# Introduction to LDA+DMFT

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CIFAR-PITP International Summer School on Numerical Methods for Correlated Systems in Condensed Matter June 1-2 2008

# Plan of the lectures

- Motivate the development of a mean field theory of correlated electron materials.
- General formulations of mean field theory: density functional theory, DMFT, spectral density functional, LDA+DMFT.
- Applications to the Mott transition.
- Some applications of LDA+DMFT
- More foundations, extensions, issues of implementation, applications.

Electronic structure calculations with dynamical mean-field theory: *G. Kotliar, S. Savrasov, K. Haule, V. Oudovenko, O. Parcollet, and C. Marianetti, Rev. Mod. Phys.* 78, 000865 (2006). Electronic structure calculations with dynamical mean-field theory: *G. Kotliar, S. Savrasov, K. Haule, V. Oudovenko, O. Parcollet, and C. Marianetti, Rev. Mod. Phys.* 78, 000865 (2006)













# Mott transition and transport crossovers in the organic compound $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu[N(CN)<sub>2</sub>]Cl





P. Limelette et al., PRL 91 (2003) 016401 See also: Kagawa et al. cond-mat/0307304

S.LHEFEREVIEWERSEY PRYS. REV. Lett. 85 (2000) p. 5420

FIG. 6: Temperature-dependence of the resistivity at different pressures. The data (circles) are compared to a DMFT-NRG calculation (diamonds), with a pressure dependence of the bandwidth as indicated. The measured residual resistivity  $\rho_0$ has been added to the theoretical curves.

#### Cerium



► Tk around 1000-2000K

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### Localization Delocalization in Actinides



Nodern understanding of this phenomena using functional approach toDMFT. K Haule S.Savrasov J Shim

# Strongly correlated materials do "big" things Competition of localization and itineracy.

- Huge volume collapses in lanthanides and actindies, eg. Ce , Pu, .....
- Metal insulator transitions as a function of pressure an composition in transition metal oxides, VO2 V2O3
- Quasiparticles with large masses m\* =1000 mel in Ce and U based heavy fermions.
- Colossal Magnetoresistance in La1-xSrxMnO3
- High Temperature Superconductivity 150 K Ca<sub>2</sub>Ba<sub>2</sub>Cu<sub>3</sub>HgO<sub>8</sub>
- Large thermoelectric response in Na<sub>x</sub>Co<sub>2</sub>O<sub>4</sub>
- 50K superconductivity in SmO1-xFxFeAs
- Many others.....



CeRhIn5: TN=3.8 K; γ ≈ 450 mJ/molK2
 CeCoIn5: Tc=2.3 K; γ ≈ 1000 mJ/molK2; • CeIrIn5: Tc=0.4 K; γ ≈ 750 mJ/molK2





#### Iron based high-Tc superconductors



Smaller <mark>c</mark> Higher T<sub>c</sub>

- a) Y. Kamihara et.al., Tokyo, JACS
  - ) X.H. Chen, et.al., Beijing, cm/0803.3790
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•2D square lattice of Fe•Fe - magnetic moment

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# Two paths for ab-initio calculation of electronic structure of strongly correlated materials



#### THE STATE UNIVERSIGN FIEld ideas can be used in both cases. RUTGERS

## Mean Field Theory : general construction principles

Spectral density functional. Effective action construction.e.g Fukuda et.al

$$Z = e^{-F[J]} = \int d\psi d\psi^+ e^{-[S(\psi^+\psi) + JA]}$$

$$\frac{\delta F}{\delta J}[J] = =a$$

$$\Gamma[a] = F[J[a]] - aJ[a]$$

 $S = S_0 + \lambda S_{\text{int}}$ 

 $J = J_0 + \lambda J_1 + \cdots$ 

 $\Gamma[a] = F_0[\mathcal{I}_0] - aJ_0 + \lambda \Gamma[a].$ 

 $\Delta \Gamma = \Gamma_{hartree} + \Gamma_{xc}$ 

$$\Delta\Gamma[a] = \int_{0}^{1} d\lambda < S_{\text{int}} > (\lambda, J(\lambda, a))$$

In practice we need good approximations to the exchange correlation, in DFT LDA. In spectral density functional theory, DMFT. Review: Kotliar et.al. Rev. Mod. Phys. **78**, 865 (2006)

$$\frac{\delta F}{\delta J_0}[J_0] = a \qquad \qquad \frac{\delta \Delta \Gamma}{\delta a} = J_0[a]$$

Kohn Sham equations

## Remarks:

- Exact functionals of an observable  $A, \Gamma_{exact}[a]$
- In practice approx are needed  $\Gamma_{mft}[a] \sim \Gamma_{exact}[a]$
- Many a's many theories.
- Introduction of a reference system. Separation into "free part" and exchange+ correlation.
- Formal expression for the correlation part of the exact functional as a coupling constant integration.
  Good approximate functionals obtained by approximating the xc part. [small parameter d helps!]
- While the construction aims to calculate <A>=a, other quantities, e.g. correlation functions, emerge as a byproduct.

### Crucial Role of the constraining field

 $J_0[a]$ 

Different reference systems [ e.g. band limit or atomic limit ] define different constraining fields.

• Different functionals (self energy functional, BK functional, Harris Foulkes functional, etc)  $\Gamma[a], \Gamma[a, J], \Gamma[a, J_0], \Gamma[J_0]$ 

## Analogy with spin systems.

# Density functional and Kohn Sham reference system

$$-\nabla^2 / 2 + V_{KS}(r) \ \psi_{kj} = \varepsilon_{kj} \psi_{kj}$$
$$\rho(r) = \sum_{kj} f(\varepsilon_{kj}) |\psi_{kj}(r)|^2$$
$$V_{KS}(r)[\rho(r)] = V_{ext}(r) + \int \frac{\rho(r')}{|r-r'|} dr' + \frac{\delta E_{xc}}{\delta \rho(r)}[\rho]$$

•Kohn Sham spectra, proved to be an excellent starting point for doing perturbation theory in screened Coulomb interactions GW. Kohn Sham Eigenvalues and Eigensates: Excellent starting point for perturbation theory in the screened interactions (Hedin 1965)



## Model Hamiltonians and DMFT

Dynamical Mean Field Theory. Cavity Construction.

A. Georges and G. Kotliar PRB 45, 6479 (1992).

 $-\sum_{i,j} J_{ij} S_i S_j - h \sum_i S_i$  $-\sum_{\langle i,j \rangle,\sigma} (t_{ij} + \mu \delta_{ij}) (c_{i\sigma}^{\dagger} c_{j\sigma}^{H_{\text{Andersön Imp}}} = \sum_{\langle i,\sigma \rangle} (V_{i\sigma} \sum_{i}^{\dagger} A_{\alpha\sigma} + c.c) + \sum_{\alpha,\sigma} \varepsilon_{\alpha} A_{\alpha\sigma}^{\dagger} A_{\alpha\sigma} + \sum_{\alpha,\sigma} \mu c_{0\sigma}^{\dagger} c_{0\sigma} + U c_{0\uparrow}^{\dagger} c_{0\downarrow} c_{0\downarrow}$ 







 $A(\omega) \qquad \Delta(\omega)$  $m_i = th[\beta \sum_j J_{ij} m_j + h] \qquad i$ 

Solving A for given bath, is not easy Impurity solvers; See P. Werner and M Jarrell's talks.

$$i\omega_n - \Sigma(i\omega_n)[\Delta] \doteq -\Delta(i\omega_n) + \frac{1}{G_{imp}(i\omega_n)[\Delta]}$$

$$i\omega_n - \Sigma(i\omega_n)[\Delta] - t(k) + \mu = \mathbf{G}_{\text{latt}}(i\omega_n, k)^{-1}$$

$$G_{imp}(i\omega_n)[\Delta] = \sum_k G_{latt}(i\omega_n, k)[\Delta]$$
 8

#### DMFT

$$H = -\sum_{ij\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + U n_{i\uparrow} n_{i\downarrow}$$

$$G_c(\tau) = -\langle Tc(\tau)c^{\dagger}(0)\rangle_{S_{\text{eff}}}$$
$$S_{\text{eff}} = -\int_0^\beta c^{\dagger}_{\sigma}(\tau)G_0^{-1}(\tau-\tau')c_{\sigma}(\tau') + \int_0^\beta d\tau U n_{\uparrow}(\tau)n_{\downarrow}(\tau)$$



#### **Evolution of the DOS.** Theory and experiments



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Evolution of the Spectral Function in Mott-Hubbard Systems with  $d^{1}$  Configuration

Qualitative Phase diagram :frustrated Hubbard model, integer filling *M. Rozenberg et.al.* 75, 105 (1995)



# Interaction with Experiments. V2O3:Anomalous transfer of spectral weight



M. Rozenberg G. Kotliar H. Kajueter G Thomas D. Rapkine J Honig and P Metcalf Phys. Rev. Lett. 75, 105 (1995)

#### Spinodals and Ising critical endpoint. Observation in $V_2O_3$ : *P. Limelette et.al. Science 302, 89 (2003)*



### LDA+DMFT

LDA+DMFT V. Anisimov, A. Poteryaev, M. Korotin, A. Anokhin and G.

Kotliar, J. Phys. Cond. Mat. 35, 7359 (1997). A Lichtenstein and M. Katsnelson PRB 57, 6884 (1988).

- The light, SP (or SPD) electrons are extended, well described by LDA .The heavy, D (or F) electrons are localized treat by DMFT.
- LDA Kohn Sham Hamiltonian already contains an average interaction of the heavy electrons, subtract this out by shifting the heavy level (double counting term)
- Kinetic energy is provided by the Kohn Sham Hamiltonian (sometimes after downfolding). The U matrix can be estimated from first principles of viewed as parameters. Solve resulting model using DMFT.

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consistent implementation

 $\Gamma_{dft}[\rho] \longrightarrow \Gamma_{lda} + dmft[G_{loc}, \rho, U]$ 



#### LDA+DMFT Self-Consistency loop



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 $\begin{aligned} & \left[ \sum_{R} LDA + DMFT \left[ \rho(r) \quad G \quad a \quad b \quad V_{KS}(r) \quad \Sigma_{ab} \right] \right] \\ & -Tr \log[i\omega_n + \nabla^2 / 2 - V_{KS} - \chi^*_{\alpha R}(r) \sum_{\alpha \beta R} \chi_{\beta R}(r)] - \\ & \int_{KS} (r)\rho(r)dr - \sum_{i\omega_n} Tr \sum (i\omega_n) G(i\omega_n) + \\ & \int_{R} V_{ext}(r)\rho(r)dr + \frac{1}{2} \int_{C} \frac{\rho(r)\rho(r')}{|r-r'|} dr dr' + E_{xc}^{LDA}[\rho] + \\ & \sum_{R} \Phi[G_{\alpha \beta R}] - \Phi_{DC} \end{aligned}$ 

#### $\Phi \text{ Sum of local 2PI graphs with local U}$ matrix and local G $<math display="block">\Phi_{DC}[G] = Un(n-1)\frac{1}{2} \qquad n = T \sum_{i} \left( G_{ab}(i\omega)e^{i0^{+}} \right)$

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### Embedding

$$\Sigma_{HH} \longrightarrow \hat{\Sigma}$$

$$\Sigma_{HH} = \begin{bmatrix} \Sigma_{22} & 0 \\ 0 & \Sigma_{11} \end{bmatrix} \longrightarrow \hat{\Sigma} = \begin{bmatrix} 0 & 0 \\ 0 & \Sigma_{HH} \end{bmatrix} \qquad \hat{H} = \begin{bmatrix} H_{LL} & H_{LH} \\ H_{HL} & H_{HH} \end{bmatrix}$$

Inversion  
$$i\omega_n \hat{O}_k - \hat{H}(k) - \hat{E} - \hat{\Sigma}(i\omega_n) \rightarrow \frac{1}{i\omega_n \hat{O}_k - \hat{H}(k) - \hat{E} - \hat{\Sigma}(i\omega_n)}$$

Integrating over BZ 
$$\hat{G}_{loc}(i\omega_n) = \sum_k \frac{1}{i\omega_n \hat{O}_k - \hat{H}(k) - \hat{E} - \hat{\Sigma}(i\omega_n)}$$
  
Truncation  $\hat{G}_{loc} = \begin{bmatrix} 0 & 0 \\ 0 & G_{HH} \end{bmatrix} \rightarrow G_{HH}$ 

$$G_0^{-1}(i\omega_n) = G_{HH}^{-1} + \Sigma_{HH}(i\omega_n)$$

### Inversion in realistic DMFT

$$[(i\omega + \mu)\hat{O}(\mathbf{k}) - \hat{h}^{(0)}(\mathbf{k}) - \mathcal{M}_{\text{int}}(i\omega)]^{-1}]$$

$$[h_{\alpha\beta}^{(0)}(\mathbf{k}) + \mathcal{M}_{\text{int},\alpha\beta}(i\omega) - \epsilon_{\mathbf{k}j\omega}O_{\alpha\beta}(\mathbf{k})]\psi_{\mathbf{k}j\omega,\beta}^{R} = 0$$

$$\mathcal{G}_{\alpha\beta}(\mathbf{k},i\omega) = \sum_{j} \frac{\psi_{\mathbf{k}j\omega,\alpha}^{R} \psi_{\mathbf{k}j\omega,\beta}^{L}}{i\omega + \mu - \epsilon_{\mathbf{k}j\omega}}.$$

$$\rho(\mathbf{r}) = T \sum_{i\omega} \sum_{\mathbf{k}j} \frac{\psi_{\mathbf{k}j\omega}^R(\mathbf{r}) \psi_{\mathbf{k}j\omega}^L(\mathbf{r})}{i\omega + \mu - \epsilon_{\mathbf{k}j\omega}} e^{i\omega 0^+}.$$

#### Generalized Kohn Sham eigenstates

$$\rho(\mathbf{r}) = T \sum_{i\omega} \sum_{\mathbf{k}j} \frac{\psi_{\mathbf{k}j\omega}^R(\mathbf{r}) \psi_{\mathbf{k}j\omega}^L(\mathbf{r})}{i\omega + \mu - \epsilon_{\mathbf{k}j\omega}} e^{i\omega 0^+}.$$

$$g_{\mathbf{k}j\omega} = \frac{1}{i\omega + \mu - \epsilon_{\mathbf{k}j\omega}}$$

1

$$\rho(r) = \sum f(\varepsilon_{kj}) |\psi_{kj}(r)|^2$$

## Comments on LDA+DMFT

- Static limit of the LDA+DMFT functional , with  $\Phi \text{=} \Phi_{\text{HF}}$  reduces to LDA+U
- Removes inconsistencies and shortcomings of this approach. DMFT retain correlations effects in the absence of orbital ordering.
- Only in the orbitally ordered Hartree Fock limit, the Greens function of the heavy electrons is fully coherent
- Gives the local spectra and the total energy simultaneously, treating QP and H bands on the same footing.

### Localization Delocalization in Actinides



Nodern understanding of this phenomena using functional approach toDMFT. K Haule S.Savrasov J Shim

# Volume Collapse Transitions: relaxing the lattice positions. Savrasov et. al.





#### DMFT Phonons in fcc $\delta$ -Pu





( Dai, Savrasov, Kotliar,Ledbetter, Migliori, Abrahams, Science, 9 May 2003) 21 (experiments from Wong et.al, Science, 22 August 2003)

### New Generation of LDA+DMFT tools K.Haule Nature (2007)







Curium has large magnetic moment and orders antif Pu does is non magnetic.





J. H. Shim, K. Haule, G. Kotliar, Nature 446, 513 (2007).

Gouder Havela Lander

# The "DMFTvalence" in the late actinides.





#### Iron based high-Tc superconductors



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- a) Y. Kamihara et.al., Tokyo, JACS
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# DMFT for LaF<sub>x</sub>O<sub>1-x</sub>FeAs

LDA+DMFT: LaOFeAs is at the verge of the metal-insulator transition (for realistic U=4eV, J=0.7eV) For a larger (U=4.5, J=0.7eV) Mott-Slater insulator

Not a one band model: all 5 bands important (for J>0.3)

K. Haule J. Shim G. Kotliar arXiv:0803.1279



# Iron Pnictides vs Cuprates

- Both are strongly correlated high temperature superconductors.
- In both cases the superconductor is proximate to a Mott insulator and is not well described by Fermi Liquid at high t.

DIFFERENCES

Pnictides

•U< Uc2

•Intrinsically Multiorbital Multiband

•Important role of Jhunds

•Frustrated Magnetism

•Single Site DMFT ?

Cuprates

•U> Uc2 , doped Mott insulator

•One band model

- •Importance of Superexchange
- •Non frustrated magnetism

•Cluster DMFT ?

Coherence-incoherence crossover in DMFT : crucial role played by J



# Coherence-incoherence crossover in DMFT : crucial role played by J



## **Experimental predictions**



### Functional formulation. Chitra and Kotliar (2001), Ambladah et. al. (1999) Savrasov and Kotliarcond- matt0308053 (2003).

$$|r\rangle = |R, \rho\rangle \qquad G = -\langle \psi(R\rho') \psi^{\dagger}(R\rho') \rangle < \langle \phi(R\rho) \rangle - \langle \phi(R\rho') \rangle \langle \phi(R\rho) \rangle = W$$

 $\Gamma[G,W] = TrLnG - Tr[G_0^{-1} - G^{-1}]G - \frac{1}{2}TrLnW + \frac{1}{2}Tr[V_C^{-1} - W^{-1}]W + E_{hartree} + \Phi[G,W]$ 

 $\Phi[G,W] \square \Phi_{\textit{EDMFT}}[G_{\textit{loc}}, W_{\textit{loc}}, G_{\textit{nonloc}} = 0, W_{\textit{nonloc}} = 0]$ 

Double loop in *Gloc and Wloc* 

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EDMFT loop G. Kotliar and S. Savrasov in New Theoretical Approaches to Strongly Correlated G Systems, A. M. Tsvelik Ed. 2001 Kluwer Academic Publishers. 259-301 . cond-mat/0208241 S. Y. Savrasov, G. Kotliar, Phys. Rev. B 69, 245101 (2004)



•Full implementation in the context of a a one orbital model. P Sun and G. Kotliar Phys. Rev. B 66, 85120 (2002).

•After finishing the loop treat the graphs involving Gnonloc Wnonloc in perturbation theory. P.Sun and GK PRL (2004). Related work, Biermann Aersetiwan and Georges PRL 90,086402 (2003).

#### THE END Thanks for your attention!!!!!

Hope it raised your interest and you want to contribute so that we can have a predictive theory of correlated materials in the very near future......

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#### Phase diagram of 115's



#### Why CelrIn<sub>5</sub>?

Ir atom is less correlated than Co or Rh (5d / 3d or 4d)
CeIrIn<sub>5</sub> is more itinerant(coherent) than Co (further away from QCP)



#### **Generalized Anderson Lattice Model**

$$\sum_{i,\sigma} \mathcal{E}_{f} f_{i\sigma}^{\dagger} f_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \sum_{\langle i,j \rangle,\sigma} (t_{ij} + \mu \delta_{ij}) (c_{i\sigma}^{\dagger} c_{j\sigma} + c_{j\sigma}^{\dagger} c_{i\sigma})$$
$$+ \sum_{\langle i,j \rangle,\sigma} V_{ij} f_{i\sigma}^{\dagger} c_{j\sigma} + c.c. = H_{ALM}$$







Buildup of coherence

Buildup of coherence in single impurity case

Slow crossover pointed out by NPF 2004

#### Very slow crossover!



Crossover around 50K

#### DMFT is not a single impurity calculation

#### Auxiliary impurity problem:



Weiss field  $\Delta(\omega)$  temperature dependent:

High-temperature  $\Delta$  given mostly by LDA

low T: Impurity hybridization affected by the emerging coherence of the lattice (collective phenomena)





Feedback effect on  $\Delta$  makes the crossover from incoherent to coherent state very slow!



Expts: F. P. Mena, D. van der Marel, J. L. Sarrao, *PRB* 72, 045119 (2005).
16. K. S. Burch *et al.*, *PRB* 75, 054523 (2007).
17. E. J. Singley, D. N. Basov, E. D. Bauer, M. B. Maple, *PRB* 65, 161101(R) (2002).

•At 300K very broad Drude peak (e-e scattering, spd lifetime~0.1eV)

•At 10K:

•very narrow Drude peak

•First MI peak at 0.03eV~250cm<sup>-1</sup>

•Second MI peak at 0.07eV~600cm<sup>-1</sup>



#### DMFT-Momentum resolved Ce-4f specture)





#### Momentum resolved total spectra $trA(\omega, \mathbf{k})$



