# Particles and Fields in Quantum Field Theory

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# Apologia

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Their work builds on a long history of results, going back to the early 1960s, by Schlieder, Reeh, Hegerfeldt, Fleming, Ruijsenaars, Jancewicz, Skagerstam, Jauch, and others.

# Talk Overview

1. Quantum Mechanics and “Ontology”
2. One particle in Galilean spacetime
3. One particle in Minkowski spacetime
4. Localizable “quanta” in Minkowski spacetime
5. An invitation

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# Quantum Systems

Consider a physical system $S$.

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# Quantum Systems

A quantum mechanical description of $S$ consists in:

1. A Hilbert space $\mathcal{H}$, rays of which represent possible states of $S$; and
2. A collection $\mathcal{E}$ of projection operators $P$ on $\mathcal{H}$, representing propositions (or "eventualities") concerning $S$.

In general, $\mathcal{E}$ will have non-trivial algebraic structure related to the physical structure of $\mathcal{S}$.

# Quantum Systems

For now, a **quantum mechanical system** will be a pair $(\mathcal{H}, \mathcal{E})$.

# Ontology of a Quantum System

What determines the “ontology” of a quantum mechanical system?

# Ontology of a Quantum System

In other words...

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What makes $(\mathcal{H}, \mathcal{E})$ a representation of system *S*?

# Ontology of a Quantum System

Trick question!

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What **can** we do?

- Study what physical systems admit a quantum mechanical description at all.
- Characterize a physical system by the algebraic structure of the associated propositions.
- Reason metaphorically about systems characterized by the same algebraic structures.

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# Affine spaces

An **affine space** is a structure $(A, \mathbf{V}, +)$, where $A$ is a collection of points; $\mathbf{V}$ is a vector space; and $+$ is a map from $A \times \mathbf{V}$ to $A$ such that:

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**AS 1** For all $p, q \in A$, there is a unique $\mathbf{u} \in \mathbf{V}$ such that $q = p + \mathbf{u}$; and

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**AS 1** For all $p, q \in A$, there is a unique $\mathbf{u} \in \mathbf{V}$ such that $q = p + \mathbf{u}$; and

**AS 2** For all $p \in A$ and all $\mathbf{u}, \mathbf{v} \in \mathbf{V}$, $(p + \mathbf{u}) + \mathbf{v} = p + (\mathbf{u} + \mathbf{v})$.

# Affine spaces

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An **affine space isomorphism** is a bijection $\phi : A \to A'$ and a vector space isomorphism $\Phi : \mathbf{V} \to \mathbf{V}'$ such that for all points $p, q \in A$, $p = q + \mathbf{u}$ if and only if $\phi(p) = \phi(q) + \Phi(\mathbf{u})$.

# Affine spaces

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# Affine spaces

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Every vector $\mathbf{u} \in \mathbf{V}$ determines an affine space isomorphism $\phi : A \to A$ defined by $\phi : p \mapsto p + \mathbf{u}$.

The collection of such isomorphisms forms a group $T$ under composition, known as the **translation group** of $A$.

# Affine spaces

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The **dimension** of an affine space $(A, \mathbf{V}, +)$ is the dimension of its associated vector space, **V**.

For any $n \in \mathbb{N}$, is a unique $n-$dimensional affine space (up to isomorphism).

# Galilean spacetime

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1. There exists a distinguished 3-dimensional subspace $\mathbf{S} \subseteq \mathbf{V}$;
2. There exists a positive definite inner product $\langle \cdot, \cdot \rangle$ on $\mathbf{S}$; and
3. There exists a linear functional $t : \mathbf{V} \rightarrow \mathbb{R}$ such that $t(\mathbf{u}) \neq 0$ iff $\mathbf{u} \notin \mathbf{S}$.

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The **spatial length** of a spacelike vector $\mathbf{u}$ is given by $\langle \mathbf{u}, \mathbf{u} \rangle^{1/2}$.

The **temporal length** of a timelike vector $\mathbf{u}$ is given by $t(\mathbf{u})$.

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Points $p, q \in A$ are said to be **spacelike related** if the vector connecting them is spacelike. Otherwise they are **timelike related.**

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The **spatial (resp. temporal) distance** between spacelike (resp. timelike) related points $p$ and $q$ is the spatial (resp. temporal) length of the vector $\mathbf{u}$ from $p$ to $q$.

# Galilean spacetime

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The collection of all points simultaneous with a point $p$, the **simultaneity slice** $\Sigma(t)$, represents space at a time $t$.

# Galilean spacetime

The spatial distance between spacelike related points represents that distance between simultaneous events.

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The spatial distance between spacelike related points represents that distance between simultaneous events.

The timelike distance between timelike related points represents the duration between non-simultaneous events.

# Galilean spacetime

We will call a Borel subset $\Delta$ of a simultaneity slice $\Sigma(t)$ a **spatial set.**

# Example: One particle in Galilean spacetime

Consider a single particle in Galilean spacetime.

# A single particle in Galilean spacetime

Classically, we represent a particle by its wordline, a curve $\gamma : \mathbb{R} \to A$ that intersects each simultaneity slice exactly once.

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Classically, we represent a particle by its wordline, a curve $\gamma : \mathbb{R} \to A$ that intersects each simultaneity slice exactly once.

Given any spatial set $\Delta$, there is an associated proposition:

$$E_\Delta = \text{“The particle is in region } \Delta \text{ (at time } t\text{).”}$$

# A single particle in Galilean spacetime

Quantum mechanically, we first fix a Hilbert space $\mathcal{H}$ of states of the particle.

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Quantum mechanically, we first fix a Hilbert space $\mathcal{H}$ of states of the particle.

We associate with each spatial set $\Delta$ a projection operator $P_\Delta$ on $\mathcal{H}$ corresponding to the proposition $E_\Delta$.

# A single particle in Galilean spacetime

The collection $\mathcal{E}$ of all such projection operators is required to have additional structure.

# A single particle in Galilean spacetime

The projection operators associated with a single simultaneity slice $\Sigma(t)$ are required to commute, and to satisfy:

$$
\begin{aligned}
P_{\Delta_1 \cap \Delta_2} &= P_{\Delta_1} P_{\Delta_2} \\
P_{\Delta_1 \cup \Delta_2} &= P_{\Delta_1} + P_{\Delta_2} - P_{\Delta_1} P_{\Delta_2} \\
P_{\Sigma(t)/\Delta} &= I - P_{\Delta}
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Note that if $\Delta_1$ and $\Delta_2$ are disjoint, then $P_{\Delta_1} P_{\Delta_2} = P_{\Delta_2} P_{\Delta_1} = 0$.

# A single particle in Galilean spacetime

There exists a (strongly continuous) unitary representation $\mathbf{a} \mapsto U(\mathbf{a})$ of the translation group on Galilean spacetime.

# A single particle in Galilean spacetime

By Stone's theorem, for any vector **a**, there exists a unique self-adjoint operator $P(\mathbf{a})$ such that

$$U(\alpha\mathbf{a}) = e^{i\alpha P(\mathbf{a})}.$$

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By Stone's theorem, for any vector **a**, there exists a unique self-adjoint operator $P(\mathbf{a})$ such that

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If **a** is timelike, we require that $P(a)$ is bounded from below.

# A single particle in Galilean spacetime

We require that for all spatial sets $\Delta$ and all vectors $\mathbf{a}$,

$$P_{\Delta+\mathbf{a}} = U(\mathbf{a})P_{\Delta}U(-\mathbf{a})$$

where $\Delta + \mathbf{a} = \{q : q = p + \mathbf{a} \text{ for some } p \in \Delta\}$.

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yada yada yada

# A single particle in Galilean spacetime

Upshot: There exists a representation of the operators described, satisfying the required properties, on $\mathcal{H}$.

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# Example: One particle in Minkowski spacetime

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There is a non-degenerate inner product $\langle \cdot, \cdot \rangle$ on $\mathbf{V}$ s.t. given any orthogonal basis $\mathbf{u}_1 \cdots \mathbf{u}_4$, one element $\mathbf{u}_1$ satisfies

$$\langle \mathbf{u}_1, \mathbf{u}_1 \rangle > 0$$

while the others satisfy

$$\langle \mathbf{u}_j, \mathbf{u}_j \rangle < 0.$$

# Minkowski spacetime

A vector $\mathbf{u} \in \mathbf{V}$ is called **spacelike** if $\langle \mathbf{u}, \mathbf{u} \rangle < 0$; **timelike** if $\langle \mathbf{u}, \mathbf{u} \rangle > 0$; and **null** if $\langle \mathbf{u}, \mathbf{u} \rangle = 0$.

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The **length** of a vector $\mathbf{u}$ is given by $|\langle \mathbf{u}, \mathbf{u} \rangle|^{1/2}$.

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A timelike vector **u** determines a **reference frame**, corresponding to a family of co-moving observers.

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Points $p, q$ are **simultaneous relative to u** if the vector relating them is orthogonal to **u**.

# Minkowski spacetime

Points $p, q$ are **simultaneous relative to u** if the vector relating them is orthogonal to **u**.

The collection of all points simultaneous with a point $p$, the **simultaneity slice** $\Sigma(\mathbf{u}, t)$, represents space at a time as determined by the family of observers.

# Minkowski spacetime

Determinations of spatial distance, temporal duration, and simultaneity can be made only relative to a reference frame.

# A single particle in Minkowski spacetime

In what follows, suppose we fix a reference frame determined by some timelike vector **u**.

# A single particle in Minkowski spacetime

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We will call a Borel subset of a simultaneity slice $\Sigma(\mathbf{u}, t)$ a **spatial set**.

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Classically, we represent a particle by its wordline, a curve $\gamma : \mathbb{R} \rightarrow A$ that intersects each simultaneity slice (relative to **any** reference frame) exactly once.

# A single particle in Minkowski spacetime

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Given any spatial set $\Delta$, there is an associated proposition:

$$E_{\Delta} = \text{“The particle is in region } \Delta \text{ (at time } t\text{).”}$$

# A single particle in Minkowski spacetime

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1. A Hilbert space $\mathcal{H}$ of states of the particle;
2. An assignment to each spatial set $\Delta$ of a projection operator $P_\Delta$, corresponding to the proposition $E_\Delta$; and

# A single particle in Minkowski spacetime

We expect a quantum mechanical description of a single particle in Minkowski spacetime to have the following ingredients.

1. A Hilbert space $\mathcal{H}$ of states of the particle;
2. An assignment to each spatial set $\Delta$ of a projection operator $P_\Delta$, corresponding to the proposition $E_\Delta$; and
3. A (strongly continuous) unitary representation $\mathbf{v} \mapsto U(\mathbf{v})$ of the translation group of Minkowski spacetime.

# A single particle in Minkowski spacetime

In addition, we suppose the following four conditions are met.

# A single particle in Minkowski spacetime

**Translation covariance**: For all vectors **a** in **V** and all subsets Δ of all instants $t$,

$$P_{\Delta+\mathbf{a}} = U(\mathbf{a})P_{\Delta}U(-\mathbf{a}).$$

# A single particle in Minkowski spacetime

**Semi-bounded energy**: For all timelike vectors **a** satisfying $\langle \mathbf{u}, \mathbf{a} \rangle > 0$, the unique operator $H(\mathbf{a})$ satisfying

$$U(t\mathbf{a}) = e^{-itH(\mathbf{a})}$$

has spectrum bounded from below.

# A single particle in Minkowski spacetime

**Localizability**: If $\Delta_1$ and $\Delta_2$ are disjoint subsets of a single instant $t$, then

$$P_{\Delta_1} P_{\Delta_2} = P_{\Delta_2} P_{\Delta_1} = \mathbf{0}.$$

# A single particle in Minkowski spacetime

**Locality**: If $\Delta_1$ and $\Delta_2$ are spacelike related subsets of instants $t_1$ and $t_2$, then

$$P_{\Delta_1} P_{\Delta_2} = P_{\Delta_2} P_{\Delta_1}.$$

# A single particle in Minkowski spacetime

Note: **Translation covariance**, **Semi-bounded energy**, and **Localizability** are all satisfied in identical form by the Galilean example.

# A single particle in Minkowski spacetime

Note: **Translation covariance**, **Semi-bounded energy**, and **Localizability** are all satisfied in identical form by the Galilean example.

Only **Locality** has changed, because the definition of **spacelike** has changed. (In the Galilean case, **Locality** is subsumed by **Localizability**.)

# A single particle in Minkowski spacetime

**Theorem (Malament (1996))**

*If the structure* $(\mathcal{H}, \Delta \mapsto P_\Delta, \mathbf{a} \mapsto U(a))$ *satisfies* ***Translation covariance***, ***Semi-bounded energy***, ***Localizability***, *and* ***Locality***, *then* $P_\Delta = \mathbf{0}$ *for all spatial sets* $\Delta$.

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# Another tack

You might think:

# Another tack

You might think: Jim!

# Another tack

You might think: Jim! You're doing this wrong!

# Number operators

**Suggestion 1**: Instead of considering projections $P_{\Delta}$, consider **(local) number operators** $N_{\Delta}$.

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**Suggestion 1**: Instead of considering projections $P_\Delta$, consider **(local) number operators** $N_\Delta$.

A local number operator $N_\Delta$ is an observable whose eigenvalues give the "number of particles" in spatial region $\Delta$.

# Number operators

Suppose we have a Hilbert space, a (strongly continuous) representation $\mathbf{a} \mapsto U(\mathbf{a})$, and assignments $\Delta \mapsto N_\Delta$ of number operators to spatial (Borel) sets.

# Number operators

Suppose we have a Hilbert space, a (strongly continuous) representation $\mathbf{a} \mapsto U(\mathbf{a})$, and assignments $\Delta \mapsto N_\Delta$ of number operators to spatial (Borel) sets.

**Translation covariance**, **Semi-bounded energy**, **Localizability**, and **Locality** carry over intact to $(\mathcal{H}, \Delta \mapsto N_\Delta, \mathbf{a} \mapsto U(\mathbf{a}))$.

# Number operators

**Number Additivity**: If $\Delta_1$ and $\Delta_2$ are disjoint subsets of the same simultaneity slice $\Sigma(t)$, then $N_{\Delta_1 \cup \Delta_2} = N_{\Delta_1} + N_{\Delta_2}$.

# Number operators

**Number conservation**: If $\{\Delta_n : n \in \mathbb{N}\}$ is a disjoint covering of a simultaneity slice $\Sigma(t)$, then $\sum_n N_{\Delta_n}$ converges to a densely defined, self-adjoint operator $N$ on $\mathcal{H}$ (independent of the covering), and for any timelike vector **a**, $U(\mathbf{a})NU(-\mathbf{a}) = N$.

# Number operators

**Theorem (Halvorson & Clifton (2001))**

*If the structure* $(\mathcal{H}, \Delta \mapsto N_\Delta, \mathbf{a} \mapsto U(a))$ *satisfies* ***Translation covariance***, ***Semi-bounded energy***, ***Localizability***, ***Locality***, ***Additivity***, *and* ***Number conservation*** *then* $N_\Delta = \mathbf{0}$ *for all spatial sets* $\Delta$*.*

# Number operators, take 2

**Suggestion 2**: Instead of considering projections local number operators on **spatial sets**, consider local number operators on **spacetime regions**.

# Algebraic QFT

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A **net of local observables** is an assignment $O \mapsto \mathcal{R}(O)$ of (von Neumann) sub-algebras of $B(\mathcal{H})$ to each bounded, open subset of Minkowski spacetime.

# Algebraic QFT

Fix a Hilbert space $\mathcal{H}$ and a (strongly continuous) unitary representation $\mathbf{a} \mapsto U(\mathbf{a})$ of the translation group on Minkowski spacetime.

A **net of local observables** is an assignment $O \mapsto \mathcal{R}(O)$ of (von Neumann) sub-algebras of $B(\mathcal{H})$ to each bounded, open subset of Minkowski spacetime.

The **global algebra** $\mathcal{R}$ is the smallest (von Neumann) algebra containing all of the local algebras.

# Algebraic QFT

**Isotony**: For any two bounded open sets of Minkowski spacetime $O_1$ and $O_2$, if $O_1 \subseteq O_2$, then $\mathcal{R}(O_1) \subseteq \mathcal{R}(O_2)$.

# Algebraic QFT

**Algebra Additivity**: Given any bounded open set $O$ of Minkowski spacetime, the set $\{\mathcal{R}(O + \mathbf{a}) : \mathbf{a} \in \mathbf{V}\}$ generates $\mathcal{R}$ as a $C^*$-algebra.

# Algebraic QFT

**Locality (Microcausality)**: Given any spacelike separated bounded open sets $O_1$, $O_2$, and any observables $A \in \mathcal{R}(O_1)$ and $B \in \mathcal{R}(O_2)$, $[A, B] = \mathbf{0}$.

# Algebraic QFT

**Vacuum**: There exists a vector $\Omega \in \mathcal{H}$, called the **vacuum**, such that for any vector $\mathbf{a} \in \mathbf{V}$, $U(\mathbf{a})\Omega = \Omega$.

# Local Number operators in AQFT

A **local number operator** associated with spacetime region $O$, $N_O$, is an element of $\mathcal{R}(O)$.

# Local Number operators in AQFT

**Necessary condition**: Given any bounded open subset $O$ of Minkowski spacetime, $N_O \in \mathcal{R}(O)$ can be a local number operator operator only if $N_O\Omega = 0$.

# Reeh-Schlieder Theorem

**Theorem (Reeh-Schlieder (1961))**

*If the structure $(\mathcal{H}, O \mapsto \mathcal{R}(O), \mathbf{a} \mapsto U(\mathbf{a}))$ satisfies **Isotony**, **Additivity**, **Semi-bounded energy**, and **Locality**, then given any bounded open set O, an operator $A \in \mathcal{R}(O)$ satisfies $A\Omega = \mathbf{0}$ only if $A = 0$.*

# Reeh-Schlieder Theorem

**Corollary**

*There are no local number operators.*

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# The Unruh effect