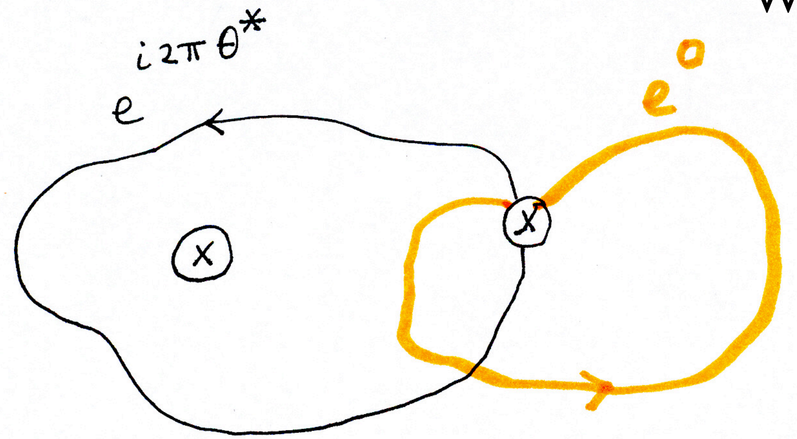
# Microscopic tests of Abelian and non-Abelian braiding statistics in the fractional quantum Hall effect

# Plan

- Abelian braiding statistics (Gun Sang Jeon, Kenneth Graham, JKJ)
- Non-Abelian statistics (Nicolas Regnault, Csaba Toke, JKJ)

# braiding statistics

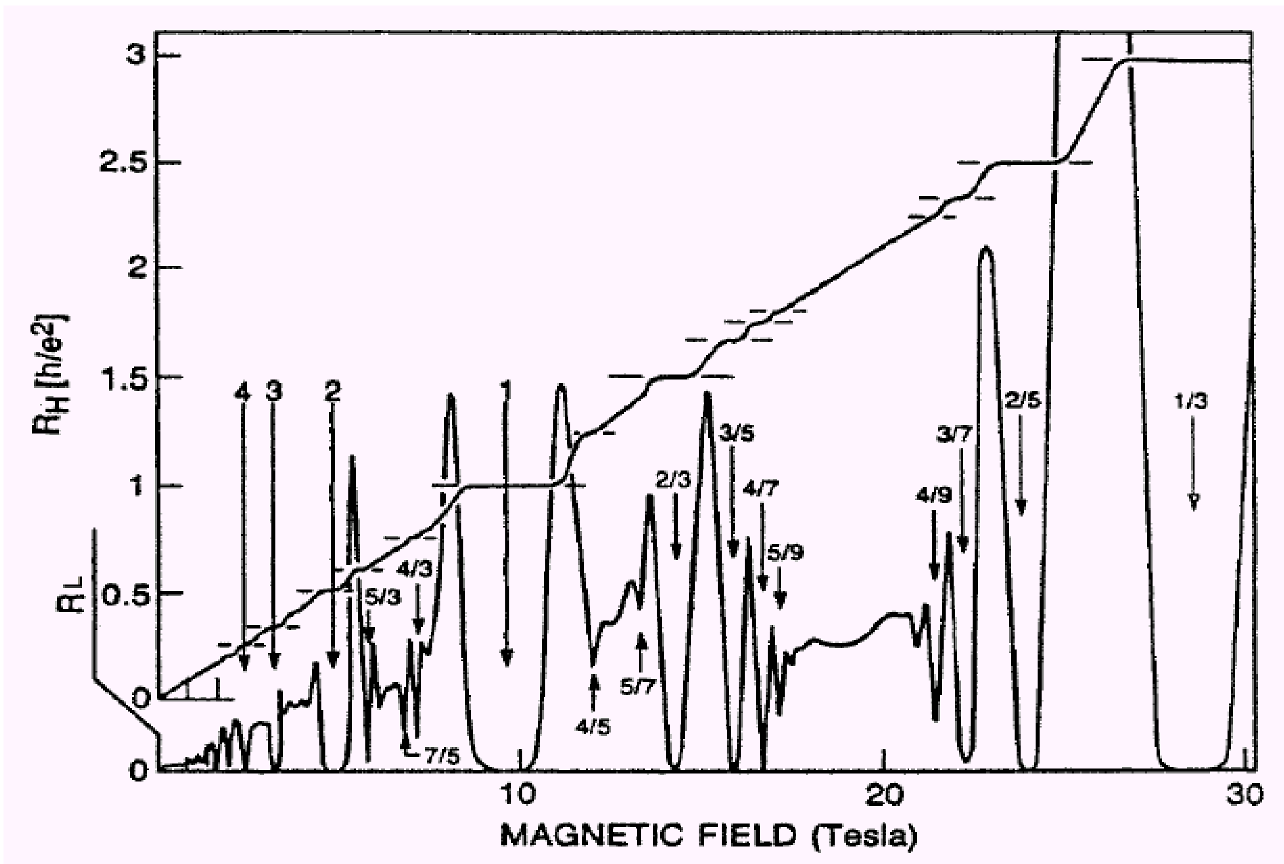
Leinaas, Myrheim (1977),
Wilczek (1982)




- Imagine particles that acquire non-trivial, path independent phases when they go around one another.
- Half a loop of a particle around another is equivalent to an exchange, which produces a phase factor $e^{i\pi\theta^*}$. Such particles are said to obey fractional "braiding statistics."
- Fractional statistics can be defined only in two dimensions, for particles that have an infinite hard core repulsion. It is a topological concept.
- Anyons can be modeled as fermions (or bosons) with flux lines bound to them.

- Just because one can define fractional braiding statistics does not mean that it exists in nature.
- All particles in nature are either bosons or fermions. However, there is no principle that precludes certain emergent quasiparticles from possessing fractional braiding statistics.
- FQHE is currently the best (the only?) candidate for its physical realization.

# FQHE: a new quantum fluid



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# Two-Dimensional Magnetotransport in the Extreme Quantum Limit

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(Received 5 March 1982)

**1/3 plateau !**

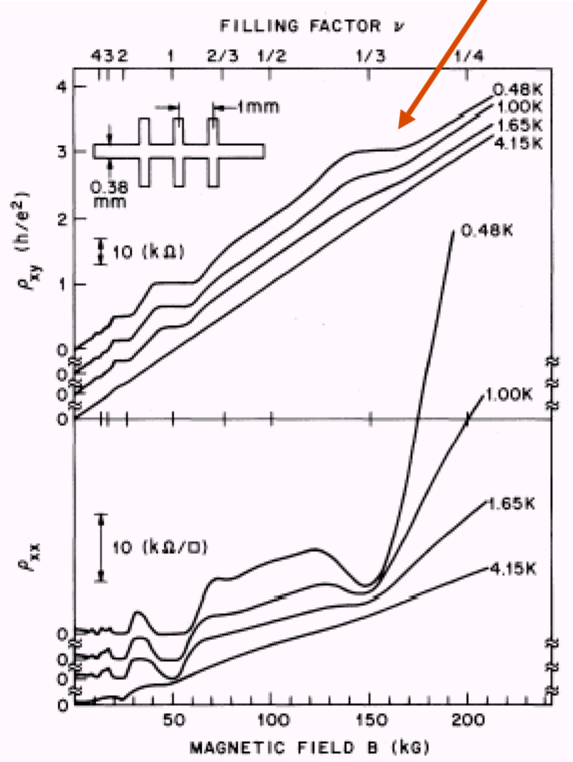



FIG. 1. $\rho_{xy}$ and $\rho_{xx}$ vs $B$, taken from a GaAs-Al$_{0.3}$-Ga$_{0.7}$As sample with $n = 1.23\times 10^{11}/\mathrm{cm}^2$, $\mu = 90\,000\ \mathrm{cm}^2/\mathrm{V\,sec}$, using $I = 1\ \mu\mathrm{A}$. The Landau level filling factor is defined by $\nu = nh/eB$.

$$R_H = \frac{h}{\frac{1}{3}e^2}$$

# Excitations $\left(\nu = \frac{1}{m}\right)$

**Laughlin, 1983**

$$\Psi^{\mathrm{ground}} = \prod_{j<k} (z_j - z_k)^m \exp[-\sum_i |z_i|^2/4]$$

$$z = x - iy$$

$$\Psi_\eta^{\mathrm{quasihole}} = \prod_j (z_j - \eta) \Psi^{\mathrm{ground}}$$

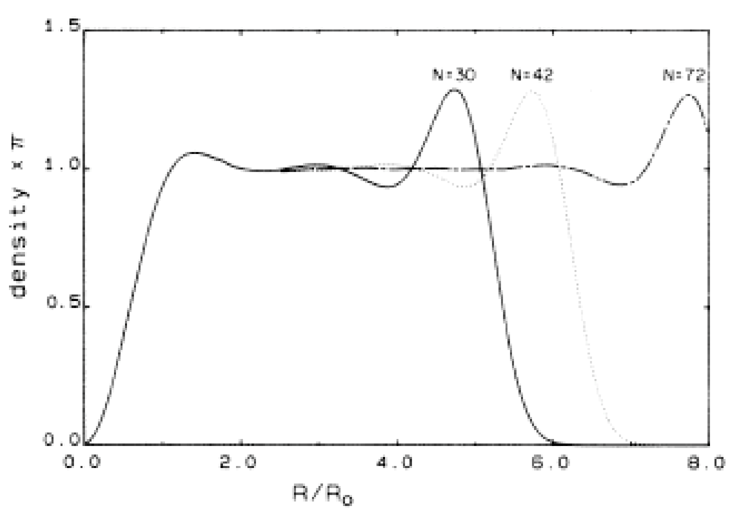
Quasihole at 1/m is modeled as a vortex.

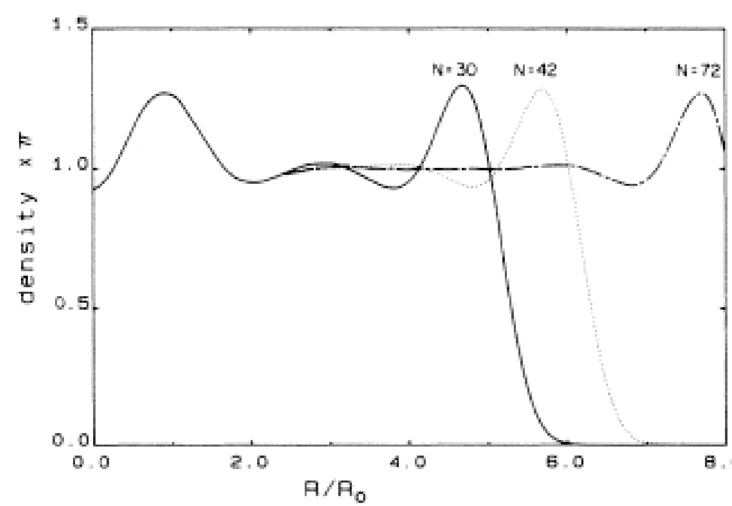
$$\Psi_\eta^{\mathrm{quasielectron}} = \exp\left[-\sum_j |z_j|^2/4\right] \prod_j \left(2\frac{\partial}{\partial z_j} - \eta^*\right) \prod_{i<k} (z_i - z_k)^m$$

"Quasielectron" looks nothing like an electron.
The name "quasiparticle" is more appropriate.

# Fractional charge

The charge excess or deficiency associated with a quasiparticle or a quasihole is precisely e/m. This can be confirmed by a direct integration of the charge density.




Morf and Halperin

# The charge of a vortex

Arovas, Schriefffer, Wilczek, 1984

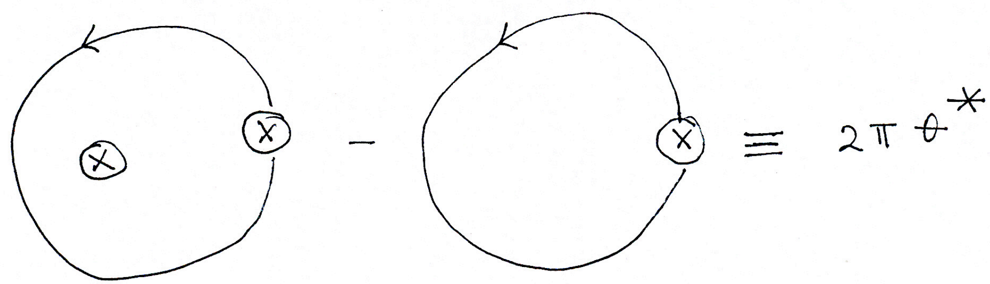
Take a vortex adiabatically in a closed loop. The Berry phase for a vortex simply counts the number of electrons enclosed in the loop:

$$\begin{aligned} \Phi^*_{\mathrm{vortex}} &= 2\pi N_{\mathrm{enc}} \\ &= 2\pi\rho A \\ &= 2\pi e^* \frac{BA}{hc} \end{aligned}$$

This implies: $e^* = \nu e \qquad (\nu = \rho hc/eB)$

# Braiding statistics

In FQHE, the Berry phase also gets a contribution from the ordinary Aharonov Bohm effect, which must be subtracted:




The braiding statistics can be evaluated with the knowledge of the many body wave function for the quasiparticles.

$$\theta^* = \oint_{\mathcal{C}} \frac{d\theta}{2\pi} \frac{\left\langle \Psi^{\eta,\eta'} \middle| i\frac{d}{d\theta}\Psi^{\eta,\eta'} \right\rangle}{\left\langle \Psi^{\eta,\eta'} \middle| \Psi^{\eta,\eta'} \right\rangle} - \oint_{\mathcal{C}} \frac{d\theta}{2\pi} \frac{\left\langle \Psi^{\eta} \middle| i\frac{d}{d\theta}\Psi^{\eta} \right\rangle}{\left\langle \Psi^{\eta} \middle| \Psi^{\eta} \right\rangle}$$

# Braiding statistics for the vortices

Halperin, 1984
Arovas, Schriefffer, Wilczek, 1984

The Berry phase for a vortex:

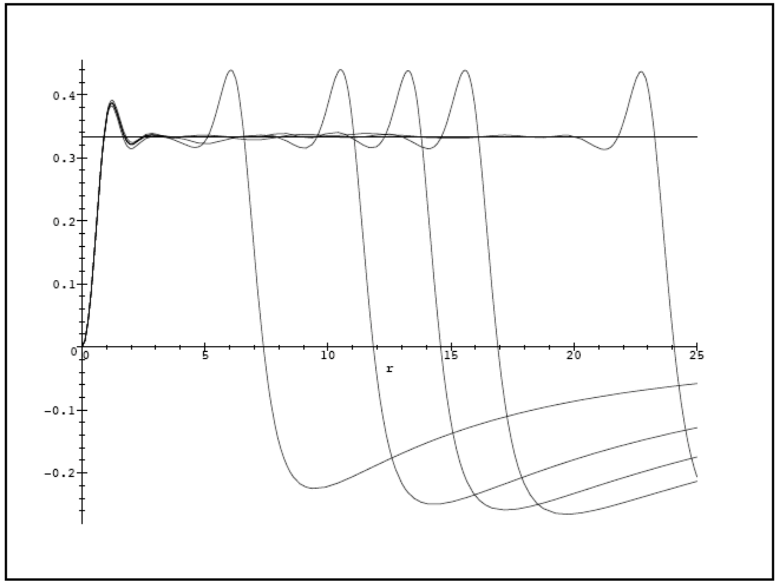
$$\Phi^*_{\mathrm{vortex}} = 2\pi N_{\mathrm{enc}}$$

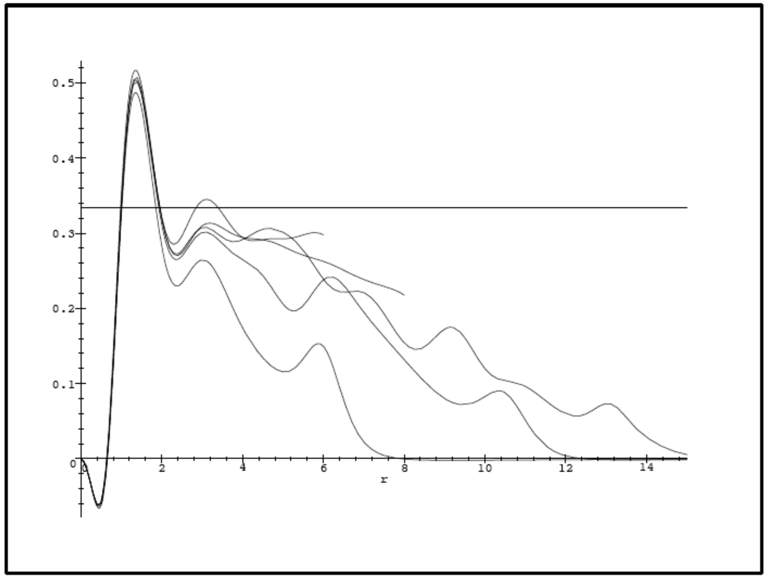
The braiding statistics thus depends on the change in the number of electrons upon the insertion of an additional vortex inside the area enclosed by the loop:

$$\Delta\Phi^* = 2\pi\theta^* = 2\pi\Delta N_{\mathrm{enc}} = -2\pi\nu$$

The 1/m quasihole happens to be a vortex, thus obeys fractional braiding statistics. This is closely related to fractional charge.

# Problems





Kjonsberg and Myrheim (1998):
Braiding statistics of quasiholes at 1/3 for N=20, 50, 75, 100, 200.

Kjonsberg and Myrheim (1998):
Braiding statistics of quasiparticles at 1/3 for N=20, 50, 75, 100, 200.
Laughlin's wave function is used.

Braiding statistics is a subtle property.
It does not follow from fractional charge.

Quasiparticles / quasiholes of other FQHE states are not vortices. How about their braiding statistics?

# Composite fermion theory

# Statement of the FQHE problem

- **Find the solutions for the quantum mechanical problem of interacting electrons in a magnetic field.**

$$H\Psi = E\Psi$$

$$H = \frac{1}{2m_b}\sum_j \left(\frac{\hbar}{i}\vec{\nabla}_j + \frac{e}{c}\vec{A}(\vec{r}_j)\right)^2 + \frac{e^2}{\epsilon}\sum_{j<k}\frac{1}{r_{jk}} + g\mu_B\vec{B}\cdot\vec{S}$$

- **In the limit of** $B \to \infty$ **the kinetic and Zeeman energies are constant.**

$$H = \sum_{j<k}\frac{1}{r_{jk}} \qquad \text{(to be solved with the lowest LL restriction)}$$

- **No small parameter. Approximate strategies are doomed by the absence of a small parameter. Usually the interaction is treated as a perturbation. Here interaction is the only energy in the problem. Perturbation theory is not possible.**

- **The FQHE problem has no normal state.**

# the degeneracy problem

• If we "switch off" the interaction between electrons, <u>all</u> lowest Landau level configurations are degenerate.

$$N = 10^9 \Rightarrow 10^{7\times 10^8} \text{ states}$$

$$(\nu = 0.4)$$

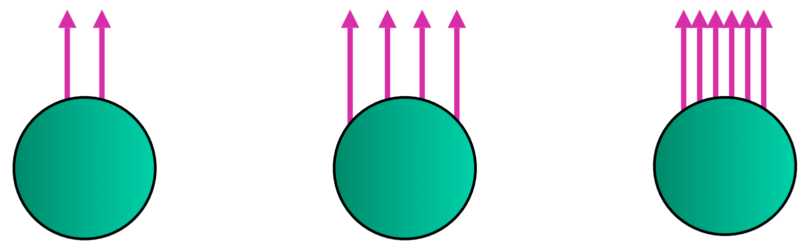
$$N = 100 \Rightarrow 10^{72} \text{ states}$$

• The repulsive Coulomb interaction between electrons picks out one of these states as the ground state. Our goal is to find this state.

• On purely theoretical grounds, with no small parameter to guide us, we have no idea where to start. The problem appears hopelessly intractable.

# Composite fermions

Jain, 1989



$^{2}CF$ $^{4}CF$ $^{6}CF$

**Composite fermion = electron + 2p quantized vortices**

A quantized vortex is a topological object. Hence also is the composite fermion.

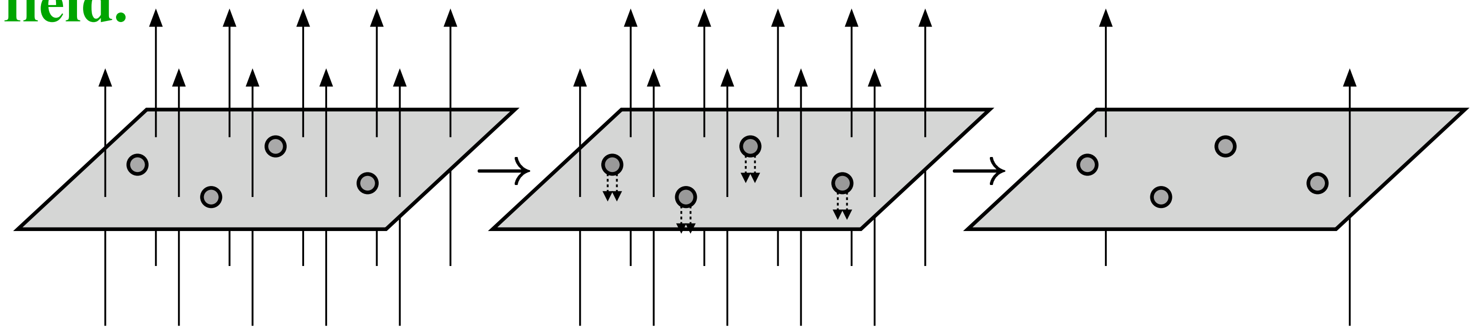
A vortex is often represented as a flux quantum. A composite fermion is often thought of (somewhat inaccurately) as:

**Composite fermion = electron + 2p flux quanta**

**In general: electron + flux = anyon (Leinaas & Myrheim; Wilczek)**

# The CF theory in a nutshell

**Electrons transform into composite fermions by capturing 2p quantized vortices. Composite fermions experience a much reduced effective magnetic field. The complex problem of strongly correlated electrons thus maps into a relatively simpler problem of weakly interacting fermions at an effective magnetic field.**



(a) (b) (c)

$$B^* = B - 2p\rho\phi_0 \quad \left(\phi_0 = \frac{hc}{e}\right)$$

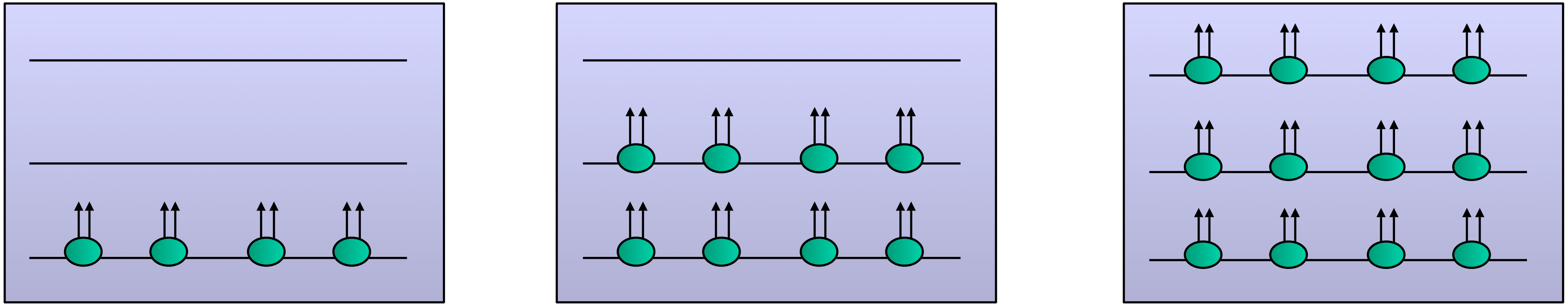
$$\nu = \frac{\nu^*}{2p\nu^* \pm 1}$$

$$\Psi_\nu = \mathcal{P}_{\mathrm{LLL}} \Phi_{\nu^*} \prod_{j<k} (z_j - z_k)^{2p}$$

# FQHE = IQHE of composite fermions

**Consider composite fermions at** $\nu^* = n$

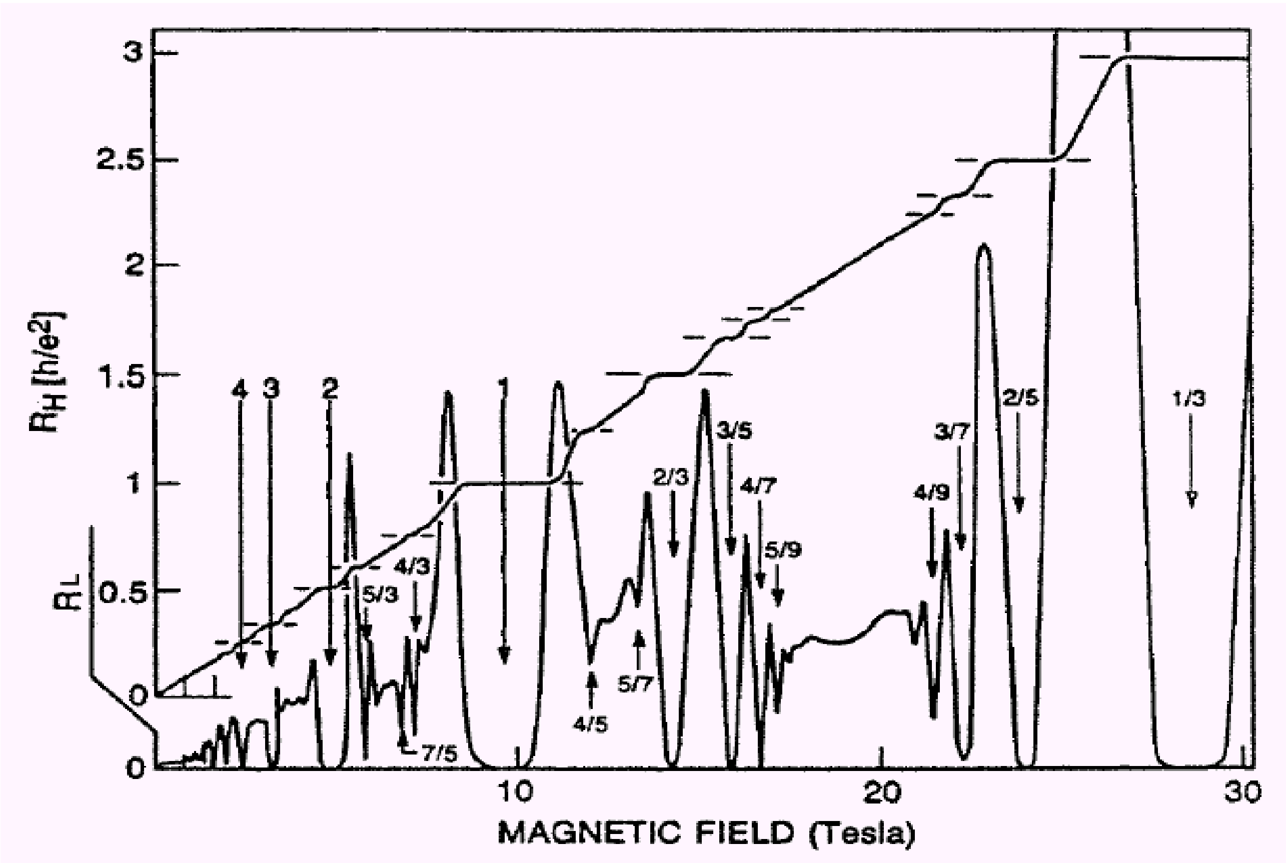
**Here** $$\boxed{\nu = \frac{\nu^*}{2p\nu^* \pm 1} = \frac{n}{2pn \pm 1}}$$



$\frac{1}{3} \Leftrightarrow 1$ $\qquad$ $\frac{2}{5} \Leftrightarrow 2$ $\qquad$ $\frac{3}{7} \Leftrightarrow 3$

**At these filling factors, when electrons transform into composite fermions, the enormous ambiguity of the electron problem completely disappears to produce a unique state with a gap.**

$$\nu = \frac{n}{2pn \pm 1}$$ **are precisely the observed fractions.**




$$\nu = \frac{n}{2n+1} = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \ldots, \frac{10}{21}$$

$$\nu = \frac{n}{2n-1} = \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \ldots, \frac{10}{19}$$

$$\nu = \frac{n}{4n+1} = \frac{1}{5}, \frac{2}{9}, \ldots, \frac{6}{25}$$

$$\nu = \frac{n}{4n-1} = \frac{2}{7}, \frac{3}{11}, \ldots, \frac{6}{23}$$

$$\nu' = 1 - \nu$$

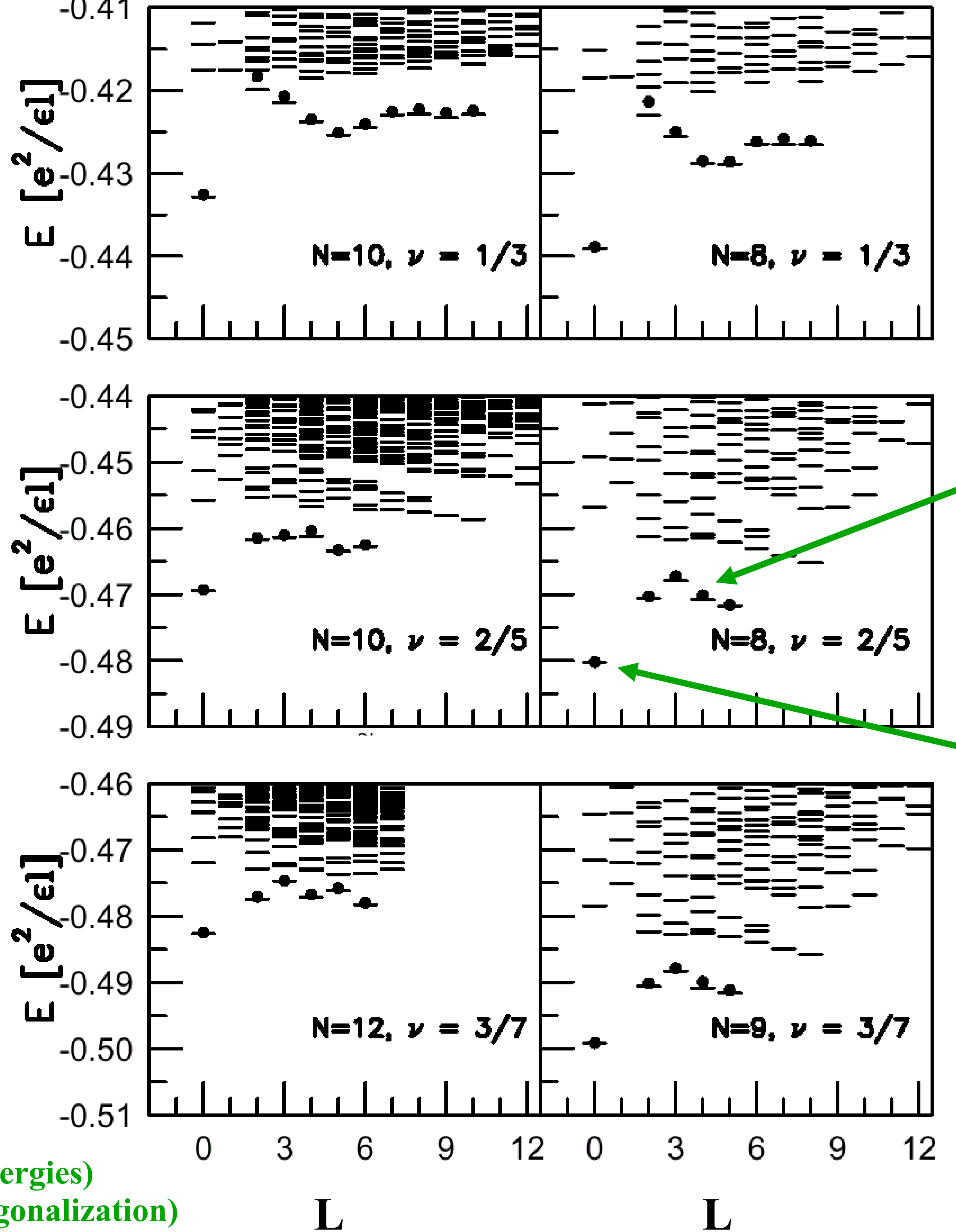
**The CF theory predicts sequences of fractions.**

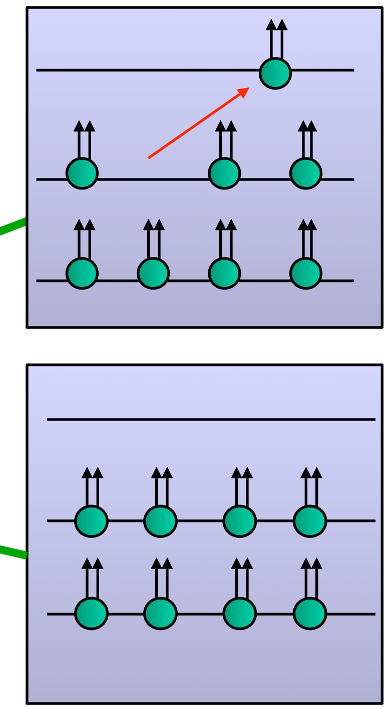
**The order of stability is correctly predicted.**

$$\nu = \frac{1}{3}$$

$$\nu = \frac{2}{5}$$

$$\nu = \frac{3}{7}$$



0.05% accuracy with no adjustable parameters!

Dev, Kamilla, Jain (CF energies)
He, Xie, Zhang (exact diagonalization)

| $\nu$ | $n$ | N | overlap |
|---|---|---|---|
| $\frac{1}{3}$ | 1 | 7 | 0.9964 |
| | | 8 | 0.9954 |
| | | 9 | 0.9941 |
| $\frac{2}{5}$ | 2 | 6 | 0.9998 |
| | | 8 | 0.9996 |
| $\frac{3}{7}$ | 3 | 9 | 0.9994 |
| $\frac{2}{3}$ | -2 | 6 | 0.9965 |
| | | 8 | 0.9982 |
| | | 10 | 0.9940 |

Fano, Ortolani, Colombo
Wu, Dev and Jain

# Statistics

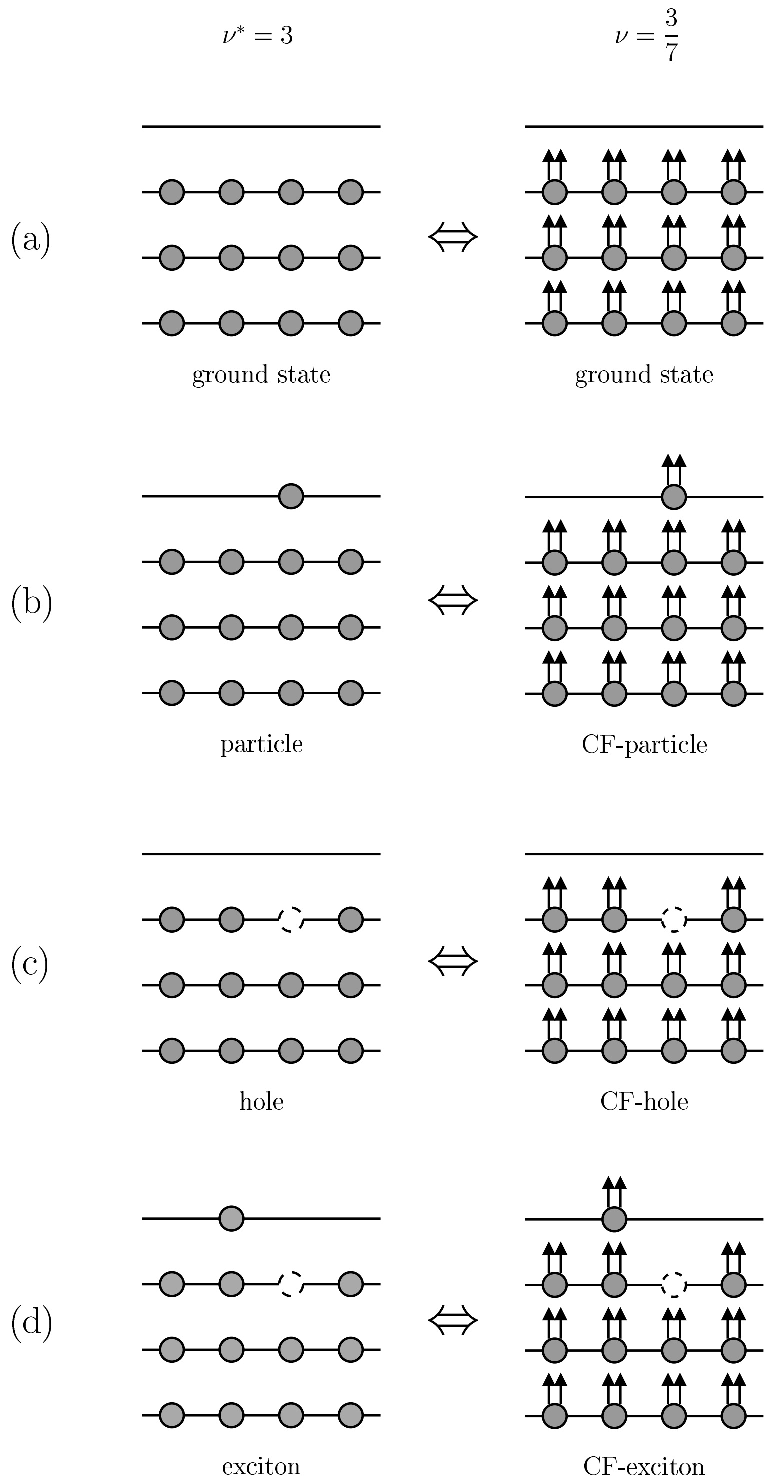
- The particle statistics has dramatic, testable consequences, both in condensed matter physics and quantum chemistry.
- Fermionic nature of electrons is crucial for explaining atomic physics (periodic table), Fermi sea, superconductivity.
- Bosonic nature of He-4 atoms is responsible for the phenomenon of superfluidity.

# Statistics of composite fermions

The fermionic statistics of composite fermions is firmly established. Much of our understanding is based on that fact.

- FQHE: composite fermions fill CF-Landau levels
- prominent sequences of fractions
- Effective magnetic field
- CF Fermi sea
- Microscopic wave functions
- Excitations; energy level counting
- etc.

- The phenomenon of the FQHE and all related experimental observations, so far, can be understood, at an impressive level of qualitative and quantitative detail, as a consequence of composite fermions.
- Fractional braiding statistics is also a direct consequence of the formation of composite fermions.




<u>The law of corresponding states</u>: The fundamental consequence of the CF theory is a one-to-one correspondence between the low energy states at $\nu$ and $\nu^*$, which allows us to understand strongly interacting FQHE system by analogy to a weakly interacting system.

<u>A "quasiparticle" is a composite fermion in an otherwise empty level, and a "quasihole" is a missing composite fermion from a full level.</u>

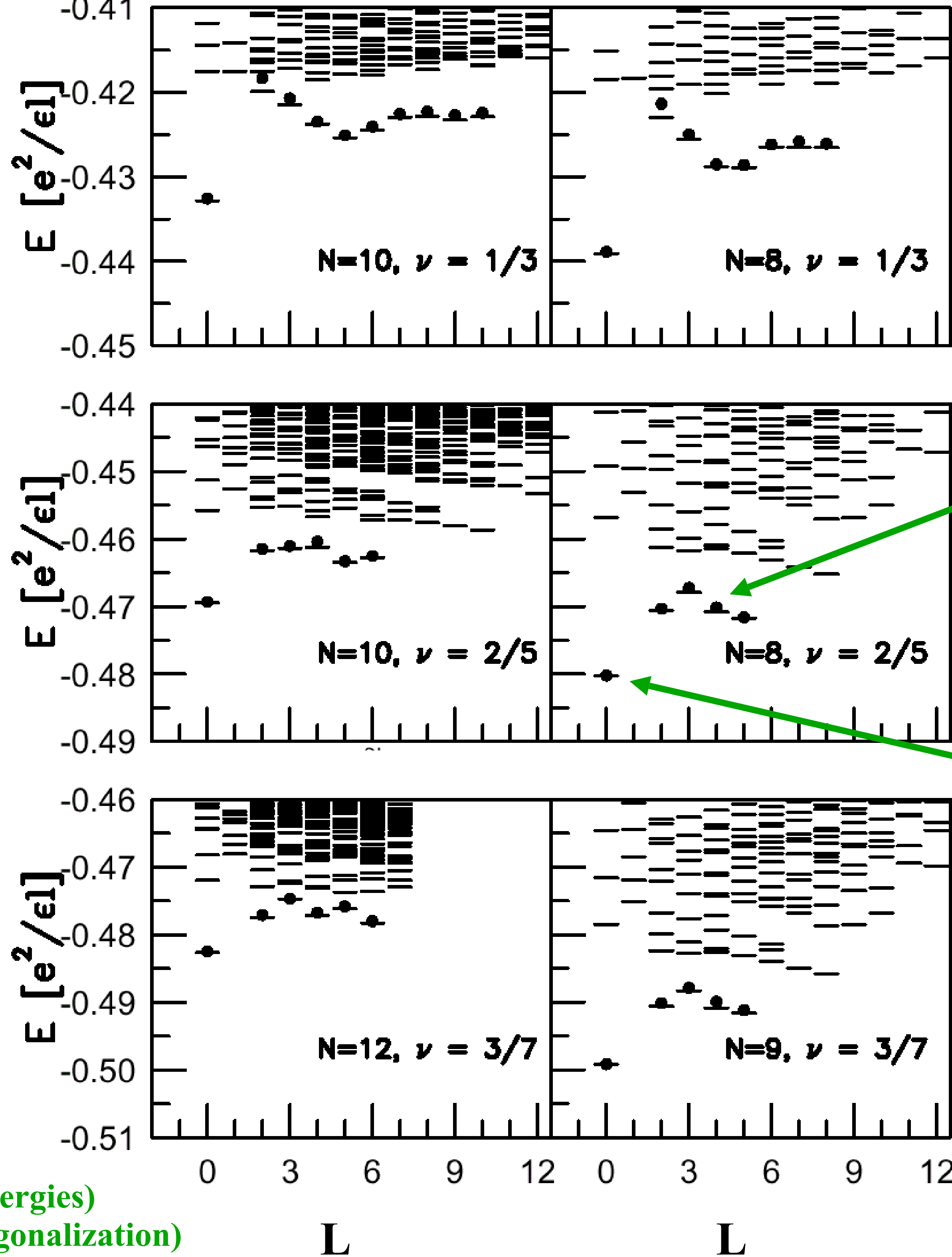
This gives us quasiparticles and quasiholes of all FQHE states at once.

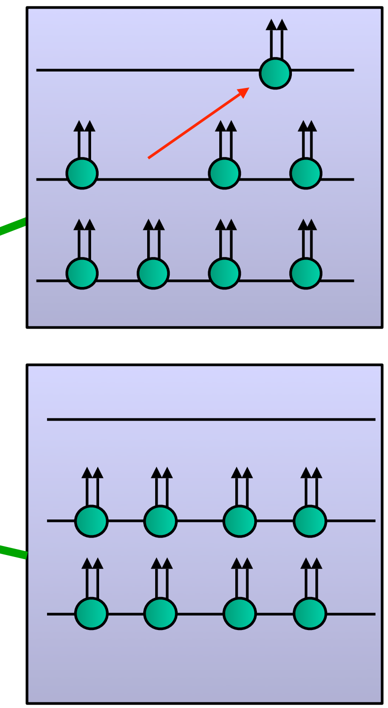
The quasihole of 1/m happens to be a vortex. But that is an exception. Other quasiparticles or quasiholes are not vortices, but have a tremendously complicated structure.

$\nu = \frac{1}{3}$

$\nu = \frac{2}{5}$

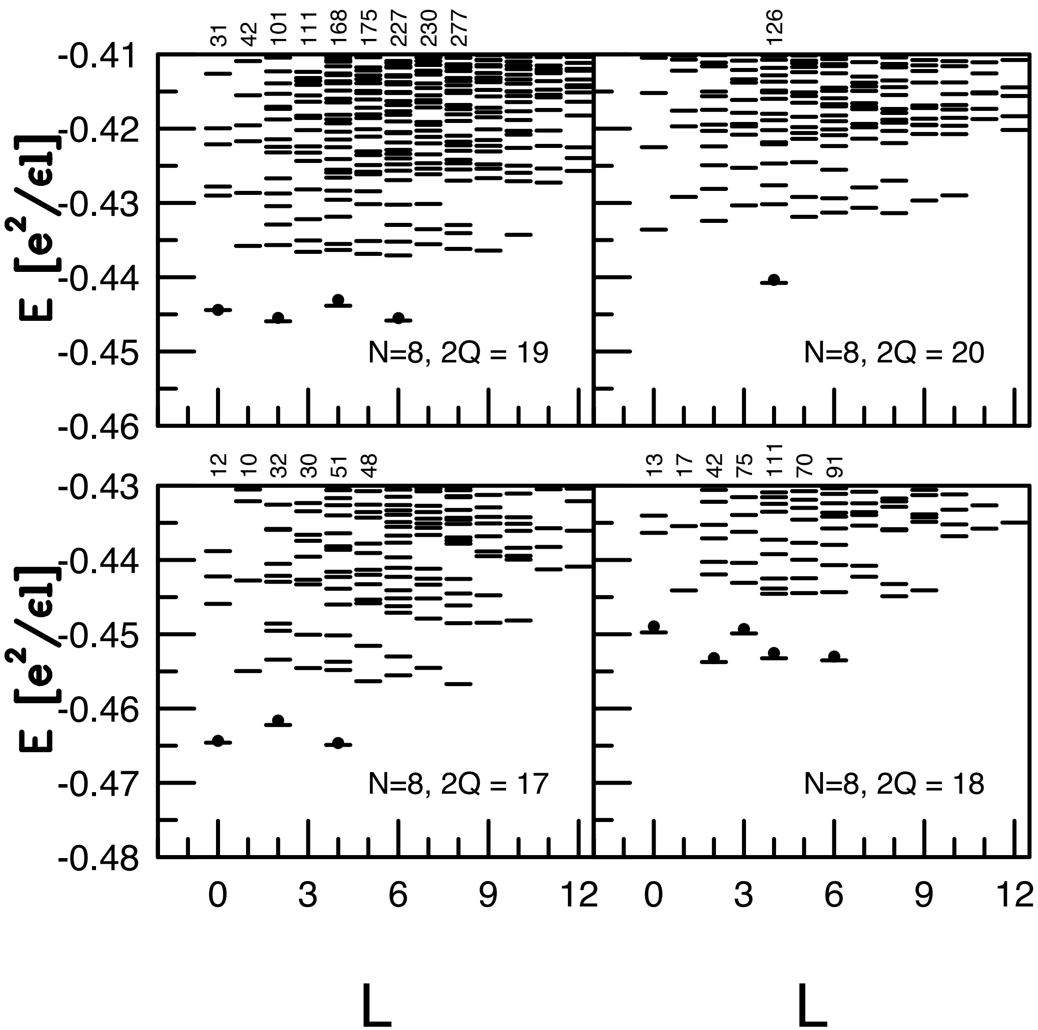
$\nu = \frac{3}{7}$

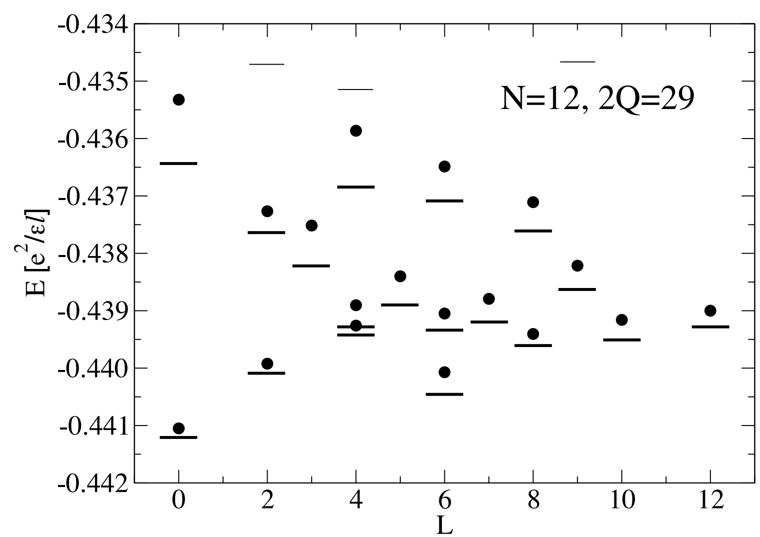


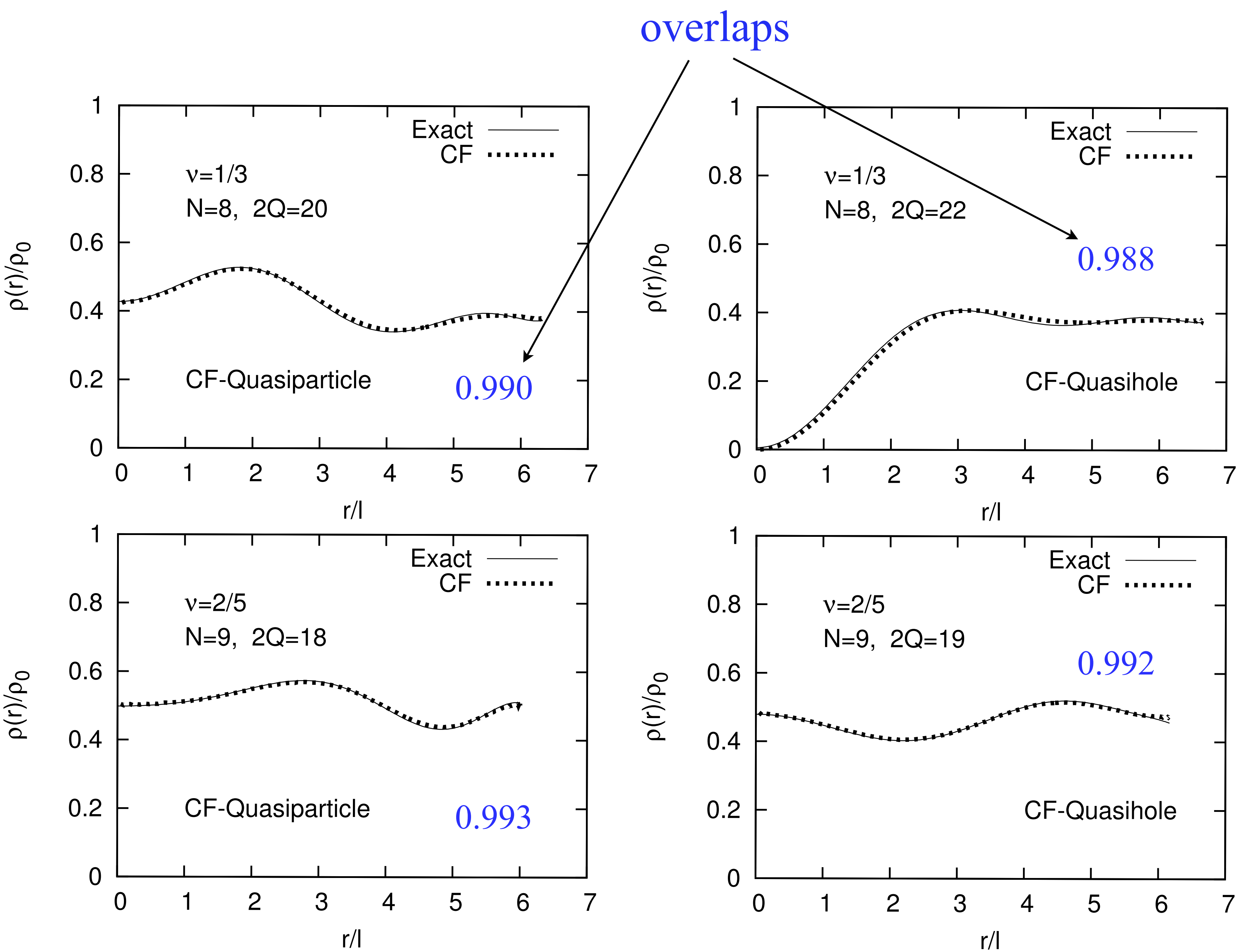
0.05% accuracy with no adjustable parameters!

Dev, Kamilla, Jain (CF energies)
He, Xie, Zhang (exact diagonalization)


31
42
101
111
168
175
227
230
277
126
-0.41
-0.42
-0.43
-0.44
-0.45
-0.46
E [e²/εl]
N=8, 2Q = 19
N=8, 2Q = 20
12
10
32
30
51
48
13
17
42
75
111
70
91
-0.43
-0.44
-0.45
-0.46
-0.47
-0.48
N=8, 2Q = 17
N=8, 2Q = 18
0
3
6
9
12
L


-0.434
-0.435
-0.436
-0.437
-0.438
-0.439
-0.440
-0.441
-0.442
E [e²/εl]
N=12, 2Q=29
0
2
4
6
8
10
12
L

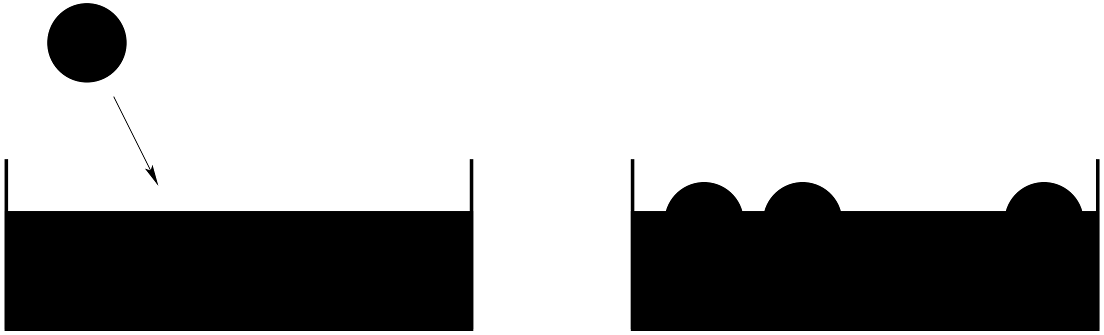
# CF quasiparticle / CF quasihole are essentially exact.



# excess charge

- The charge excess associated with a CF-quasiparticle is equal to the charge of an electron + the charge of 2p vortices (which produce a correlation hole around it).
- It has a precise fractional value. Fractional charge.

$$\text{For} \quad \nu = \frac{n}{2pn+1}\,, \qquad e^* = -1 + 2p\frac{n}{2pn+1} = -\frac{1}{2pn+1}$$



## CF-quasiparticles and CF-quasiholes obey fractional braiding statistics.

- The phase associated with a complete loop of a CF-quasiparticle is given by:

$$\Phi^* = -2\pi \frac{BA}{\phi_0} + 2\pi 2p N_{\rm enc}$$

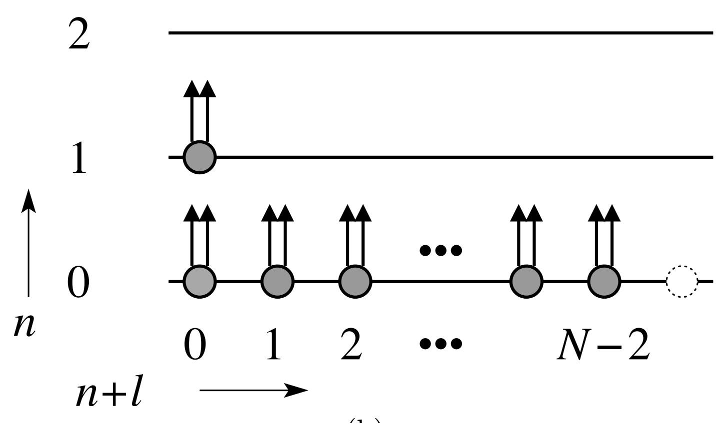
(gives B*)

- Berry phase difference: $$\Delta\Phi^* = 2\pi 2p \Delta N_{\rm enc} = 2\pi \frac{2p}{2pn+1} \equiv 2\pi\theta^*$$
- Fractional statistics $$\theta^* = \frac{2p}{2pn+1}$$

The fractional statistics is thus a statement about the change in the effective magnetic field due to order one changes in the density.

Does the heuristic result hold up in a microscopic calculation?

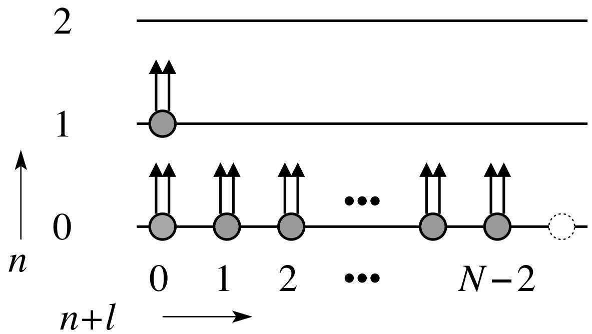
# CF quasiparticle at 1/3

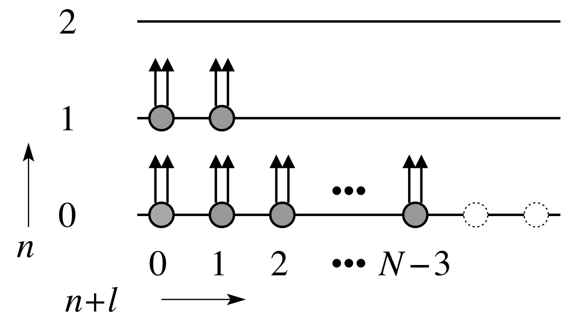


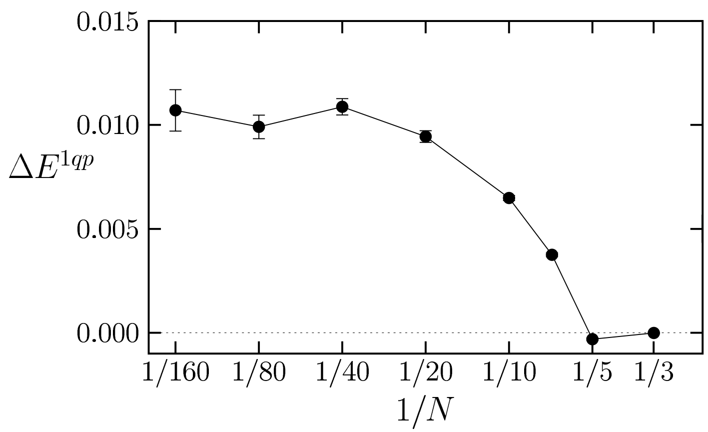

$$\Psi_{CF}^{1qp} = \mathcal{P}\prod_{j<k}(z_j-z_k)^2 \begin{vmatrix} z_1^* & z_2^* & \ldots \\ 1 & 1 & \ldots \\ z_1 & z_2 & \ldots \\ \vdots & \vdots & \ldots \\ z_1^{N-2} & z_2^{N-2} & \ldots \end{vmatrix} = \sum_{i=1}^{N} \frac{6\sum_k'(z_i-z_k)^{-1}}{\prod_j'(z_j-z_i)}\Psi^{GS}$$

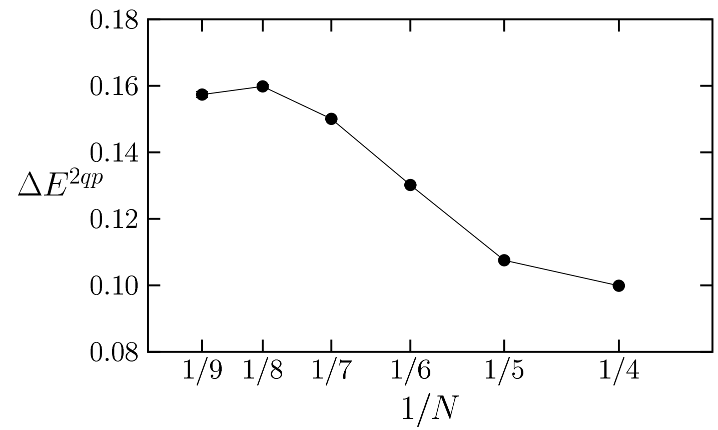
The form of the wave function is different from Laughlin's, and more complicated.

# Comparing the two trial wave functions



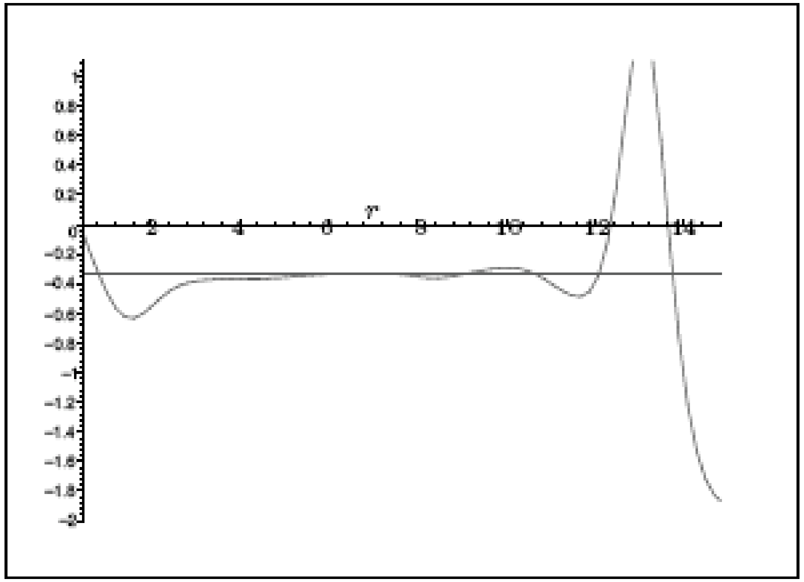



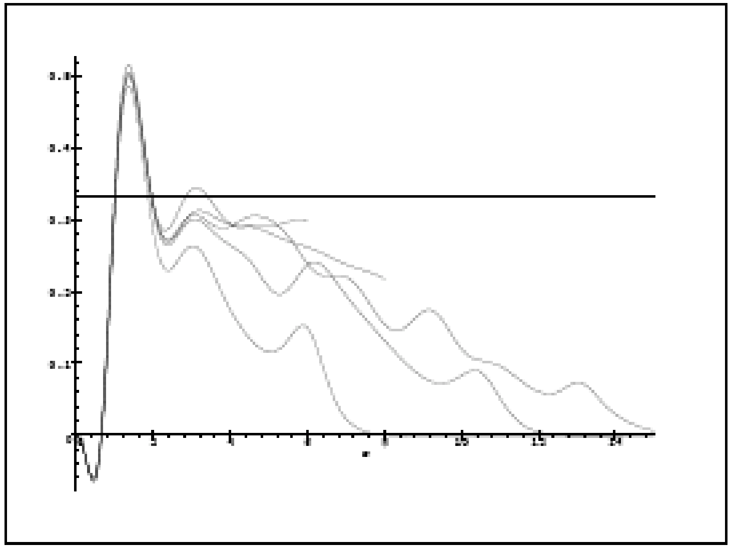




| N | D | CF | Laughlin |
|---|---|---|---|
| 3 | 3 | 1 | 1 |
| 4 | 11 | 0.9969 | 0.9987 |
| 5 | 46 | 0.9930 | 0.9967 |
| 6 | 217 | 0.9941 | 0.9885 |
| 7 | 1069 | 0.9828 | 0.9651 |
| 8 | 5529 | 0.9671 | 0.9365 |

Jeon and Jain, 2003
Kasner and Apel, 1993
Girlich and Hellmund, 1994
Bonesteel and Melik-Alaverdian, 1998




CF quasiparticle



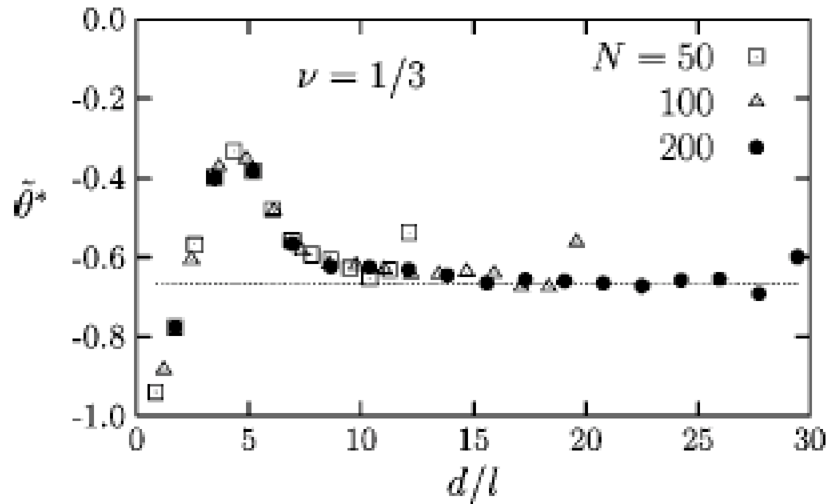
Laughlin

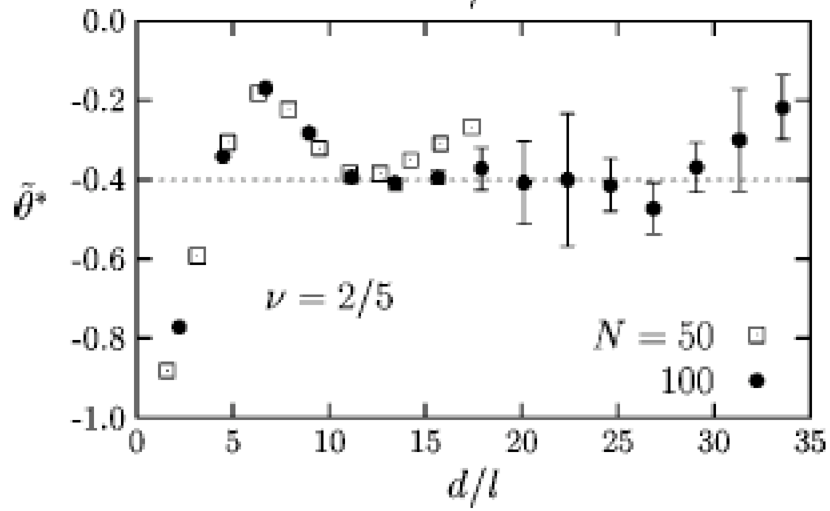
Kjonsberg and Leinaas (1999): Braiding statistics of CF quasiparticles at 1/3 is meaningful.

$$\Psi^{\eta,\eta'}_{1/(2p+1)} = \mathcal{P}_{LLL} \begin{vmatrix} \phi^{(1)}_{\eta}(\boldsymbol{r}_1) & \phi^{(1)}_{\eta}(\boldsymbol{r}_2) & \dots \\ \phi^{(1)}_{\eta'}(\boldsymbol{r}_1) & \phi^{(1)}_{\eta'}(\boldsymbol{r}_2) & \dots \\ 1 & 1 & \dots \\ z_1 & z_2 & \dots \\ \cdot & \cdot & \dots \\ \cdot & \cdot & \dots \\ z_1^{N-3} & z_2^{N-3} & \dots \end{vmatrix} \times \prod_{i<k=1}^{N} (z_i - z_k)^{2p} \exp\left(-\sum_j |z_j|^2/4l^2\right).$$

$$\Psi^{\eta,\eta'}_{2/5} = \mathcal{P}_{LLL} \begin{vmatrix} \phi^{(2)}_{\eta}(\boldsymbol{r}_1) & \phi^{(2)}_{\eta}(\boldsymbol{r}_2) & \dots \\ \phi^{(2)}_{\eta'}(\boldsymbol{r}_1) & \phi^{(2)}_{\eta'}(\boldsymbol{r}_2) & \dots \\ \bar{z}_1 & \bar{z}_2 & \dots \\ \bar{z}_1 z_1 & \bar{z}_2 z_2 & \dots \\ \vdots & \vdots & \dots \\ \bar{z}_1 z_1^{N/2-3} & \bar{z}_2 z_2^{N/2-3} & \dots \\ 1 & 1 & \dots \\ z_1 & z_2 & \dots \\ \vdots & \vdots & \dots \\ z_1^{N/2-1} & z_2^{N/2-1} & \dots \end{vmatrix} \times \prod_{i<k=1}^{N} (z_i - z_k)^2 \exp\left(-\sum_j |z_j|^2/4l^2\right).$$

$$\bar{\phi}^{(n)}_{\eta}(\boldsymbol{r}) = (\bar{z} - \bar{\eta})^n \exp\left[\frac{\bar{\eta} z}{2l^{*2}} - \frac{|\eta|^2}{4l^{*2}} - \frac{1}{4l^{*2}}|z|^2\right].$$




The CF-quasiparticles obey well defined braiding statistics provided they are separated by >10 magnetic lengths.

Jeon, Graham, Jain, PRL 2003, PRB 2004

# Non-Abelian braiding statistics

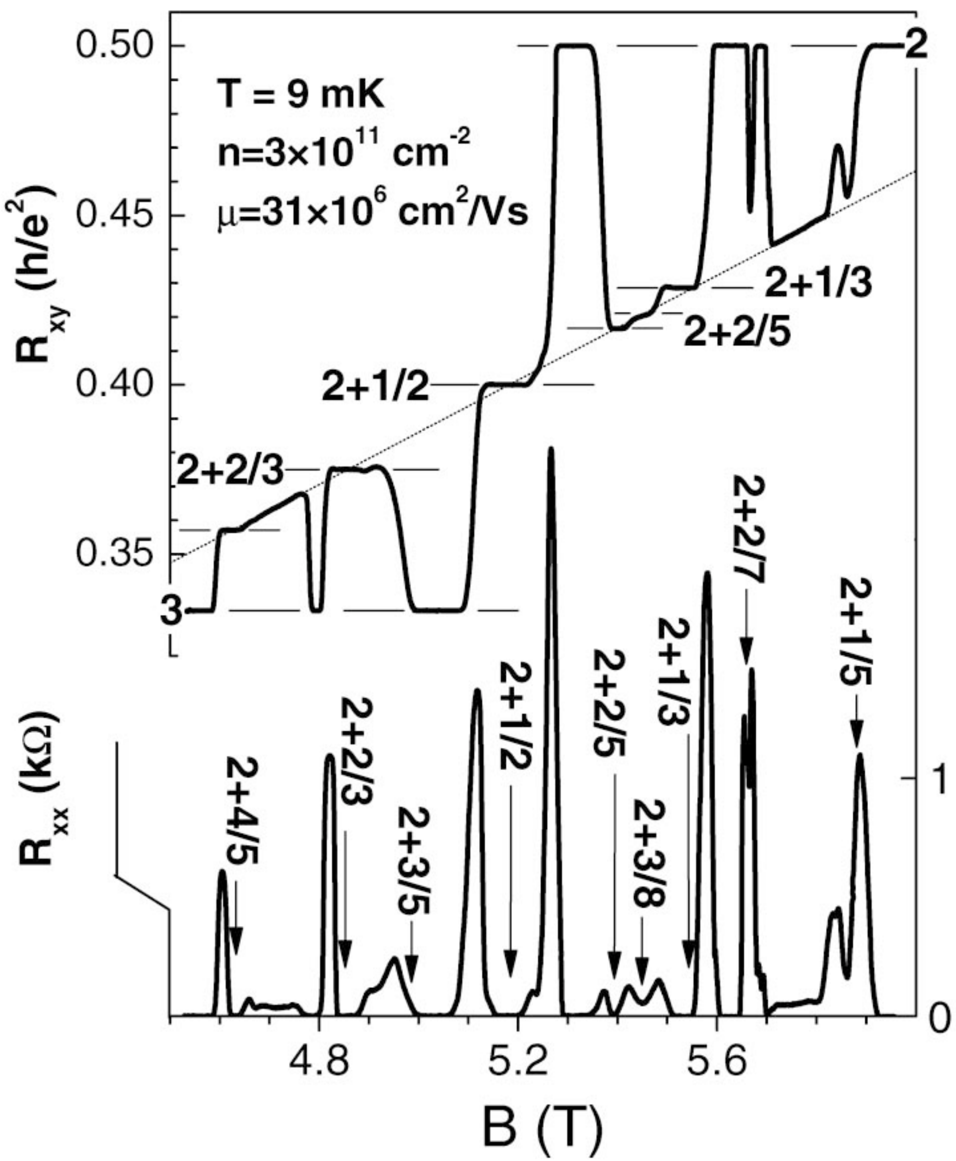
Proposal I: The quasiparticles of "paired CF states" obey non-abelian braiding statistics.

Moore and Read, 1991

Proposal II: These can be used for topological quantum computation.

Das Sarma, Nayak, Freedman, 2004
Stern and Halperin, 2005

# 5/2 FQHE




Willett et al.
Pan et al.
Xia et al.

# Pfaffian wave function

- A CF Fermi sea is seen at 1/2, but a FQHE is seen at 5/2.
- It is proposed that the 5/2 CF Fermi sea is unstable to a BCS-like p-wave pairing of composite fermions, which opens a gap (Greiter, Wen, Wilczek).
- The paired state is described by a "Pfaffian" wave function (Moore and Read):

$$\Psi^{\mathrm{Pf}}_{1/2} = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)^2 \exp\left[-\frac{1}{4}\sum_k |z_k|^2\right]$$

$$\mathrm{Pf}\ M_{ij} = A(M_{12}M_{34}\cdots M_{N-1,N})$$

Compare with:

$$\Psi_{\mathrm{BCS}} = A[\phi_0(\boldsymbol{r}_1, \boldsymbol{r}_2)\phi_0(\boldsymbol{r}_3, \boldsymbol{r}_4)\cdots\phi_0(\boldsymbol{r}_{N-1}, \boldsymbol{r}_N)]$$

comparison with the exact 5/2 state:

| $N$ | 8 | 10 | 14 | 16 |
|---|---|---|---|---|
| overlap | 0.87 | 0.84 | 0.69 | 0.78 |

Scarola, Jain, Rezayi, 2002

# Pfaffian quasiholes

vortex: $$\prod_j (z_j - \eta)\Psi^{\rm Pf}_{1/2} = \prod_j (z_j - \eta){\rm Pf}\left(\frac{1}{z_i - z_j}\right)\Phi_1^2$$ (charge 1/2)

$$\Psi^{\rm Pf}_{1/2}(\eta, \eta') = {\rm Pf}\,(M_{ij})\ \Phi_1^2$$

2 quasiholes: $$M_{ij} = \frac{(z_i - \eta)(z_j - \eta') + (i \leftrightarrow j)}{(z_i - z_j)}$$ (charge 1/4 for each)

2n quasiholes: $$M_{ij} = \frac{\prod_{\alpha=1}^{n}(z_i - \eta_\alpha)(z_j - \eta_{\alpha+n}) + (i \leftrightarrow j)}{(z_i - z_j)}$$

The wave function for 2n quasiholes is not fully specified by their positions. There are $2^{n-1}$ linearly independent wave functions for a given set of positions.
(Naively, one would have thought $(2n)!/[2(n!)^2]$.)

# The origin of non-Abelian braiding statistics

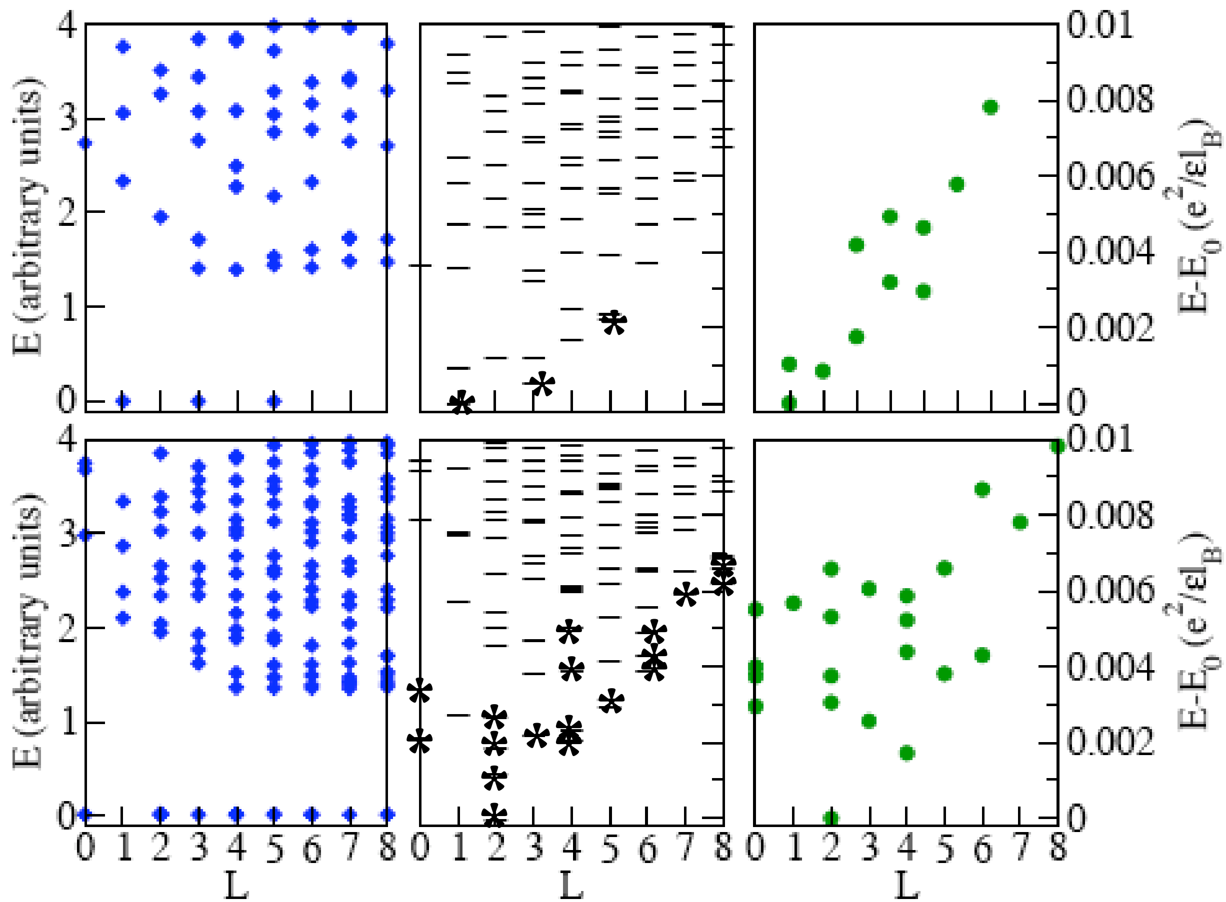
- The initial 2n-quasihole wave function is a linear superposition of several linearly independent wave functions.
- After a braiding operation, the wave function ends up, in general, in a different linear superposition, which is related to the initial one by a Berry matrix.
- Consecutive braidings do not commute. Hence, non-Abelian braiding statistics.

- The Pfaffian wave function for the ground state and the quasiholes is exact for a model three-body interaction (V3) that penalizes states in which three particles occupy the smallest angular momentum state.
- Non-Abelian statistics is on solid grounds for this interaction.
- Question: How well does it represent the solutions of the Coulomb interaction?

# Testing the Pfaffian quasihole wave functions

N=10
2 quasiholes

N=10
4 quasiholes




$$L = 1, 3, 5$$

$$L = 0^2, 1^0, 2^4, 3^1, 4^4, 5^2, 6^3, 7^1, 8^2, 9^0, 10^1$$

No one-to-one correspondence between the solutions of the model and the Coulomb interactions. Lack of adiabatic connection.

# Overlaps

## two quasiholes

| $L$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $N=8$ | 0.64 | - | 0.48 | - | 0.52 | - | - | - |
| $N=10$ | - | 0.05 | - | 0.56 | - | 0.61 | - | - |
| $N=12$ | 0.59 | - | 0.30 | - | 0.49 | - | 0.39 | - |
| $N=14$ | - | 0.39 | - | 0.13 | - | 0.39 | - | 0.27 |

## four quasiholes

| $N$ | $L=0$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 0.78 | 0.54 | 0.65 | 0.47 | 0.36 | 0.45 | - | 0.21 | - | - | - |
| 10 | 0.67 | 0.48 | 0.49 | 0.47 | 0.21 | 0.34 | 0.26 | 0.32 | - | 0.02 | - |
| 12 | 0.42 | 0.32 | 0.27 | 0.32 | 0.17 | 0.28 | 0.21 | 0.23 | 0.23 | 0.24 | 0.07 |

$$\mathcal{O} = \sum_{i,j}^{\mathcal{N}} \left| \left\langle \Psi^{(3)}_{\mathrm{4-qh,i}} \middle| \Phi^{\mathrm{C}}_{\mathrm{4-qh,j}} \right\rangle \right|^2 / \mathcal{N}$$

Regnault, Toke, and Jain, 2006

The overlaps are quite poor by FQHE standards.

# effect of disorder

- We ask the question how quasiholes and quasiparticles are localized by disorder.
- For this purpose, we study the two quasihole / quasiparticle state in the presence of one and two delta function impurities.
- For V3 interaction, a single delta function binds both quasiholes, producing a vortex, but might produce two charge 1/4 quasiholes for the Coulomb interaction.

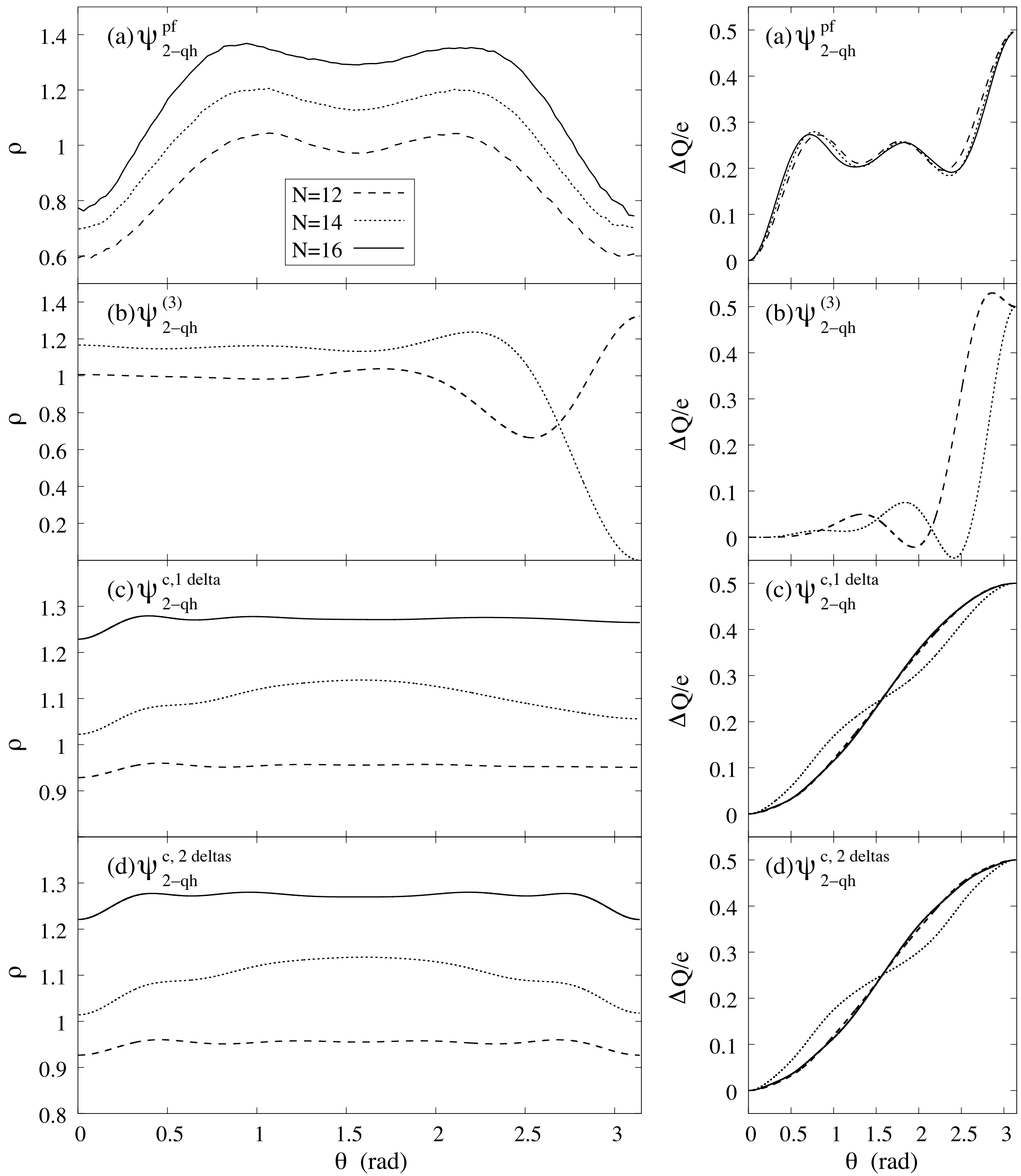
# two quasiholes in the presence of delta function impurities

two delta functions: Pfaffian w.f.

two delta functions:V3

single delta function: Coulomb

two delta functions: Coulomb



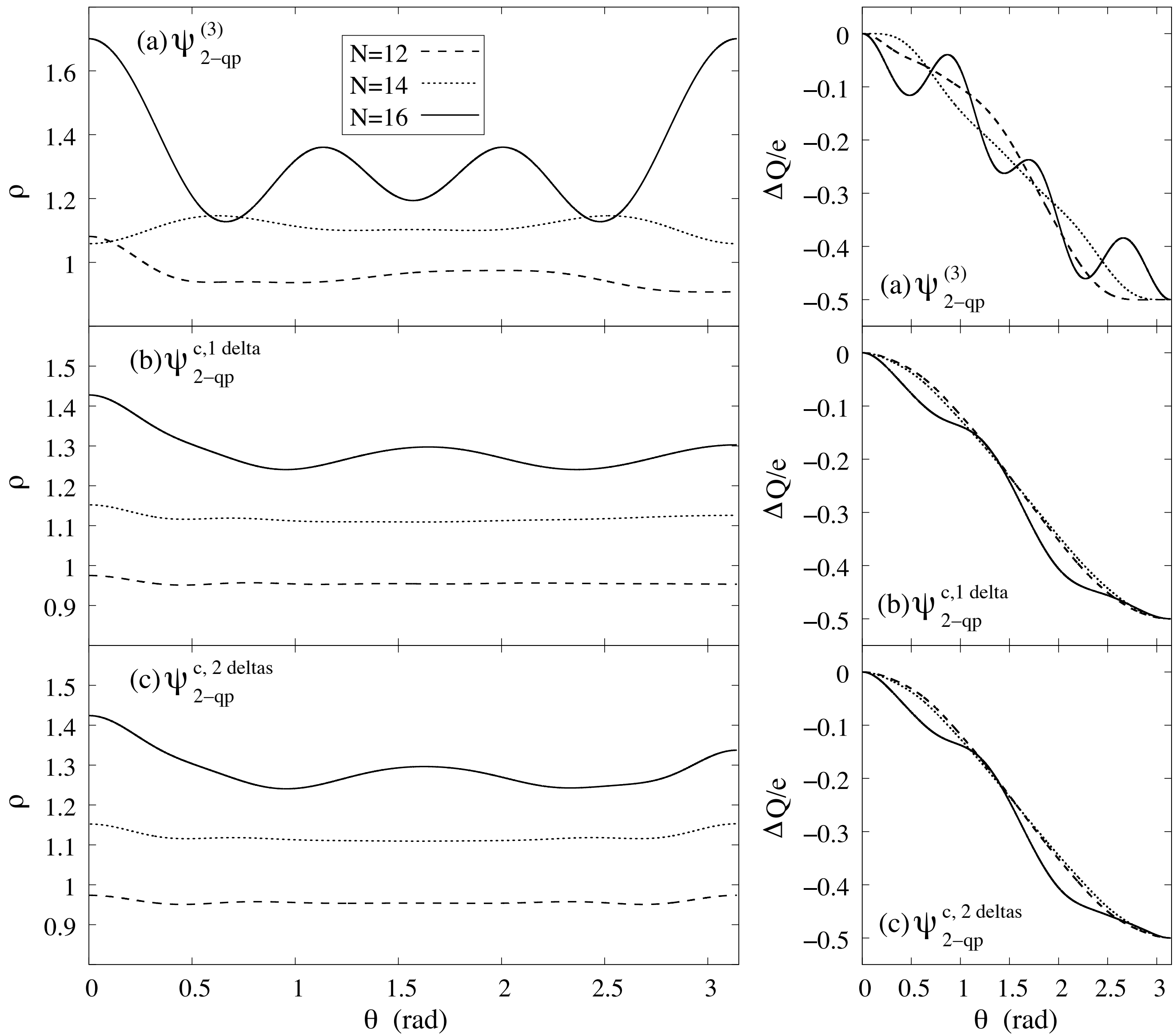

Regnault, Toke, and Jain, 2006

# two quasiparticles in the presence of delta function impurities

two delta functions: V3

single delta function: Coulomb

two delta functions: Coulomb




Regnault, Toke, and Jain, 2006

# Conclusions

- The FQHE state is a candidate for the realization of abelian fractional statistics. CF quasiparticles exhibit (theoretically) well-defined fractional braiding statistics in the low-density limit, which may have measurable consequences.
- A difficulty in measuring the fractional braiding statistics is that the statistical phase sits atop a large Aharonov-Bohm phase.
- The validity of the Pfaffian (or V3) model and the notion of non-Abelian braiding statistics for quasiholes and quasiparticles at 5/2 remain unconfirmed for the actual Coulomb state.