The state is not abolished, it withers away

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Outline

1. The old QFT
2. QFT and relativity
3. Abolishing the State (Heisenberg‘s minimal length)
4. Abolishing the State (Wheeler-Feynman ED)
5. Conclusion

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Functional Methods in QFT (1927/28)

Wolfgang Pauli
(1900-1958)

Pascual Jordan
(1902-1980)

Vito Volterra
(1860-1940)

Quantum Mechanics

dynamical variables are q numbers $\hat{x}\ \hat{p}$

obeying the commutation relation $[\hat{x}, \hat{p}] = i\hbar$

which is fulfilled by the differential operators

$$\hat{x} = x \qquad \hat{p} = -i\hbar\frac{\partial}{\partial x}$$

inserting these into the Schrödinger equation

$$\left[\frac{\hat{p}^2}{2m} + V\left(\hat{x}\right)\right]\psi = E\psi$$

gives the differential equation

$$\left[-\frac{\hbar^2}{2m}\Delta + V(x)\right]\psi(x) = E\psi(x)$$

The analogous procedure in QFT

dynamical variables are q functions $\hat{F}_{\mu\nu}(x)$

obeying the commutation relation

$$\left[\hat{F}_{\mu\nu}(x), \hat{F}_{\rho\sigma}(x')\right] = i\hbar c \Delta_{\mu\nu,\rho\sigma}(x' - x)$$

which is fulfilled by the functional differential operators

$$\hat{F}^{-}_{\mu\nu}(x) = \frac{1}{2}\left[\hat{F}_{\mu\nu}(x) - \hat{F}_{\mu\nu}(-x)\right] = F^{-}_{\mu\nu}(x)$$

$$\hat{F}^{+}_{\mu\nu}(x) = \frac{1}{2}\left[\hat{F}_{\mu\nu}(x) + \hat{F}_{\mu\nu}(-x)\right] = \frac{i\hbar c}{2}\frac{\delta}{\delta F^{-}_{\mu\nu}(x)}$$

The analogous procedure in QFT

inserting these into the “Schrödinger equation” (t=0)

$$\int \left[F^{\mu\alpha}\left(\vec{x}\right) F^{0}_{\alpha}\left(\vec{x}\right) - \frac{1}{4}\delta^{\mu 0} F_{\alpha\beta}\left(\vec{x}\right) F^{\alpha\beta}\left(\vec{x}\right) \right] \Psi d^3\vec{x} = J^{\mu}\Psi$$

gives a differential equation for the state functional $\Psi\left[F^{-}_{\mu\nu}(x')\right]$

“These equations play an analogous role for an ‘isolated’ radiation field, as the Schrödinger differential equation for a specific quantum state of an isolated mechanical system.” (Jordan and Pauli, 1928)

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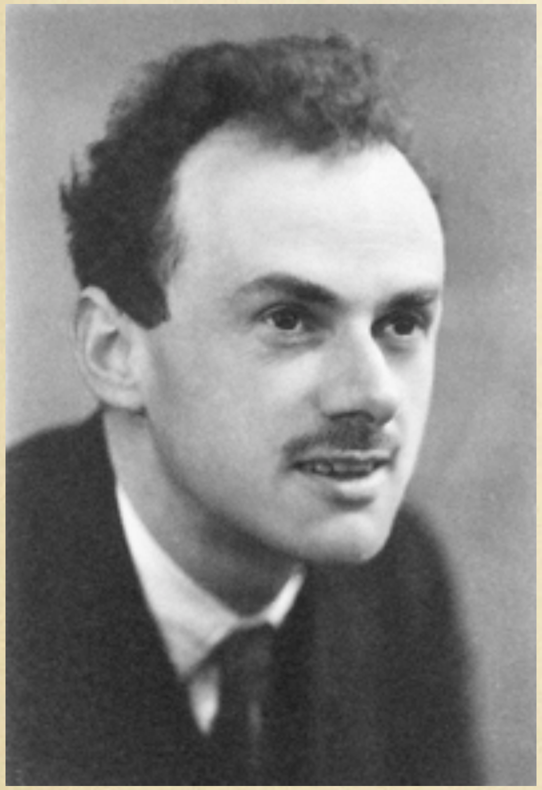
Making Quantum Theory (more) relativistic



P.A.M. Dirac
(1902-1984)

Main proponent of relativistic quantum theory from the start

1926: Relativistic Matrix Mechanics

1928: Dirac equation

1930: Textbook “Principles”

(based on relativistic notion of state)

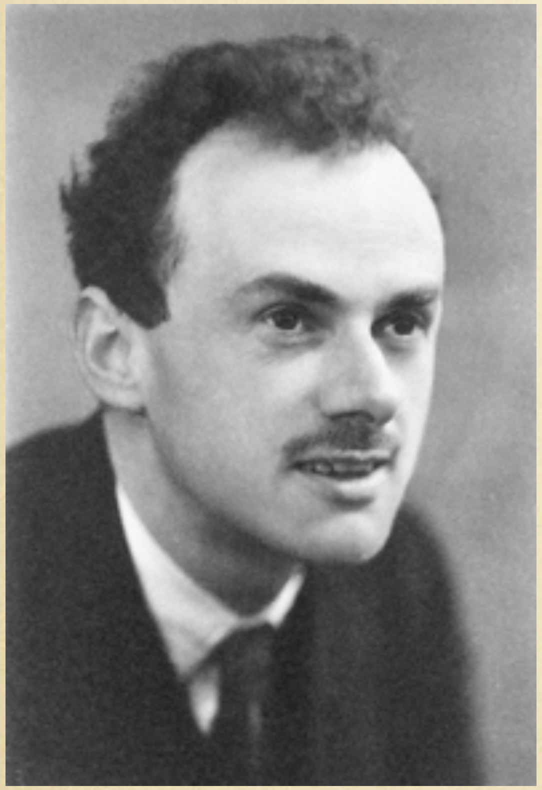
1932: Interaction Representation

(allows for covariant commutation rels)

1933: Lagrangian Quantum Theory

(to avoid intrinsic non-covariance of Hamiltonian formulation)

Making Quantum Theory (more) relativistic



1932: Interaction Representation

(allows for covariant commutation rels)

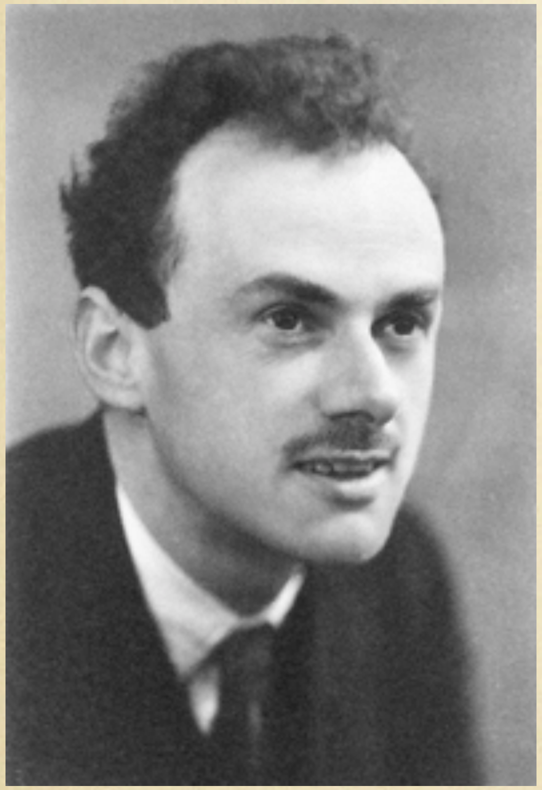
Central equation is now time-dependent Schrödinger equation

$$i\frac{\partial\varphi}{\partial t} = \overline{H_1}\varphi = \left(\int H_1 dV\right)\varphi$$

Starting point for covariant time evolution operator (Heisenberg, 1937)

$$\varphi(t_1) = \prod_{\Omega} e^{-iH_1 dV\, dt}\varphi(t_0) = \prod_{\Omega} e^{-iH_1 d\omega}\varphi(t_0)$$

Making Quantum Theory (more) relativistic



1933: Lagrangian Quantum Theory

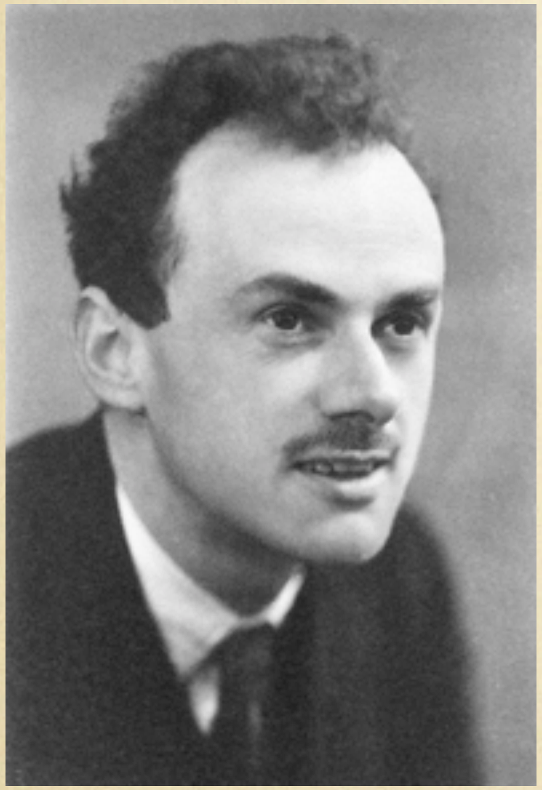
(to avoid intrinsic non-covariance of Hamiltonian formulation)

"A little consideration shows, however, that one cannot expect to be able to take over the classical Lagrangian equations in any very direct way. [...]
We must therefore seek our quantum Lagrangian theory in an indirect way."

Indirect Way:

Hamilton Principal function (time integral of Lagrangian evaluated on actual path) as generator of canonical trafo relating dynamical variables at different times

Making Quantum Theory (more) relativistic



Leads to vaguely defined analogous quantum equation for matrix elements:

$$\langle q'_t \mid q'_T \rangle = e^{iS(q'_t, q'_T)/\hbar}$$

Is to be taken as starting point for "generalized transformation functions," where there is no longer an initial and final time, but only arbitrary three-dimensional hypersurfaces.

Making Quantum Theory (more) relativistic

Drive towards more covariant formulation viewed as starting point, not as remedy for divergence difficulties of QFT

In general leads to formulations that emphasize transitions between initial and final states, rather than energies of stationary states

In the late 1940s, fully covariant formulation is developed by Tomonaga and Schwinger, based on differential (time) evolution on an arbitrary foliation of space-time

Soon superseded by formulations based on transitions between free asymptotic states (S matrix, Feynman diagrams and path integrals)

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Heisenberg‘s remedy: Fundamental Length

Werner Heisenberg
(1901-1976)

Fundamental Length: Breakdown of continuous space-time, serves as cutoff for QFT divergences

Critical Length: Explosive many-particle events (in cosmic rays) below a certain wavelength (breakdown of perturbation theory for dimensionful coupling constant)

Combined: Explosions prevent precise space-time measurements

Conclusion: Only free, asymptotic states

Fundamental Length implies: No more differential equations, no description of microscopic space-time evolution (positivist credo of 1925)

Already in 1937: Replace “differential” time evolution operator, with integral operator that only relates free, asymptotic states (all one needs for cosmic ray scattering)

Algebraic theory (as matrix mechanics was originally)

Starting point for S Matrix theory

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Feynman's remedy: Action at a distance

Richard Feynman
(1918-1988)

John Wheeler
(1911-2008)

Rewrite electrodynamics without fields in terms of (advanced and) retarded interactions at a distance

Allows elimination of self-energy

Theory is formulated in terms of (Fokker) action

$$S_{int} = \sum_{a<b} \frac{e_a e_b}{c} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta\left((x_a^\mu - x_b^\mu)^2 \right) \dot{x}_a^\nu \dot{x}_{b,\nu} d\tau_a d\tau_b$$

Conclusion: Only free asymptotic states

Development of path integral formalism based on Dirac‘s Lagrangian approach

Forms starting point for quantizing theories with only an action, no Lagrangian

But these (as Fokker action) always include integration over all time

Only allows for interpretation as transitions between free, asymptotic states

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Conclusions

The early QFT (also in attempts at a functional formulation) was structured on QM, with stationary states and their energy values at its center

The drive towards more covariant formulations led to an emphasis on processes and transitions between states

In the 1930s and early 1940s, there were attempts to solve the difficulties of QFT, by abolishing the notion of instantaneous state altogether and focusing only on transitions between asymptotic, free states

These attempts failed, but provided mathematical tools that became central to renormalized QFT, where the notion of state was consequently marginalized