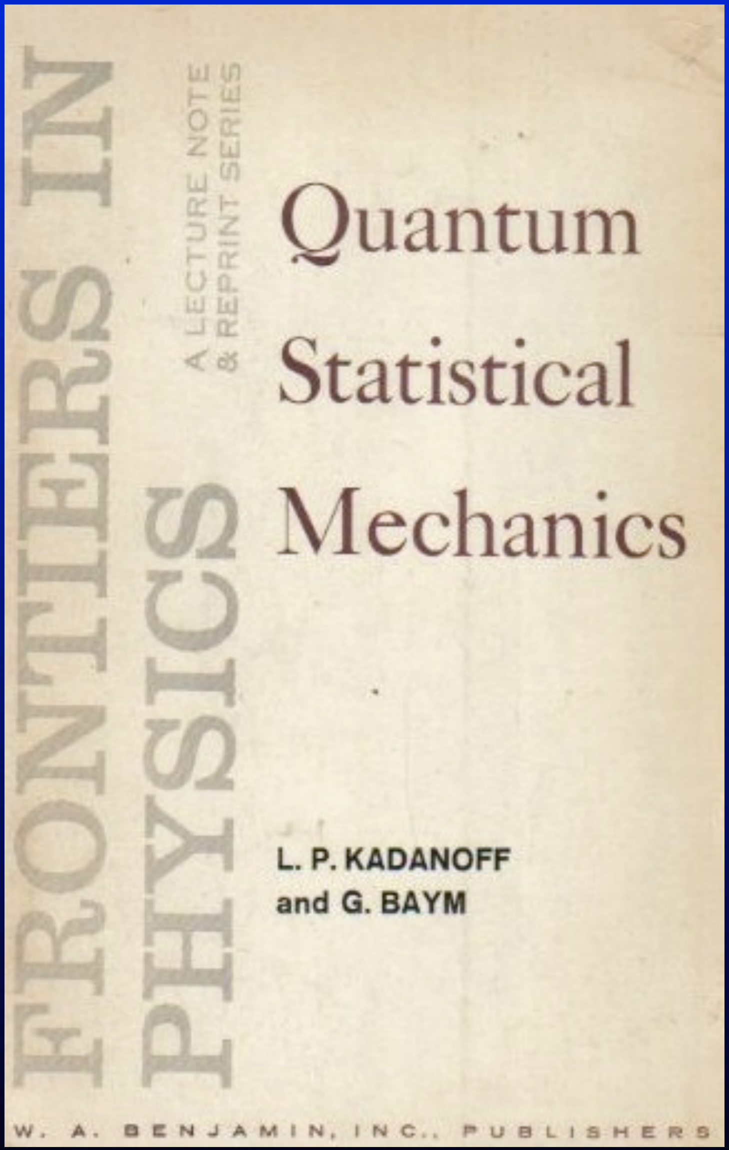
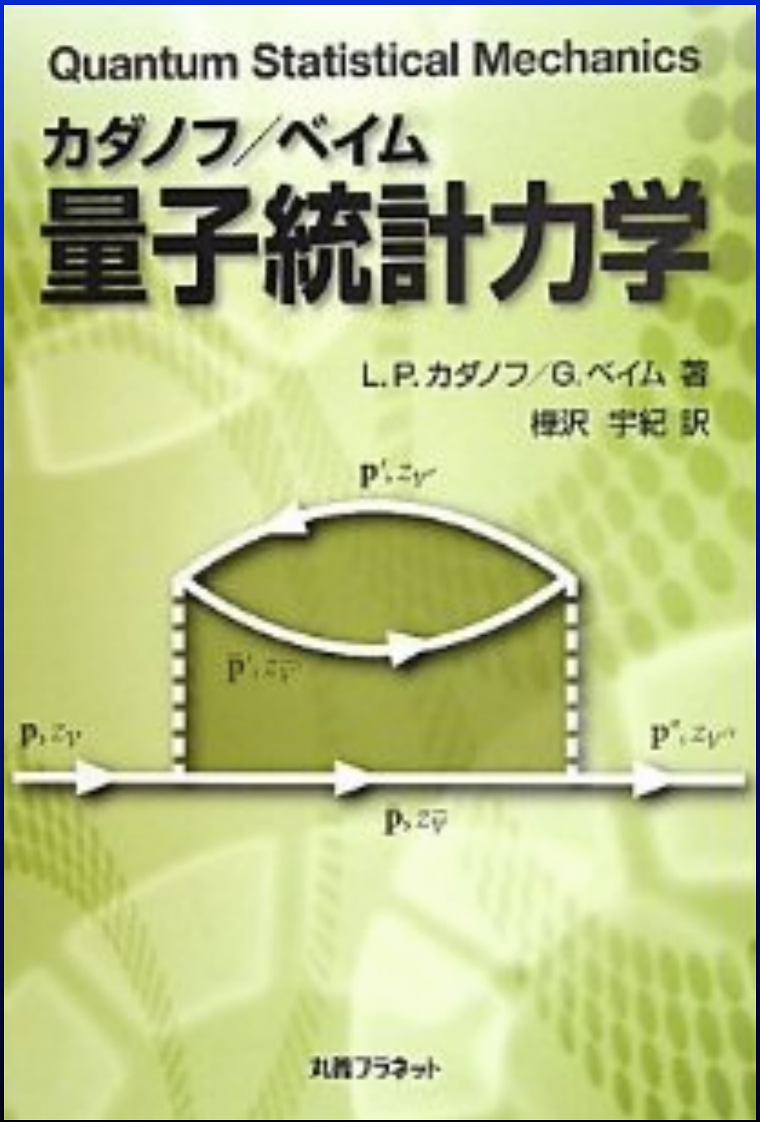


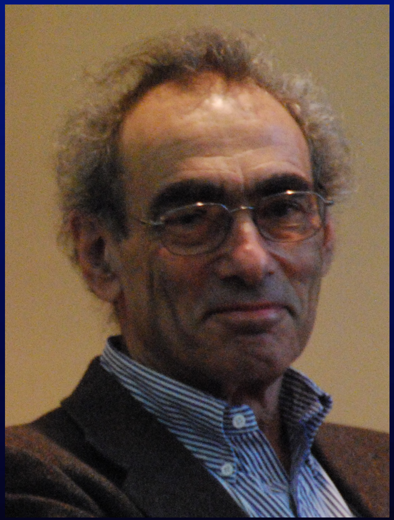
Leo Kadanoff



2011

Niels Bohr Institute, Copenhagen





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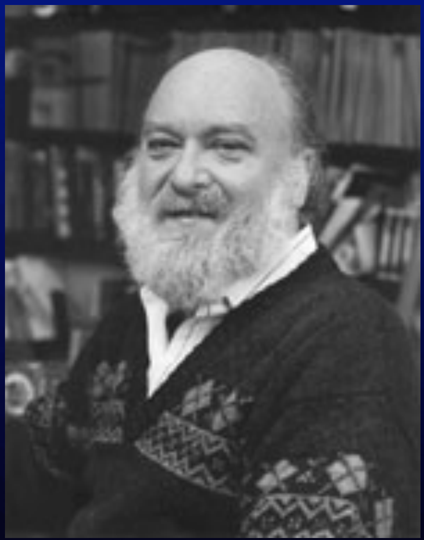

Leo Kadanoff

... and in Moscow at the time, quite independently

L.D. Landau, N.N. Bogoliubov, A. A. Abrikosov, L.P. Gorkov, L.E. Dzyaloshinskii, A.B. Migdal, L. Pitaevskii, D.N. Zubarev, L. Keldysh

N.N. Bogoliubov & S.V. Tyablikov, *Retarded and advanced Green functions in statistical physics*, Sov. Phys Doklady, **4**, 589 (1959).

D.N. Zubarev D. N., *Double-time Green functions in statistical physics*, Soviet Physics Uspekhi **3**(3), 32 (1960).

L.V. Keldysh, *Diagram technique for nonequilibrium processes,* ZhETF 47, 1515 (1964) [Sov. Phys. JETP 20, 1018 (1965)].

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BOOKS:

А.А. Абрикосов, Л.П Горьков & И.Е Дзялошинский, *Квантовые полевые теоретические методы в статистической физик,* 1961 (English *edition*, A. A. Abrikosov, I. E. Dzyaloshinskii, and L. P. Gor'kov, *Methods of Quantum Field Theory in Statistical Physics* 1963)..
Also ZhETF 36, 900 (1959) [(Sov Phys. JETP 9, 636 (1959)].

V.L. Bonch-Bruevich & S.V. Tyablikov, *The Green Function Method in Statistical Mechanics* (1962).

Preserving the conservation laws, gauge invariance, Ward identities

How does one guarantee that approximations to correlation functions (e.g., 2 particle Green's functions) maintain gauge invariance and satisfy particle number, momentum, and energy conservation?

$$\frac{\partial \rho(rt)}{\partial t} + \nabla \cdot \vec{j}(rt) = 0$$

operator particle-conservation law

Time-ordered (T) correlation functions should obey

$$\frac{\partial}{\partial t}\langle T\left(\rho(rt)\mathcal{O}(r't')\right)\rangle + \nabla \cdot \langle T(\vec{j}(rt)\mathcal{O}(r't'))\rangle = \langle[\rho(rt), \mathcal{O}(r't)]\rangle\delta(t-t').$$

($\mathcal{O}$ is arbitrary operator) But even many reasonable approximations do not!!! Not enough to conserve particle number, momentum, and energy at the vertices in diagrams.

THE
PHYSICAL REVIEW
A journal of experimental and theoretical physics established by E. L. Nichols in 1893
SECOND SERIES, VOL. 124, NO. 2 OCTOBER 15, 1961

Conservation Laws and Correlation Functions

GORDON BAYM AND LEO P. KADANOFF
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark
(Received May 15, 1961)

In describing transport phenomena, it is vital to build the conservation laws of number, energy, momentum, and angular momentum into the structure of the approximation used to determine the thermodynamic many-particle Green's functions. A method for generating conserving approximations has been developed. This method is based on a consideration, at finite temperature, of the equations of motion obeyed by the one-particle propagator G, defined in the presence of a nonlocal external scalar field U. Approximations for $G(U)$ are obtained by replacing the $G_2(U)$ which appears in these equations by various functionals of $G(U)$. If the approximation for $G_2(U)$ satisfies certain simple symmetry conditions, then the $G(U)$ thus defined obeys all the conservation laws. Furthermore, the two-particle correlation function, generated as $(\delta G/\delta U)_{U=0}=\pm L$, in terms of which all linear transport can be described, will obey all the conservation laws as well as several essential sum rules, such as the longitudinal f-sum rule.

Several examples of conserving approximations are described. The Hartree approximation, $G_2(U)=G(U)G(U)$, generates the random-phase approximation for L. The Hartree-Fock approximation for $G(U)$ leads to a natural generalization of the random-phase approximation in which hole-particle ladder diagrams are summed. Another conserving approximation for $G(U)$ is obtained by expanding the self-energy to first order in the many-particle scattering matrix $T(U)$. This T is obtained by summing ladder diagrams in which the sides of the ladder are composed of $G(U)$'s. The resulting L equation, which involves coefficients proportional to $|T|^2$, is analogous to the linearized version of the usual Boltzmann equation. Finally, in order to obtain a description of collisions in a plasma, the self-energy is expanded to first order in a dynamically shielded potential, $V_s(U)$. This potential is obtained by summing bubbles composed of two $G(U)$'s. The resulting L equation is similar in structure to a Boltzmann equation in which the collision cross section is proportional to $|V_s|^2$.

PHYSICAL REVIEW VOLUME 127, NUMBER 4 AUGUST 15, 1962

Self-Consistent Approximations in Many-Body Systems

GORDON BAYM*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark
(Received March 26, 1962)

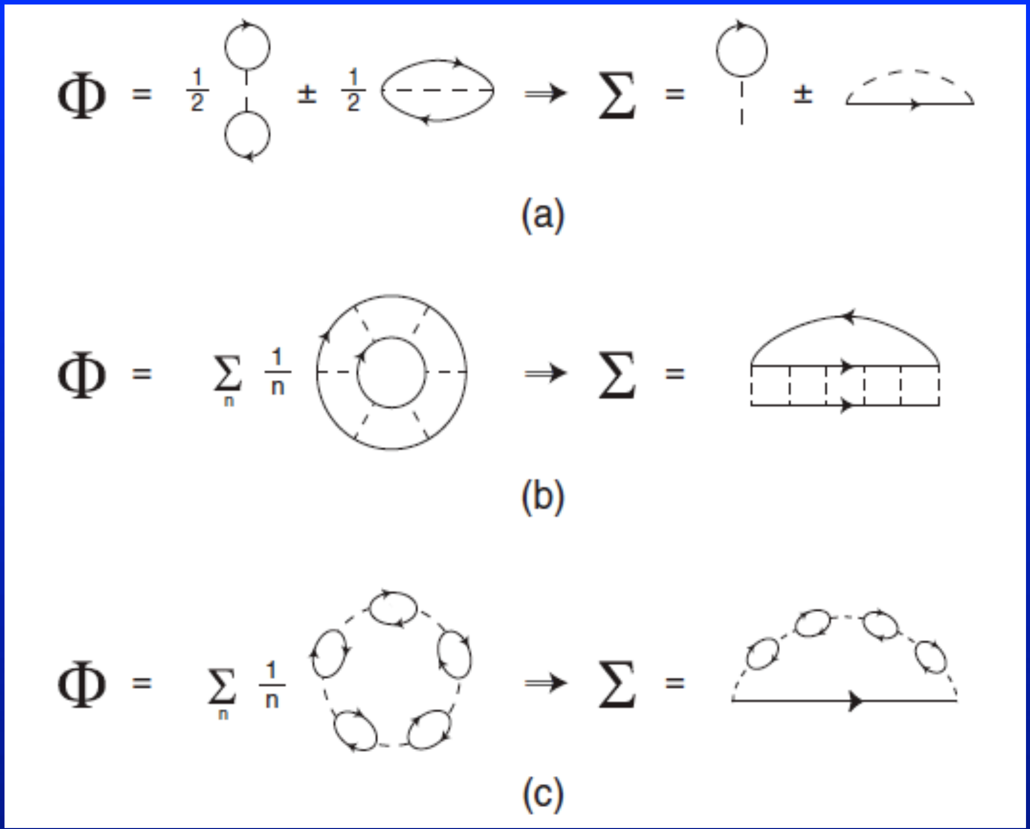
This paper investigates the criteria for maintenance of the macroscopic conservation laws of number, momentum, and energy by approximate two-particle correlation functions in many-body systems. The methods of generating such approximations are the same as in a previous paper. However, the derivations of the conservation laws given here clarify both why the approximation method works and the connection between the macroscopic conservation laws and those at the vertices.

Conserving nonequilibrium approximations are based on self-consistent approximations to the one-particle Green's function. The same condition that ensures that the nonequilibrium theory be conserving also ensures that the equilibrium approximation has the following properties. The several common methods for determining the partition function from the one-particle Green's function all lead to the same result. When applied to a zero-temperature normal fermion system, the approximation procedure maintains the Hugenholtz-Van Hove theorem. Consequently, the self-consistent version of Brueckner's nuclear matter theory obeys this theorem.

Generate approximations starting with a functional of the single particle Green's function G. $\Phi(G)$

Variation of Φ with respect to G gives the self-energy $\delta\Phi[G] = \mathrm{tr}\,\Sigma\delta G$

solve $G^{-1}(p,z) = z - p^2/2m - \Sigma(p,z)$ self-consistently for G



Hartree-Fock

Ladders (t-matrix)

Rings (shielded potential, GW)

All conservation laws obeyed by approximate correlation functions, including Galilean invariance.

All ways of calculating thermodynamics (e.g., from G, or coupling constant integration) give same results:

Partition function $\ln Z = \pm[\Phi[G] - \mathrm{tr}\,\Sigma G + \mathrm{tr}\,\log(-G)]$

Leads to non-equilibrium theory with same correlations as in equilibrium

Non-equilibrium Green's functions on round trip contour

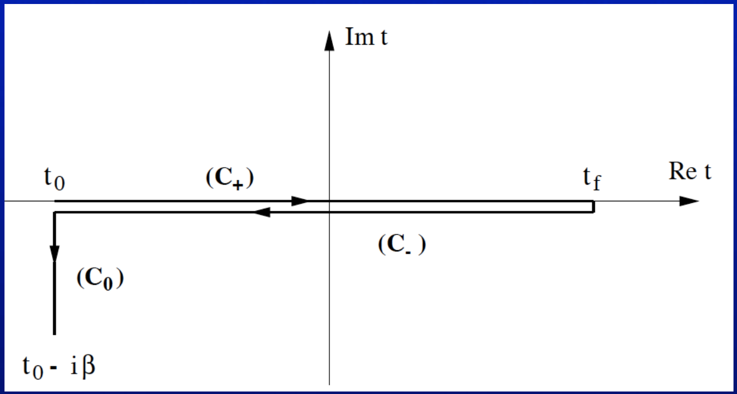
Green's function

$$G(12;U) = -i\,\frac{\mathrm{tr}\left[e^{-\beta(H-\mu N)}T\left(e^{-i\int dt H_{\mathrm{ext}}}\psi(1)\psi^{\dagger}(2)\right)\right]}{\mathrm{tr}\left[e^{-\beta(H-\mu N)}T\left(e^{-i\int dt H_{\mathrm{ext}}}\right)\right]}$$

in presence of external driver, e.g.

$$H_{\mathrm{ext}}(t) = \int d^3r\, U(r,t)\rho(r,t)$$

on contour in complex time plane



Equation of motion on contour:

$$\left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} - U(rt)\right)G(rt,r't') - \oint \Sigma(rt,\bar{r}\bar{t})G(\bar{r}\bar{t},r't') = \delta(r-r')\delta(t-t').$$

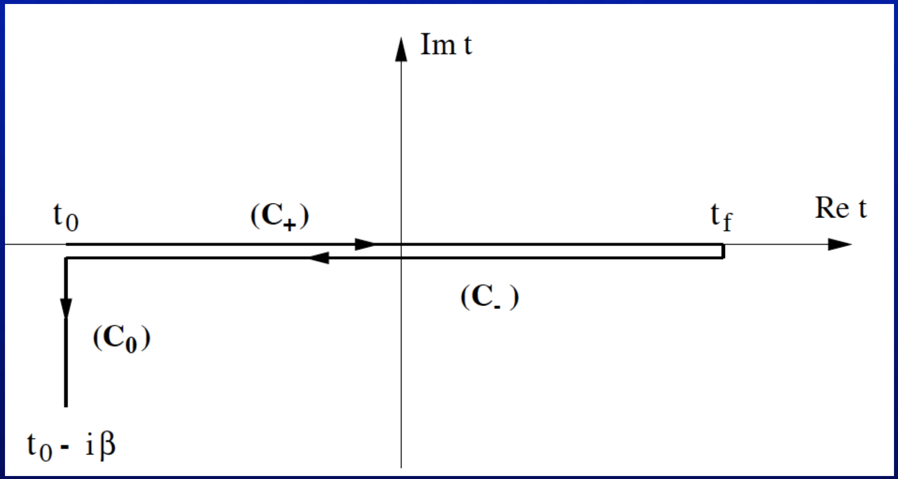
Equilibrium correlations built in on time interval (0, -i β)

Non-equilibrium Green's functions on round trip contour

$$G(12;U) = -i\,\frac{\mathrm{tr}\left[e^{-\beta(H-\mu N)}T\left(e^{-i\int dt H_{\mathrm{ext}}}\psi(1)\psi^\dagger(2)\right)\right]}{\mathrm{tr}\left[e^{-\beta(H-\mu N)}T\left(e^{-i\int dt H_{\mathrm{ext}}}\right)\right]}$$

$$\left(i\frac{\partial}{\partial t}+\frac{\nabla^2}{2m}-U(rt)\right)G(rt,r't')-\oint \Sigma(rt,\bar{r}\bar{t})G(\bar{r}\bar{t},r't')=\delta(r-r')\delta(t-t').$$

On the contour



t_u on upper contour
t_l on lower contour

Matrix Green's functions

$$\begin{pmatrix} G(t_u,t'_u) & G(t_u,t'_\ell) \\ G(t_\ell,t'_u) & G(t_\ell,t_\ell) \end{pmatrix}$$

Matrix self-energy

$$\begin{pmatrix} \Sigma(t_u,t'_u) & \Sigma(t_u,t'_\ell) \\ \Sigma(t_\ell,t'_u) & \Sigma(t_\ell,t_\ell) \end{pmatrix}$$

Keldysh approach

or work in terms of the correlation functions

$$g^<(p\omega RT) = \int drdte^{-ip\cdot r+i\omega t}\langle\psi^\dagger(1')\psi(1)\rangle$$

$$g^>(p\omega RT) = \int drdte^{-ip\cdot r+i\omega t}\langle\psi(1)\psi^\dagger(1')\rangle$$

$r = r_1 - r'_1, t = t_1 - t'_t$ and $R = (r_1 + r'_1)/2, T = (t_1 + t'_1)/2$

Using equation of motion for G(11') on round-trip contour construct Eqs. for $g^<$ (and $g^>$) for slowly varying disturbances:

$$\frac{\partial g^<}{\partial t} + \frac{\partial g^<}{\partial \omega}(U + Re\Sigma) + \frac{p}{m}\cdot\nabla_R g^< - \nabla_R(U + Re\Sigma)\cdot\nabla_p g^<$$
$$+\frac{\partial Re\, g}{\partial \omega}\frac{\partial \Sigma^<}{\partial t} - \frac{\partial Re\, g}{\partial t}\frac{\partial \Sigma^<}{\partial \omega} + \nabla_p Re\, g\cdot\nabla_R\Sigma^< - \nabla_R Re\, g\cdot\nabla_p\Sigma^<$$
$$= -\Sigma^> g^< + \Sigma^< g^>$$

Generalized non-equiliubrium Boltzmann (or Kadanoff-Baym) equations

Keldysh remarks on the original motivation of his work:

I agree completely that idea of contour was in the air and not only in Harvard. My 1964 paper ... after very brief introduction starts with the reference "Following the method of Konstantinov and Perel' [Konstantinov, O. V., and Perel, V. I., 1960, Zh. Eksp. Teor. Fiz., 39, 197; [Sov. Phys. JETP, 1961, 12, 142].]... . In this paper published in 1960 authors used exactly the same evolution contour which Schwinger uses in his paper in the J. Math. Phys. Unfortunately their perturbation theory is very much different from the standard field theoretical and is deprived of universality, elegance and many other advantages of the Schwinger-Feynman-Dyson technique. To improve the Konstantinov-Perel's theory was the original motivation of my study.

Leonid Keldysh reminiscences about 1962 notes by Bob Mills

There were neither PCs nor Xerox in 1960s. And the preprints were relatively new information tool. Preprints were printed in some restricted number and sent to the narrow list of colleges supposed to be interested in the subject. My name was absolutely unknown to the Quantum Many Body community. So Mills did not sent his paper to me or anybody around me. So I was absolutely unaware of its existence until after submitting my paper to JETP. I came to the Landau seminar ... After seminar I. Dzyaloshinskii told me "somewhere I have heard something about Green's Function matrices". And later called to me and told about that Mills preprint. But warned "be careful: pre-print two years old but still nothing published. Probably something is wrong". There was no possibility to make any copy to check that warning. However I was not worried after it became clear that preprint is only about thermodynamically equilibrium states and thus did not overlap with my paper. Still that was the real problem for me: can I refer to the text which is neither published nor sent to me? Finally I decided to interrupt publishing and insert the reference. I wonder whether did exist that time any other references to this preprint beside that in my paper.

By early 1960's had unified quantum field theory of many-body systems encompassing:

interacting quantum many-body systems in equilibrium and perturbed out of equilibrium, from condensed matter to nuclear

BCS paired systems and Bose-Einstein condensation and other macroscopic quantum phenomena. Broken symmetry.

Relativistic quantum field theory at finite temperature (rediscovered in 70's by particle theorists!); phase transitions in the early universe, quark-gluon plasmas, ...

Developed in U.S., Soviet Union, and Japan, albeit with little cross-fertilization

"the recent developments in the many-body problem ... have tended to change it from a quiet corner of theoretical physics to a major crossroad." Pines ca. 1961


THANK YOU