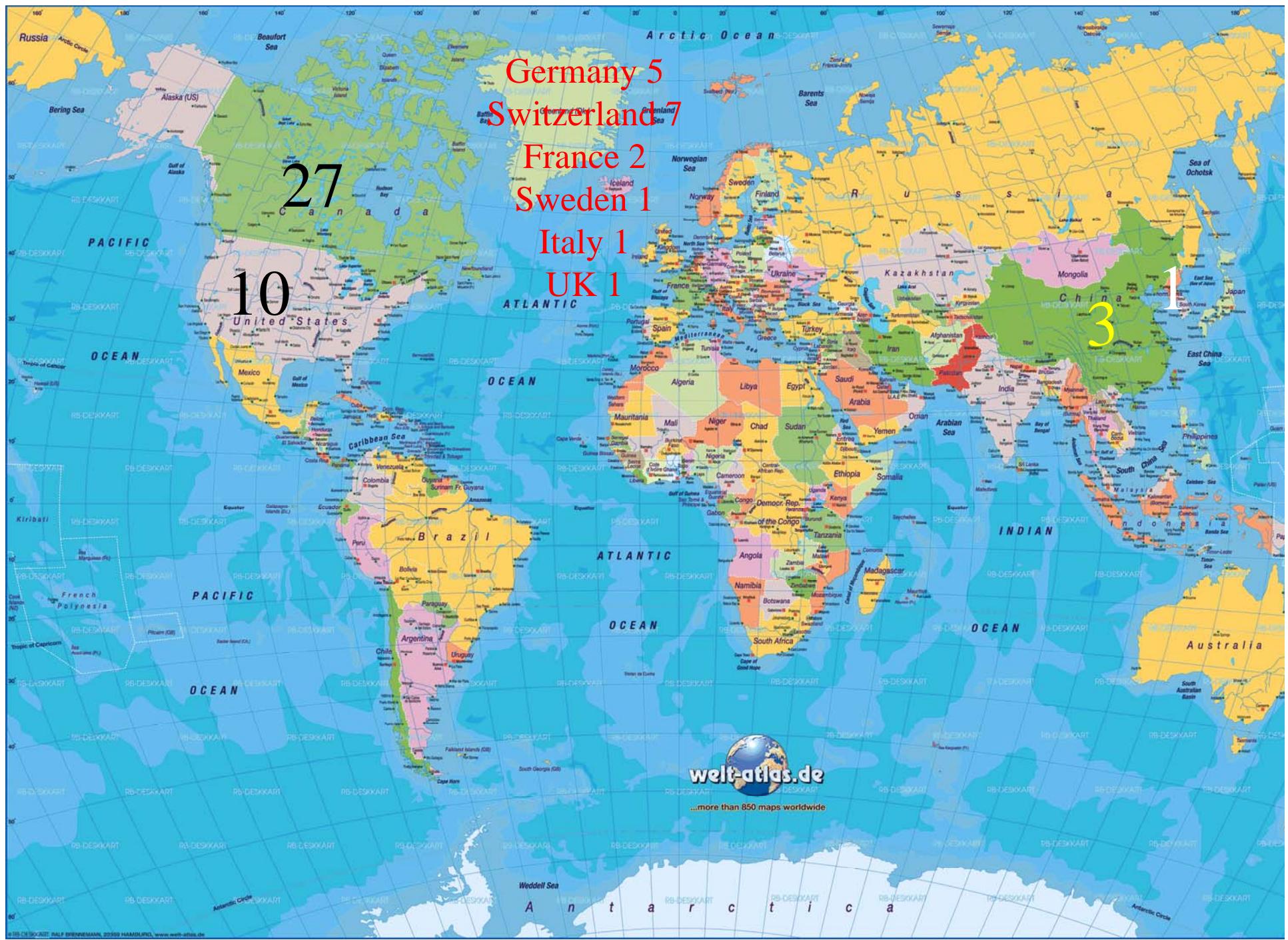




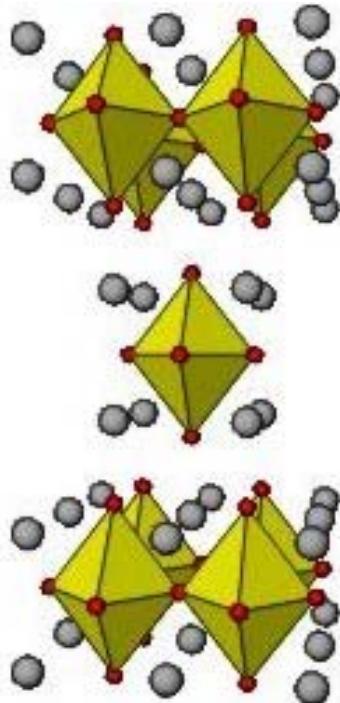
CIFAR-PITP

International Summer School on Numerical Methods for Correlated Systems in Condensed Matter

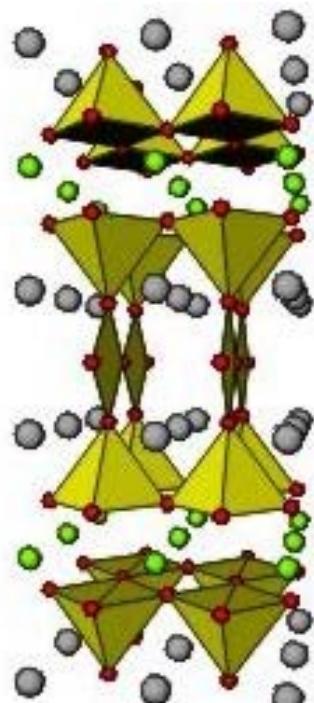




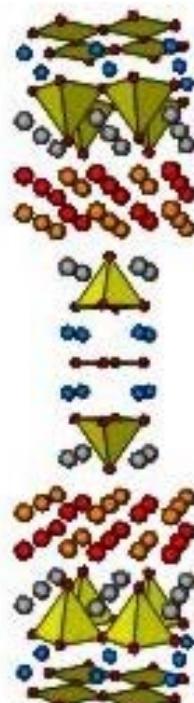
Condensed systems



$\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$



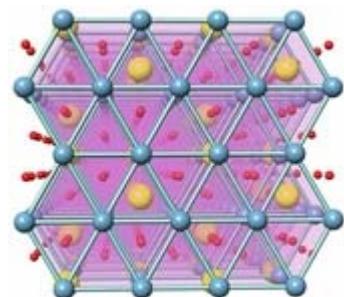
$\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$

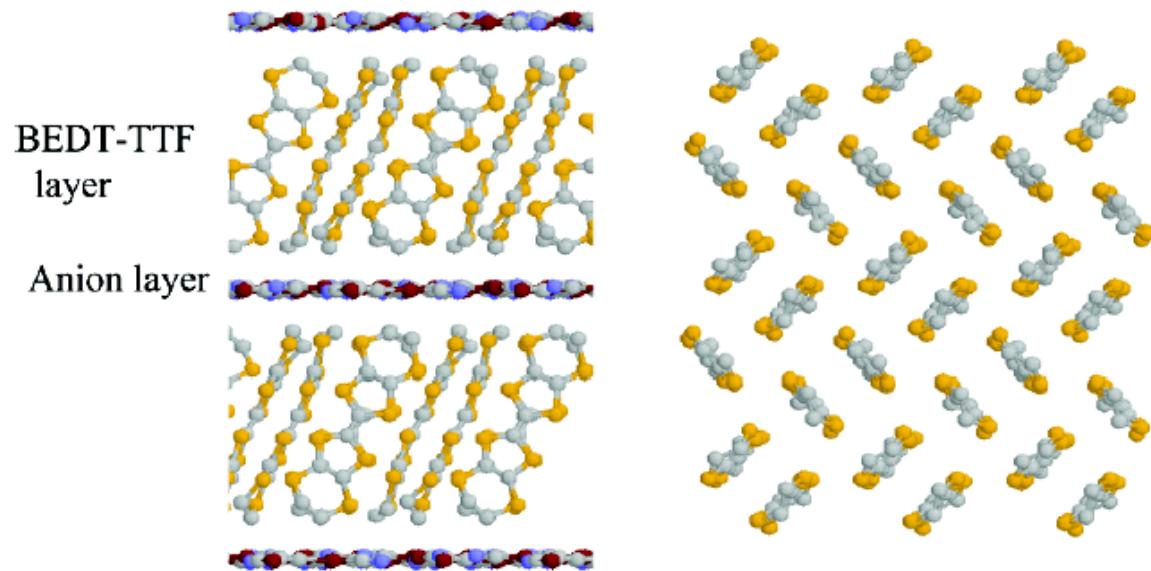


$\text{Tl}_2\text{Ca}_2\text{Ba}_2\text{Cu}_3\text{O}_{10}$

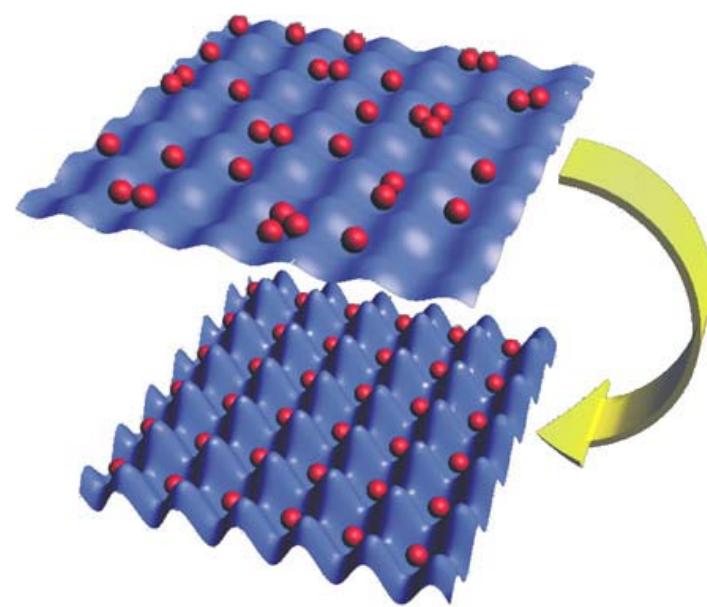
- High-temperature superconductors

- Cobaltates

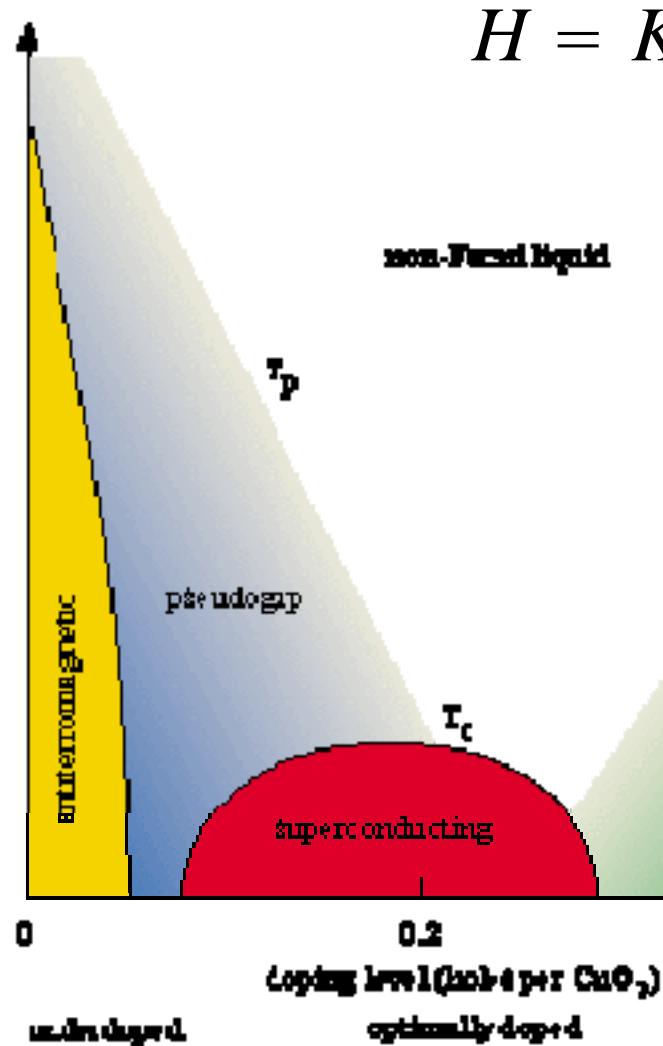




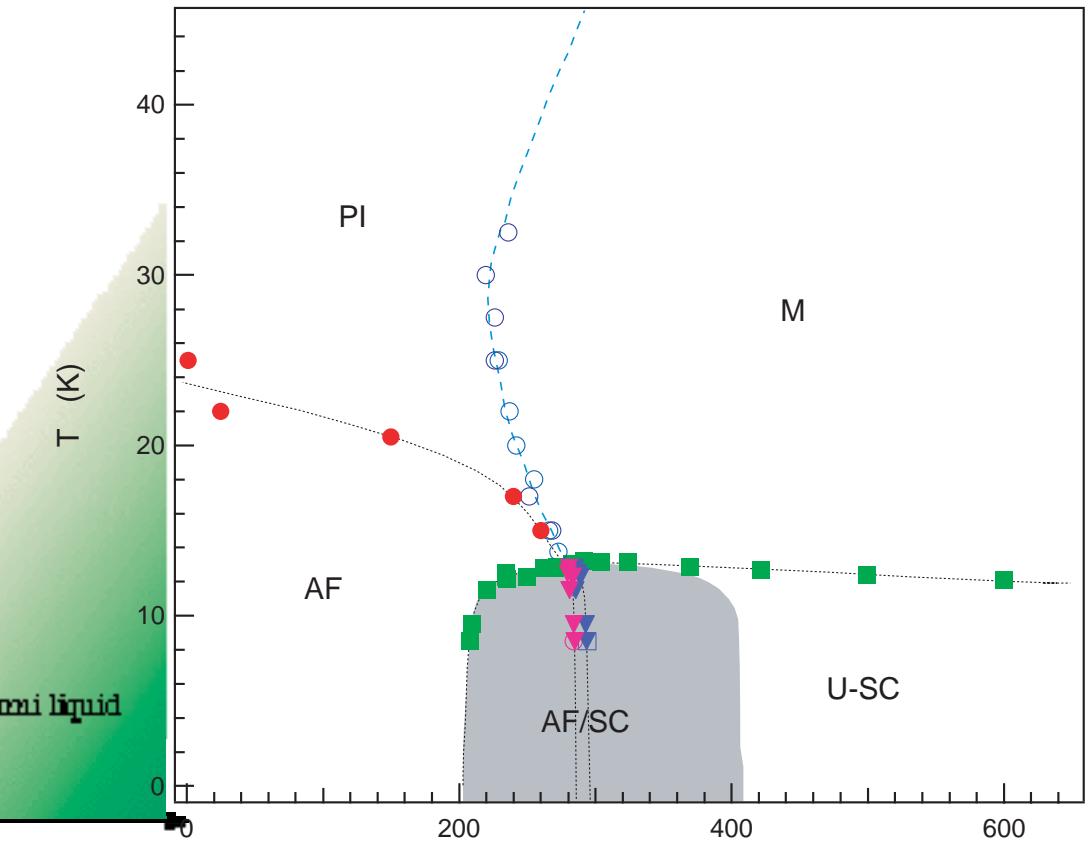
- Organics
- Cold atoms in optical lattices



The theory of everything



$$H = K + V_{e-e} + V_{e-i} + V_{i-i} + V_{s-o}$$



Battlogg and Varma,
Physics World, Feb. 2000

BEDT (X=Cu^P[N(CN)₂]Cl)
 S. Lefebvre et al. PRL **85**, 5420 (2000),
 P. Limelette, et al. PRL **91** (2003) UNIVERSITÉ DE SHERBROOKE

The theory of everything

$$H = K + V_{e-e} + V_{e-i} + V_{i-i} + V_{s-o}$$

- 10-1000 eV vs 10 meV (3-5 orders of magnitude)



The theory of everything

$$H = K + V_{e-e} + V_{e-i} + V_{i-i} + V_{s-o}$$

- 10-1000 eV vs 10 meV (3-5 orders of magnitude)
- Broken symmetry (lattice)
- Born-Oppenheimer approximation
- DFT
- Effective low energy Hamiltonian

Obtain « best » one-particle basis

- Hohenberg-Kohn theorem
- Kohn-Sham equations
- Exchange-correlation potential (s-p)
 - Michel Côté
 - Matthias Ernzerhof
 - Xavier Gonze
- LDA + U

abinit.org

Effective Hamiltonians (d-f)

- Hubbard
- t-J
- Anderson
 - André-Marie Tremblay

Solve Effective (or model) Hamiltonian

- Monte Carlo methods
 - Matthias Troyer
- QMC evaluation of Green function (Worm)
- Stochastic series expansion
 - Massimo Boninsegni
- Exact diagonalization
 - David Sénéchal
- Density Matrix Renormalization group
 - Steve White
 - Uli Schollwöck (Time dependence, Quantum Information)
- Numerical Renormalization group
 - Uli Schollwöck
- Continuous-time Quantum Monte Carlo
 - Philipp Werner

ALPS Matthias Troyer

DMFT

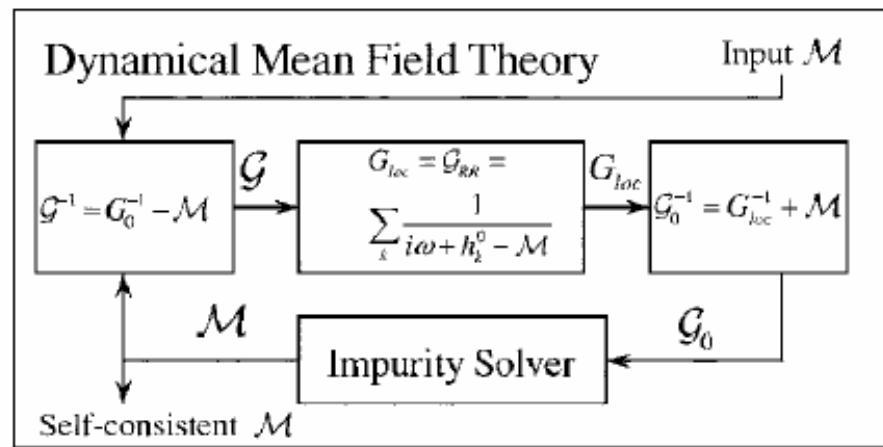


FIG. 5. Illustration of the self-consistent cycle in DMFT. Starting from a bath function, the impurity solver delivers a local M , which in turn defines a Kohn-Sham Green's function G defined for all R and R' . Only the diagonal part of the Kohn-Sham Green's function is important for the self-consistency condition which gives G_{loc} , which in turn provides the new bath function closing the iteration loop.

Kotliar et al. RMP (2006)

Variational cluster perturbation theory and DMFT as special cases of SFT

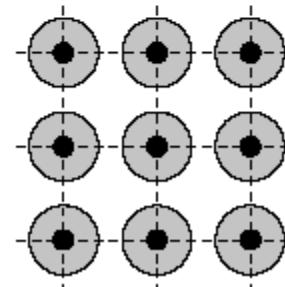
M. Potthoff *et al.* PRL **91**, 206402 (2003).

DCA,
Jarrell
et al.

Savrasov,
Kotliar,
PRB (2001)

$$\sum_{\omega_n} \sum_{\mu\nu} \left[\frac{N}{N_c} \left(\frac{1}{G_{0\mathbf{t}'}^{-1} - \Sigma'(i\omega_n)} \right)_{\mu\nu} - \sum_{\tilde{\mathbf{k}}} \left(\frac{1}{G_{0\mathbf{t}}^{-1}(\tilde{\mathbf{k}}) - \Sigma'(i\omega_n)} \right)_{\mu\nu} \right] \frac{\delta \Sigma'_{\nu\mu}(i\omega_n)}{\delta \mathbf{t}'} = 0.$$

DMFT



Georges
Kotliar, PRB
(1992).
M. Jarrell,
PRL (1992).
A. Georges,
et al.
RMP (1996).

Quantum Cluster methods

- Variational Cluster Approximation
- Cellular Dynamical Mean-Field Theory
 - David Sénéchal
 - Gabi Kotliar
- Dynamical Cluster Approximation + Hirsch-Fye algorithm
 - Mark Jarrell
- Continuous-time Quantum Monte Carlo
 - Philipp Werner

LDA+DMFT

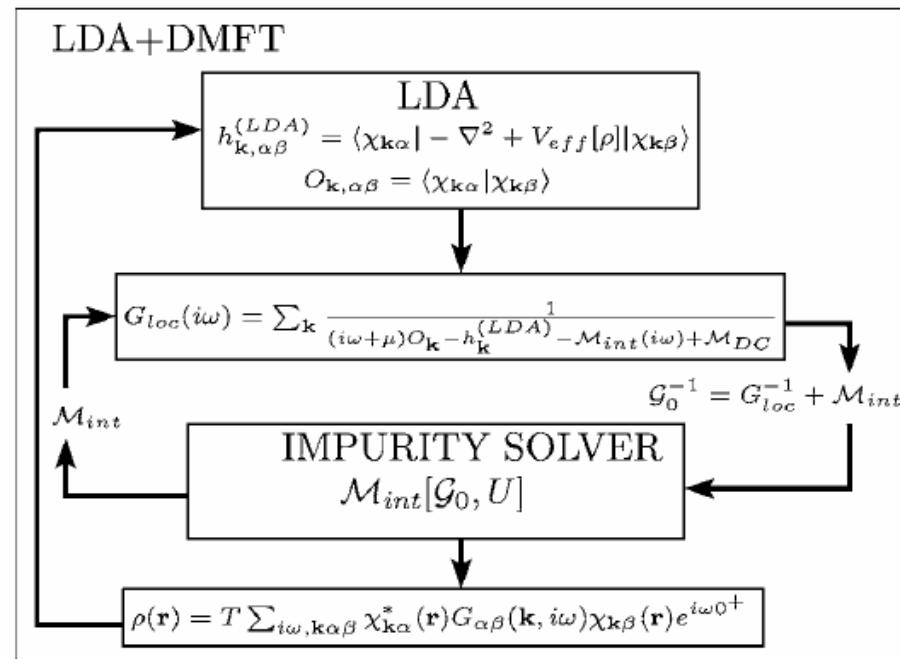


FIG. 9. Illustration of the self-consistent cycle in spectral density-functional theory within the LDA+DMFT approximation: the double iteration cycle consists of the inner DMFT loop and outer (density plus total energy) loop.

Kotliar et al. RMP (2006)

A unified perspective

- Inversion method
 - Gabi Kotliar
 - Exchange-correlation potential
 - Luttinger-Ward or Baym Kadanooff functional
 - Spectral density functional
- Self-Energy functional

Kotliar et al. RMP (2006)

A unified perspective

- Inversion method

TABLE I. Parallel between the different approaches, indicating the physical quantity which has to be extremized, and the field which is introduced to impose a constraint (constraining field). BL and AL correspond to the band and atomic limit reference systems, respectively.

Method	Physical quantity	Constraining field
Baym-Kadanoff	$G_{\alpha\beta}(\mathbf{k}, i\omega)$	$\Sigma_{\text{int},\alpha\beta}(\mathbf{k}, i\omega)$
DMFT (BL)	$G_{\text{loc},\alpha\beta}(i\omega)$	$\mathcal{M}_{\text{int},\alpha\beta}(i\omega)$
DMFT (AL)	$G_{\text{loc},\alpha\beta}(i\omega)$	$\Delta_{\alpha\beta}(i\omega)$
LDA+DMFT (BL)	$\rho(r)$, $G_{\text{loc},ab}(i\omega)$	$V_{\text{int}}(r)$, $\mathcal{M}_{\text{int},ab}(i\omega)$
LDA+DMFT (AL)	$\rho(r)$, $G_{\text{loc},ab}(i\omega)$	$V_{\text{int}}(r)$, $\Delta_{ab}(i\omega)$
LDA+ U	$\rho(r)$, n_{ab}	$V_{\text{int}}(r)$, λ_{ab}
LDA	$\rho(r)$	$V_{\text{int}}(r)$

Guest speakers

- Electron-Phonon interactions
 - Mona Berciu
- Ab Initio Molecular Electronics
 - Hong Guo

Overall view



Direct ED, QMC, SSE, Worm, NRG, DMRG

Quantum Clusters CPT, VCA, DCA, CDMFT

Final exam

- Friday June 6
- Take home is possible (but not recommended).

Groups for hands-on training and posters

- Today 13:30, first time.
- Group 1 and week 1: Ali to Joseph (30)
- Group 2 and week 2: Julien to Zhanyu (28)
- CAUTION: Times not always the same.

Registration

- Tuesday: Passport or birth certificate
 - 13:00 A to D
 - 13:15 E to J
 - 13:30 K to P
 - 13:45 Q to Z

Québec 1608-2008

- Self-organized process

1713

