QED and the man who didn‘t make it

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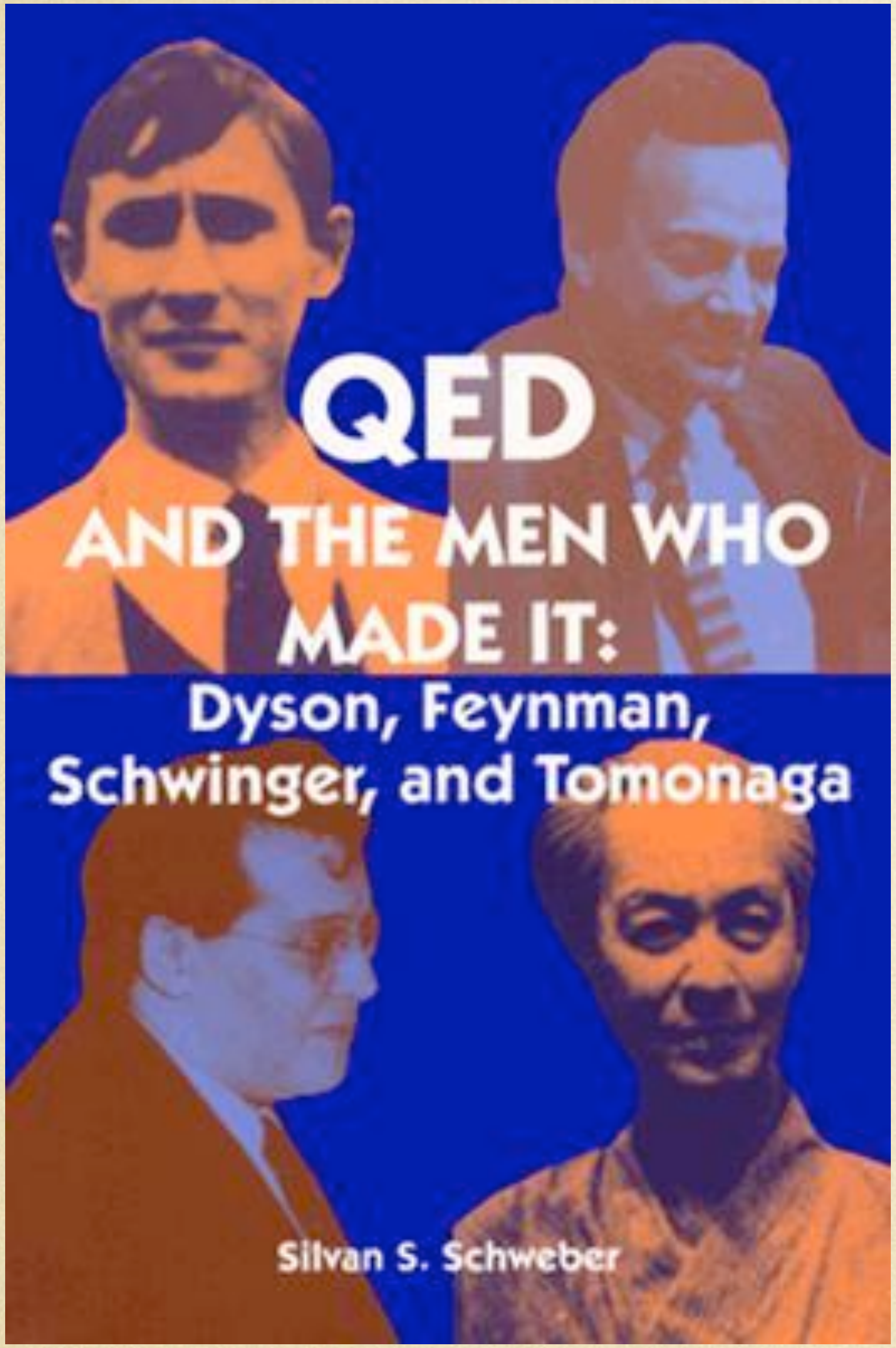
Sidney Michael
Dancoff
(1913-1951)



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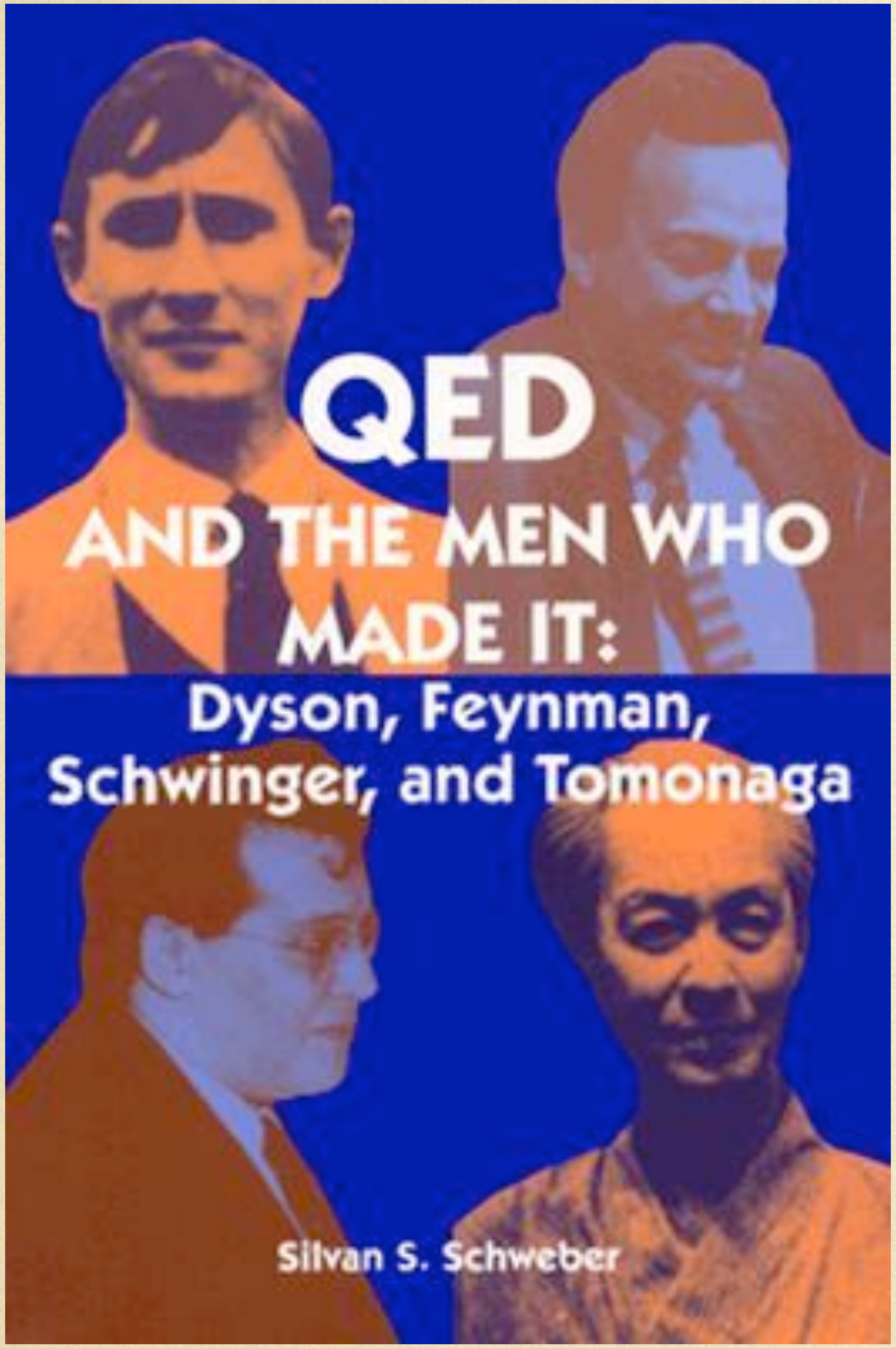

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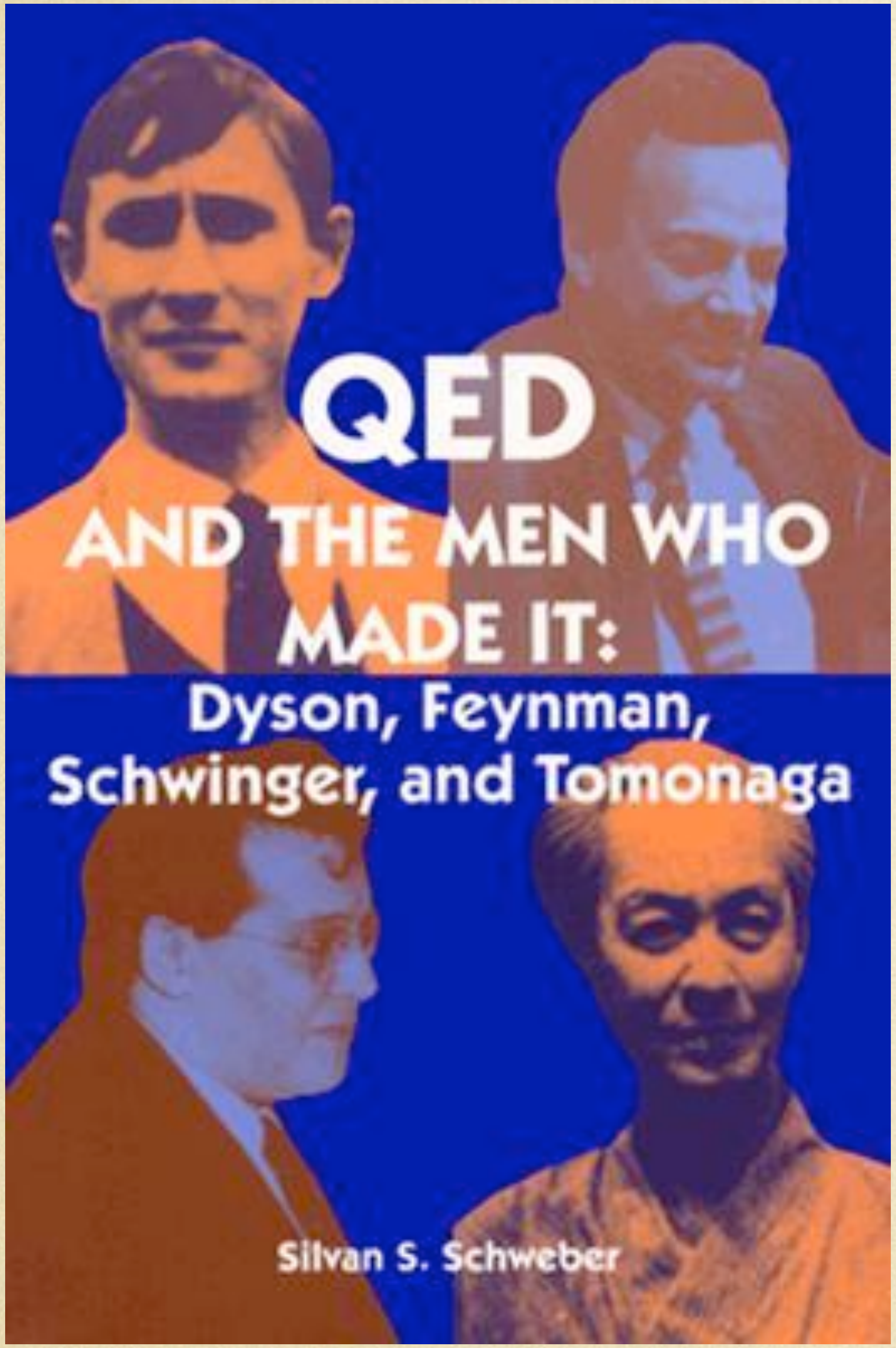
What motivated these calculations?



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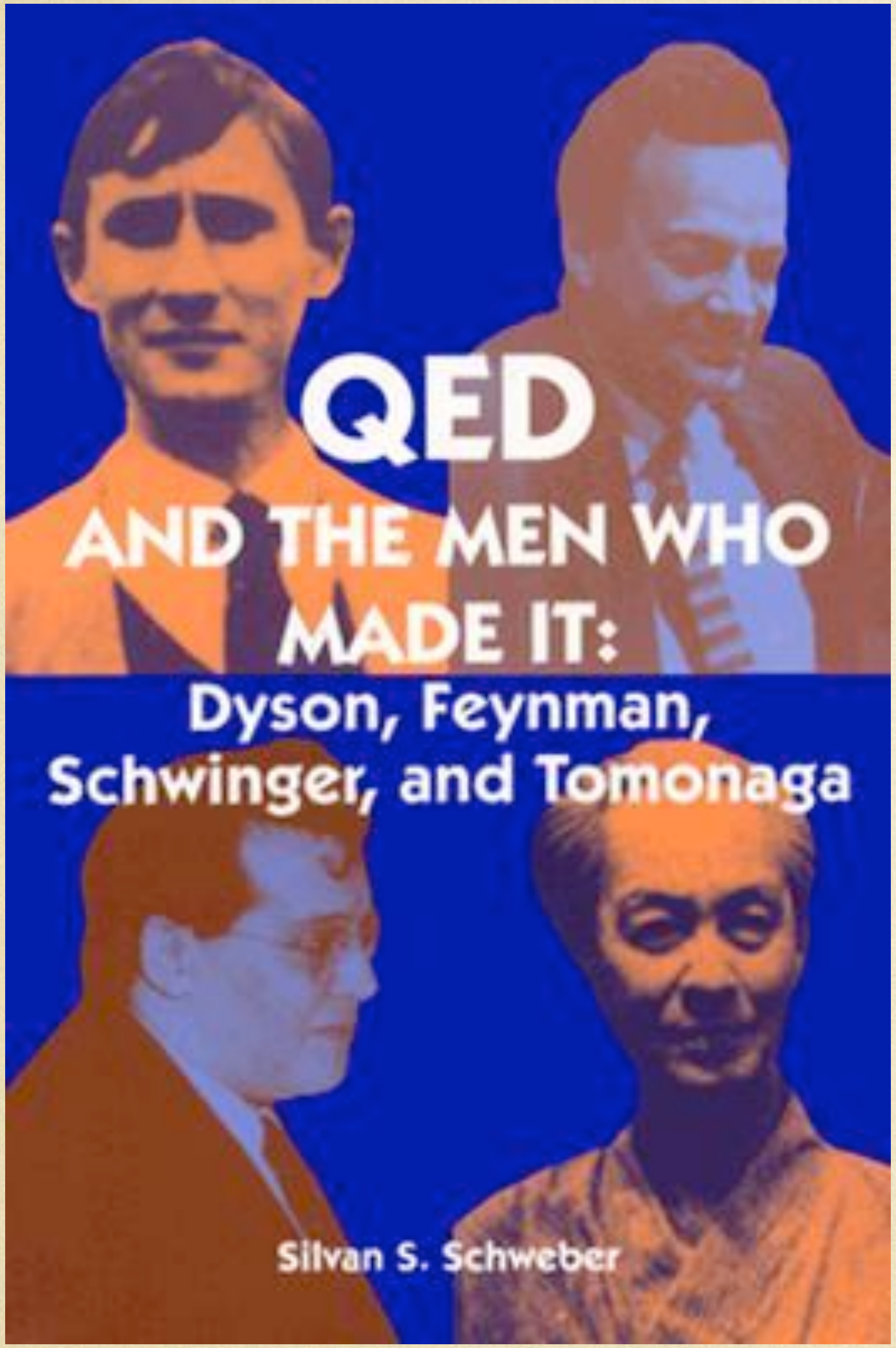
Is this really what he did?



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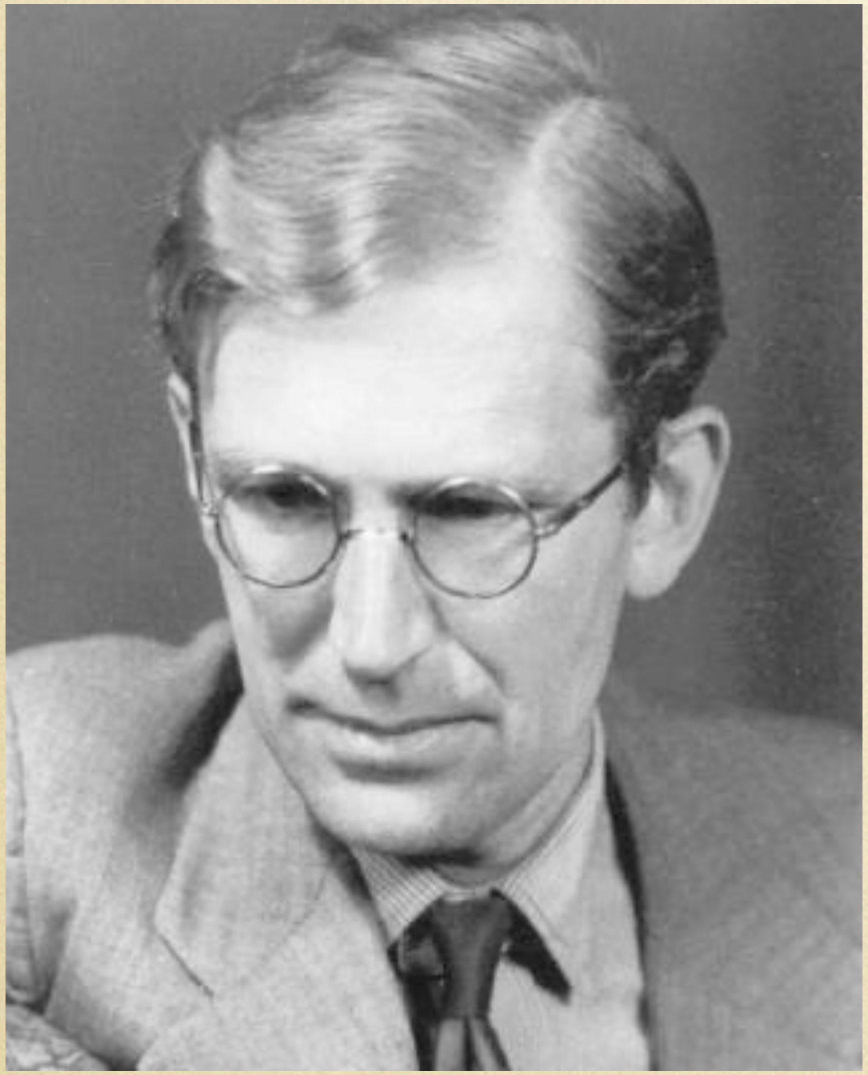
Can we understand why this mistake was made?

The Infrared Divergence in QED (1931)

$$I_\nu d\nu\, d\omega = \left(\frac{Z\epsilon^2}{2mv^2}\right)^2 \frac{\epsilon^2}{hc} \frac{v^2}{c^2} \frac{16}{3\pi} \frac{d\nu}{\nu} d\omega\, (k/k' + k'/k - 2\cos\theta)^{-1}. \quad \ldots(20)$$



Nevill Mott (1905-1996)

„The formula (20) is clearly not correct for all ν, since it becomes infinite for ν→0, making the total number of electrons scattered in any solid angle infinite. This [...] must be due to the method of successive approximations that we used [...] to solve this equation.“

The infrared divergence solved (1937)

Felix Bloch
(1905-1983)

Arnold
Nordsieck
(1911-1971)

Wolfgang Pauli
and
Markus Fierz
(1912-2006)

Canonical transformation of system (non-relativistic) electron+radiation

Interaction replaced by electromagnetic mass of electron (which is then dropped)

$$e\,\mathbf{p}\cdot\mathbf{A} \rightarrow \frac{\mathbf{p}^2}{2\mu}$$

Theory of free photons and free „dressed“ electrons

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Transitions between free electron and (arbitrary number of) photons through scattering potential

Demonstration that in any scattering event an arbitrary number of „soft" photons is emitted...

The Pauli-Fierz divergence appears

...but the resulting radiative corrections to the scattering cross section are now **ultraviolet** divergent.

„This was surprising since – as opposed to the case of the self-energy – one is dealing with long waves."

(Pauli to Klein, 9 August 1937)

A non-relativistic artefact?

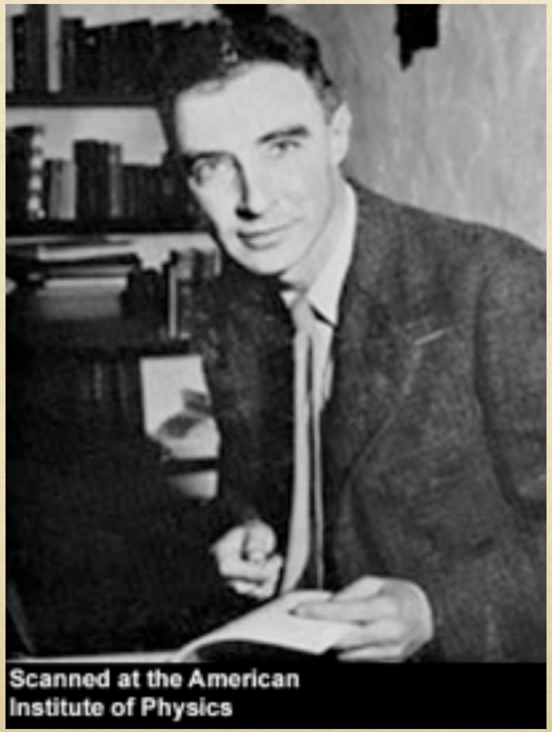
Treating the electron non-relativistically is inconsistent as soon as high photon energies are involved

But a relativistic generalization of the Bloch-Nordsieck-Pauli-Fierz (BNPF) transformation proved impossible

A Perturbative Understanding

Werner Braunbek (1901-1977)

J.Robert Oppenheimer (1904-1967)

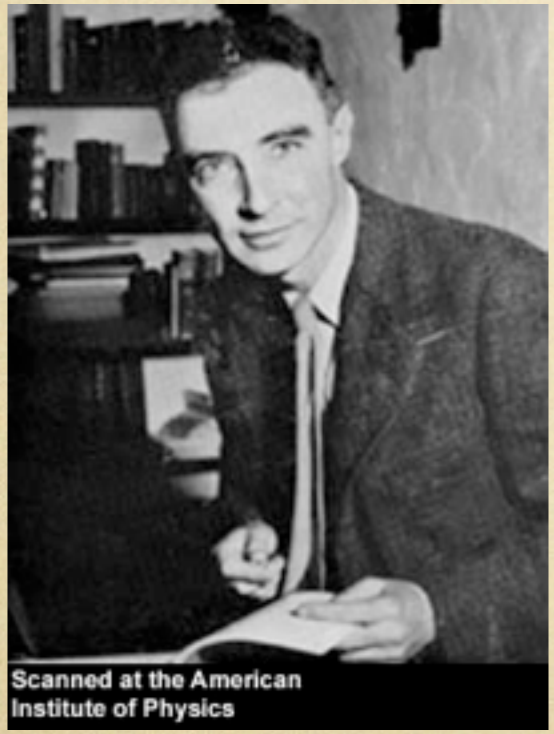


infrared divergence from real, emitted soft photons cancelled by radiative corrections from virtual, low-energy photons

UV divergence (in non-relativistic case) caused by those terms that cancel the IR divergence

This made a relativistic, perturbative treatment of the radiative corrections feasible

Mass renormalization?



no isolation of electromagnetic mass thru canonical trafo

instead: subtraction (or ignoring) of divergent self-energy of the scattered electron, which shows up in the density of final states in scattering

Non-relativistic „mass operator“: $\hat{p}^2/2m$

no off-diagonal terms

only of kinematic relevance (density of final states) in scattering

self-energy subtraction equivalent to mass renormalization

Relativistic Dirac mass operator: $\gamma_0 mc^2$

spin off-diagonal terms

transitions from positive to negative energy states

(in hole theory: pair creation)

also affects transition amplitudes in scattering

Even if Dancoff had calculated the radiative corrections to the transition amplitudes correctly, the divergences would not have been cancelled.

The Coulomb Term in QED

1927: Dirac quantizes the radiation field

1929: Heisenberg/Pauli and Fermi quantize the full electromagnetic field (QED)

1930: Oppenheimer and Fermi demonstrate that QED is equivalent to Dirac radiation theory plus Coulomb interaction - Coulomb self-interaction dropped as infinite constant

1936: Established as standard formulation of QED (Heitler's textbook) - identified as special gauge choice (Coulomb gauge) only in the second edition (1944)

The Coulomb Term in Hole Theory

„In the theory constructed here, there are no processes in which the number of electrons changes. Therefore the additional additive terms [the Coulomb self-interaction] do not present a difficulty, since one is only interested in energy differences.“

Heisenberg and Pauli in 1929

Conclusions

The non-covariance of 1930s QED prevented the relativistic generalization of the Bloch-Nordsieck-Pauli-Fierz canonical transformation

The subtraction of self-energies instead of a mass operator was not sufficient to cancel all divergences from radiative corrections

The standard procedure of going to Coulomb gauge favored Dancoff's incorrect calculation of those radiative corrections