What happens to a boundary observable if the boundary changes?

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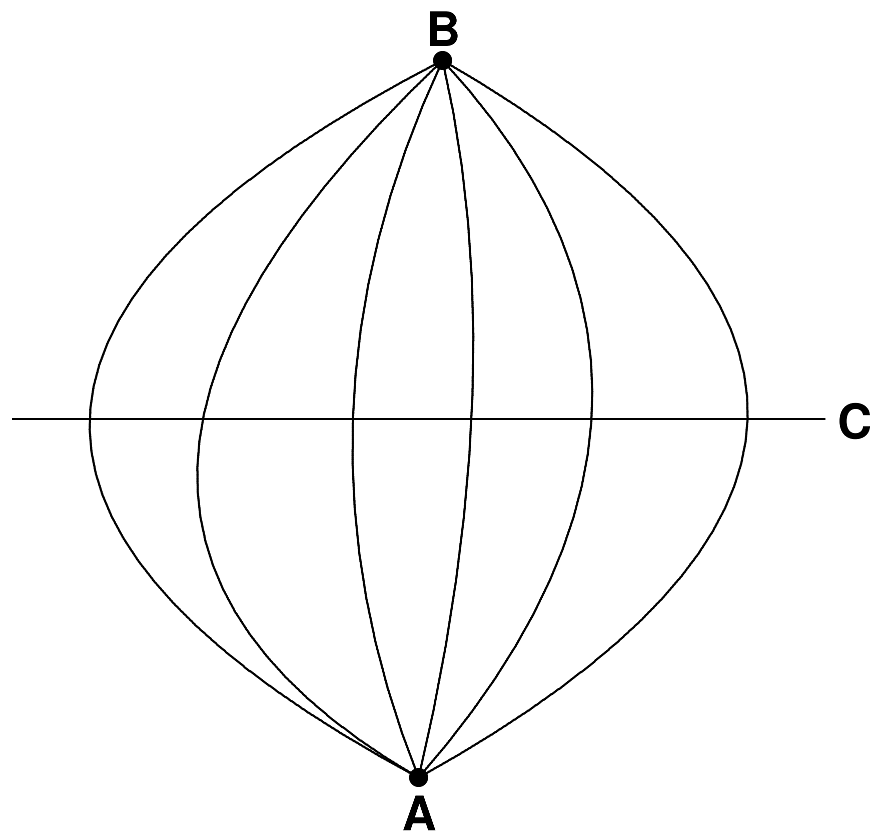
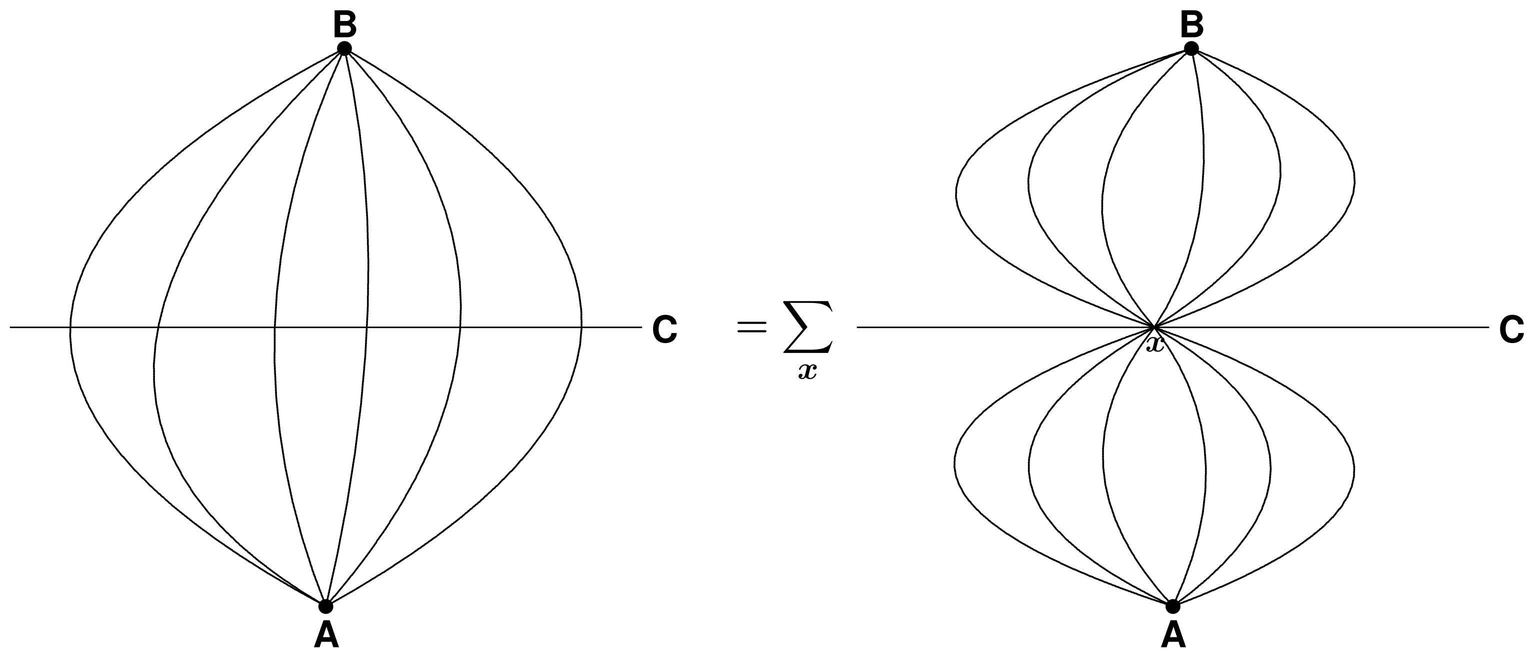
Boundaries and path integrals



– Path integral gives transition amplitude from **A** to **B**

– Path integral on line **C** gives a wave function (as function of location on **C**)

– Combine integral from **A** to **C** and integral from **C** to **B**
by integrating over points on **C** $\Rightarrow$ sum over intermediate states



Sum over intermediate states

Path integral:

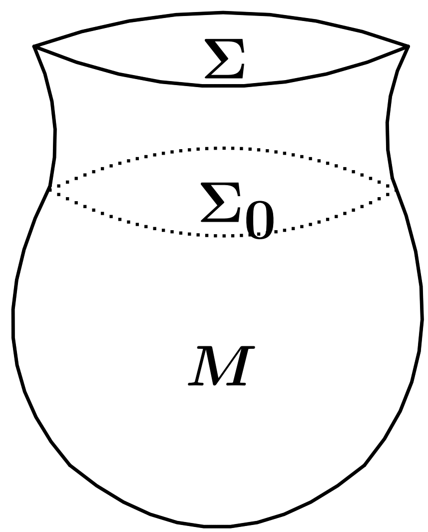
$$\boldsymbol{Z} = \int_{\boldsymbol{x}(0)=\mathrm{A}}^{\boldsymbol{x}(1)=\mathrm{B}} [\boldsymbol{dx}]\boldsymbol{e}^{\boldsymbol{iI}[\boldsymbol{x}(\boldsymbol{t})]}$$

Need to specify "half of the phase space" on **C** (e.g., positions or momenta)

Action generically requires a boundary term $\boldsymbol{I_{bdry}}$

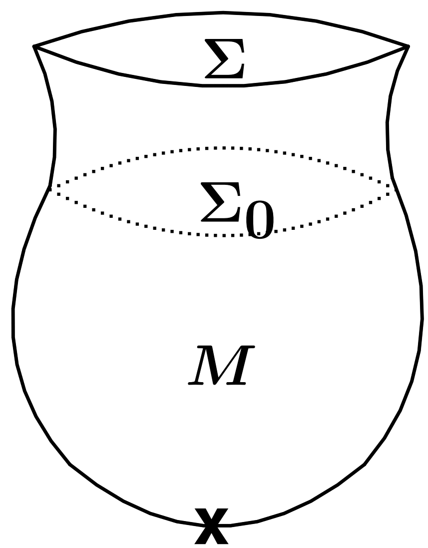
$$\delta \boldsymbol{I} = \textit{e.o.m} + \textit{boundary terms that must be cancelled off}$$

For free field theories, can show rigorously that $\boldsymbol{I_{bdry}}$ is needed for "sewing"



Integrating out the bulk

"Operator-state correspondence"



Insert operator: path integral gives associated state

In CFT, all states can be obtained this way

$\Rightarrow$ boundary state captures bulk information

Path integrals with gauge symmetries

New feature with gauge symmetries: new degrees of freedom

Chern-Simons theory:

$$I_{CS} = \frac{k}{4\pi}\int_M \mathrm{Tr}\left\{A \wedge dA + \frac{2}{3}A \wedge A \wedge A\right\} + \textit{boundary term}$$

$$A = g^{-1}\bar{A}g + g^{-1}dg$$

Then

$$I_{CS}[A] = I_{CS}[\bar{A}] + kI^{+}_{WZW}[g^{-1}, \bar{A}]$$

with

$$I^{+}_{WZW}[g^{-1}, \bar{A}_z] = \frac{1}{4\pi}\int_{\partial M} \mathrm{Tr}\left(g^{-1}\partial_z g\, g^{-1}\partial_{\bar{z}} g - 2g^{-1}\partial_{\bar{z}} g \bar{A}_z\right) + \frac{1}{12\pi}\int_M \mathrm{Tr}\left(g^{-1}dg\right)^3$$

Witten: WZW term is needed for correct sewing

New boundary degrees of freedom

$$\mathcal{H} = \mathcal{H}_{bulk} \otimes \mathcal{H}_{bdry}$$

Where do these new degrees of freedom matter?

– Black hole entropy: associated with boundary at infinity or at horizon
– Other causal horizons?
– Perhaps asymptotic null infinity for asymptotically flat spaces
– Boundaries in entanglement entropy
– Nice observables in quantum gravity (especially for closed universe)

Least ambiguous case: BTZ black hole

- Described by Chern-Simons theory
- No bulk degrees of freedom
- But standard Bekenstein-Hawking entropy
- Induced WZW theory at infinity $\Rightarrow$ correct entropy

For other black holes

- Boundary action not known (nonlocal?)
- But horizon symmetries determine density of states

But what if the boundary changes?

“Objective” changes:

– Black hole evaporation
– Unruh/Rindler observer stops accelerating
– “Quantum switch” for entanglement entropy

“Subjective” changes:

– Physicist decides to ask a different question

- Calculations with boundary can be viewed as conditional probabilities
- Implicit in original Everett “relative state” interpretation
- But “relative” part not yet very well defined . . .

Black hole evaporation

– For black hole in asymptotically AdS (or flat?) space,
can choose "boundary states" at infinity—see Aron Wall's talk

– For black hole in closed universe, states must be at horizon
(or "stretched horizon")

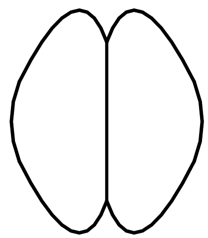
- Black hole evaporates $\Rightarrow$ boundary shrinks $\Rightarrow$ Hilbert space shrinks
- Complete evaporation $\Rightarrow$ boundary states disappear
- What does this mean for unitarity?

Can we couple boundary states to bulk in an enlarged Hilbert space?
What happens to this space during evaporation?

Tempting to say only "accessible" part of $\mathcal{H}_{bdry}$ shrinks
. . . but then how big is "full" $\mathcal{H}_{bdry}$? And where?

Other issues

– How “physical” is entanglement entropy?
– How do boundary observables on overlapping boundaries relate?



– Is there a complete (but not overcomplete) set of boundary observables?

Donnelly and Freidel: observables associated with local subsystems;
(classical) boundary observables from gauge degrees of freedom

Oeckl: “general boundary formulation” of QFT;
quantum states associated to arbitrary surfaces