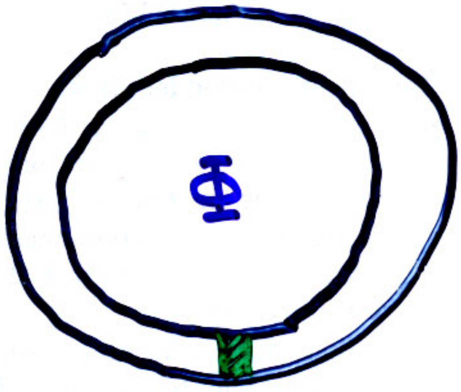
SUPERPOSITION AND ENTANGLEMENT OF STATES OF SOLID-STATE QUBITS

A. Flux-mode SQUID:

Superposition:

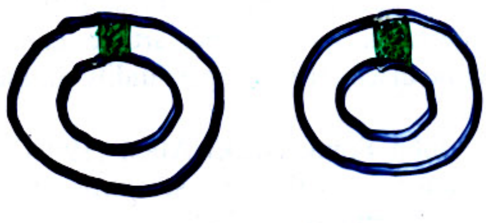


$$\Psi(\underset{\sim}{J}; \Phi) \sim$$

$$a\,\psi_L(\underset{\sim}{J})\chi_L(\Phi) + b\,\psi_R(\underset{\sim}{J})\chi_R(\Phi)$$

$$\langle \psi_L(J) | \psi_R(J) \rangle = 0$$

Entanglement:



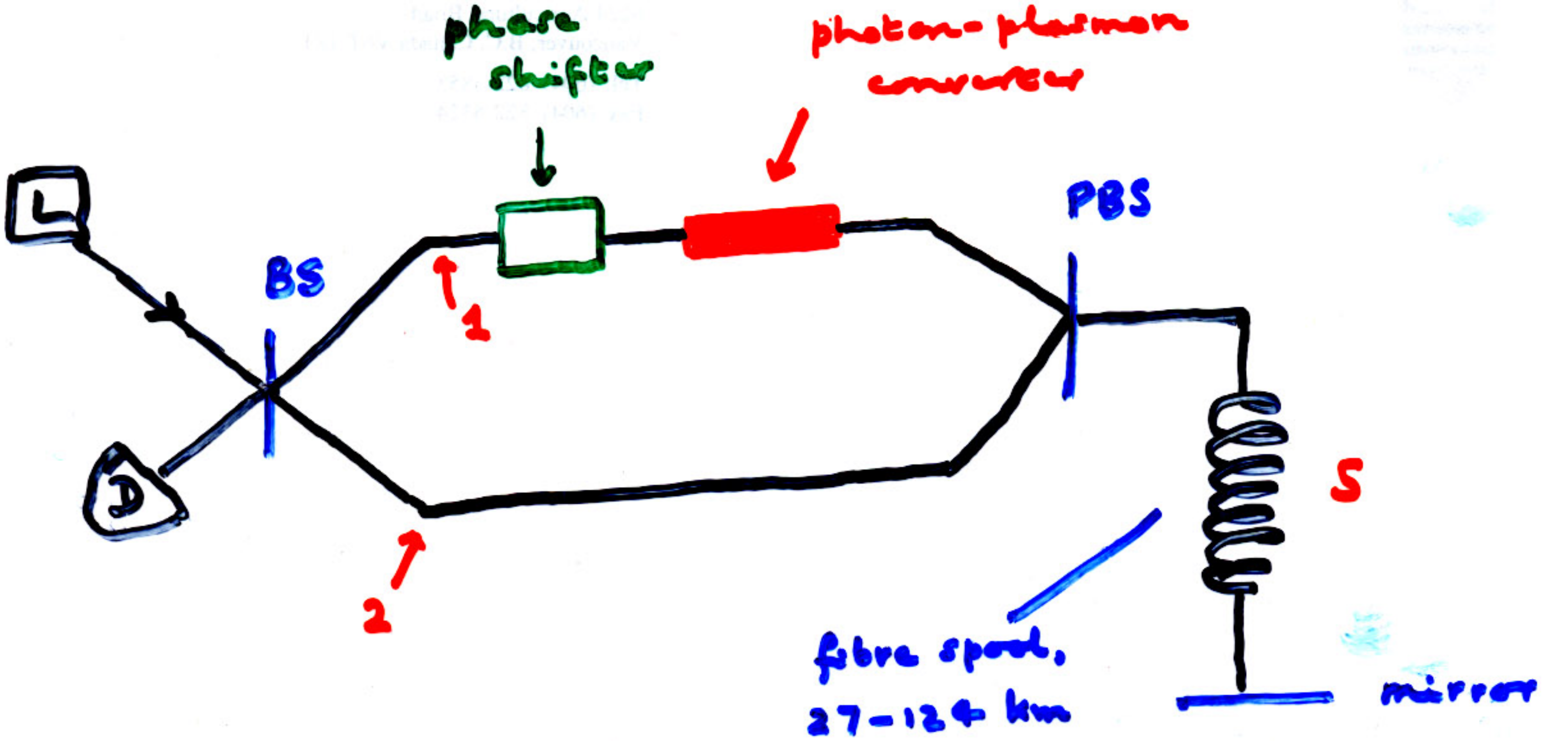
$$\Psi(J_1\Phi_1, J_2\Phi_2)$$

$$\sim a\,\psi_L(J_1)\chi_L(\Phi_1)\psi_R(J_2)\chi_R(\Phi_2) -$$

$$b\,\psi_R(J_1)\chi_R(\Phi_1)\psi_L(J_2)\chi_L(J_2)$$

again, $\langle \psi_L(J_1) | \psi_R(J_1) \rangle = \langle \psi_L(J_2) | \psi_R(J_2) \rangle = 0$

B. Surface plasmons (S. Fasel et al., NJP 8, 13 (2006))



Path A : L → 1 → PPC → S → 2 → D

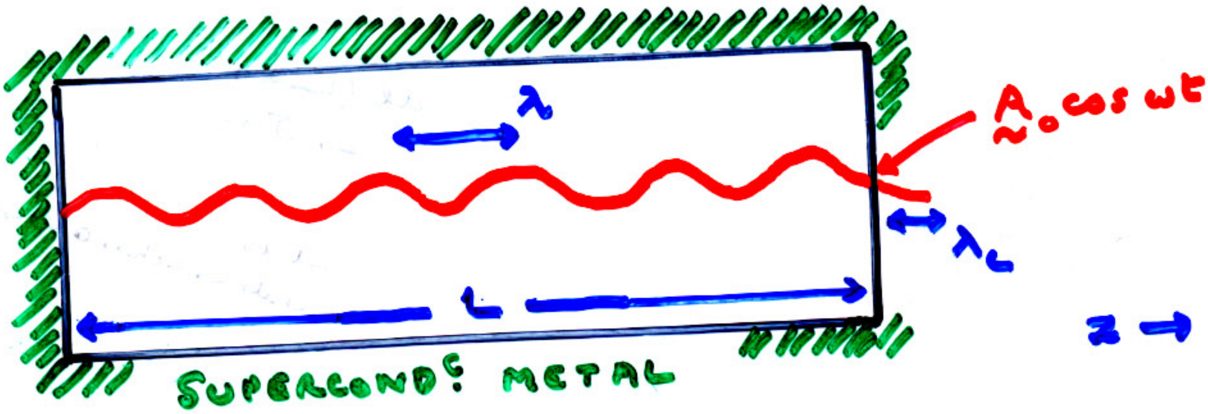
Path B : L → 2 → S → PPC → 1 → D

The time difference at which the photon is converted into a surface plasmon reaches a max. value > 1 msec !

Nevertheless, "fringe visibility" > 99%. !

?!

DIGRESSION: PHOTON CONFINED BY WALLS



The electrons in the walls are essential to confine the photon. Does this mean that they are strongly entangled with it? Assume $\omega \ll \omega_p$ ← plasma freq of sup^r

i.e.:

$$\Psi \sim \underbrace{a\overset{\text{vac}}{|0\rangle} + b\overset{1\gamma}{|1\rangle}}_{\text{naive description}} \Rightarrow \Psi \sim a|0\rangle|\chi_0\rangle + b|1\rangle|\chi_1\rangle$$

states of electrons

How orthogonal are χ_0 and χ_1?

Ans:

$$\mathcal{H}_{wall} = \int d\underset{\sim}{r} \left\{ + \frac{J^2(r)}{\epsilon_0 \omega_p^2} + \underline{J}(r)\cdot\underline{A}(r) + (\underline{\nabla}\times\underline{A})^2/2\mu_0 + \epsilon_0 \dot{\underline{A}}^2 \right\}$$

$$\Rightarrow \underset{\sim}{J}(\underset{\sim}{r}) = -\epsilon_0 \omega_p^2 \underset{\sim}{A}(\underset{\sim}{r})$$

so:

$$\Psi\{J(r), A(r)\} = \tilde{\delta}(J(r) + \epsilon_0 \omega_p^2 A(r))\, \Psi\{A(r)\}$$

so, qn. of orthogonality of electron states reduces to that of states of EM field in walls.

Normal modes of EM field corr. to Fourier comps. A_k, with freq.

$$\omega_k = (\omega_p^2 + c^2 k^2)^{1/2}$$

Now in a "classical" superpos^n of $|0\rangle$ and $|1\rangle$,

$$\langle A(z,t)\rangle \sim A_0 \exp - K(\omega) z \cos \omega t$$

$$[(\omega_p^2 - \omega^2)/c^2]^{1/2}$$

$$\rightarrow \langle A_k\rangle(t) \sim L_w^{-1/2} \frac{A_0}{K(\omega) + ik}$$

But, if $|k\rangle \equiv \dfrac{|0\rangle_k + \epsilon |1\rangle_k}{\sqrt{1+\epsilon^2}}$

$\epsilon_k^2 = \langle A_k\rangle^2 / \langle A_k^2\rangle_0$, and $\langle A_k^2\rangle_0 = L_w^{-1}\left(\dfrac{\hbar}{\epsilon_0 \omega_k}\right)$

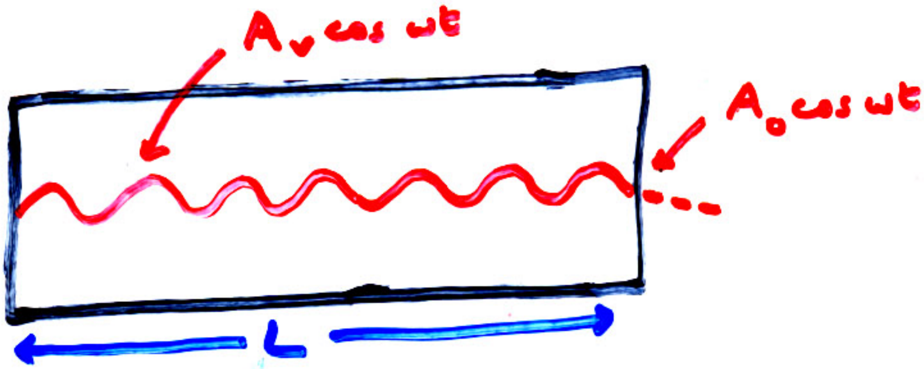
$\Rightarrow$ degree of orthogonality of whole system

$$\sim \sum_k \epsilon_k^2 = L_w^{-1} \sum_k \frac{\epsilon_0}{\hbar} \frac{\omega_k A_0^2}{K^2(\omega) + k^2} = \frac{\epsilon_0}{\hbar} A_0^2 L_w^{-1} \sum_k \frac{c^2 \omega_k}{\omega_k^2 - \omega^2}$$

$$= \frac{\epsilon_0}{\hbar} A_0^2 c^2 \frac{1}{2\pi} \int dk \frac{\omega_k}{\omega_k^2 - \omega^2} \sim \frac{\epsilon_0}{\hbar} A_0^2 c \ln\left(\frac{\lambda_L}{\lambda_D}\right)$$

Debye s. l.

So:



degree of orthogonality of electrons in walls,

$$1 - \left(|\langle \chi_0 | \chi_1 \rangle|^2\right) \approx \frac{\epsilon_0 A_0^2 c}{\hbar} \times \ln \text{term}$$

But, from usual boundary condition,

$$A_0 = \left(ik/\kappa(\omega)\right) A_v$$

and, from equipartition, for a single-photon state,

$$L \cdot \epsilon_0 \omega^2 A_v^2 = \hbar \omega$$

Hence,

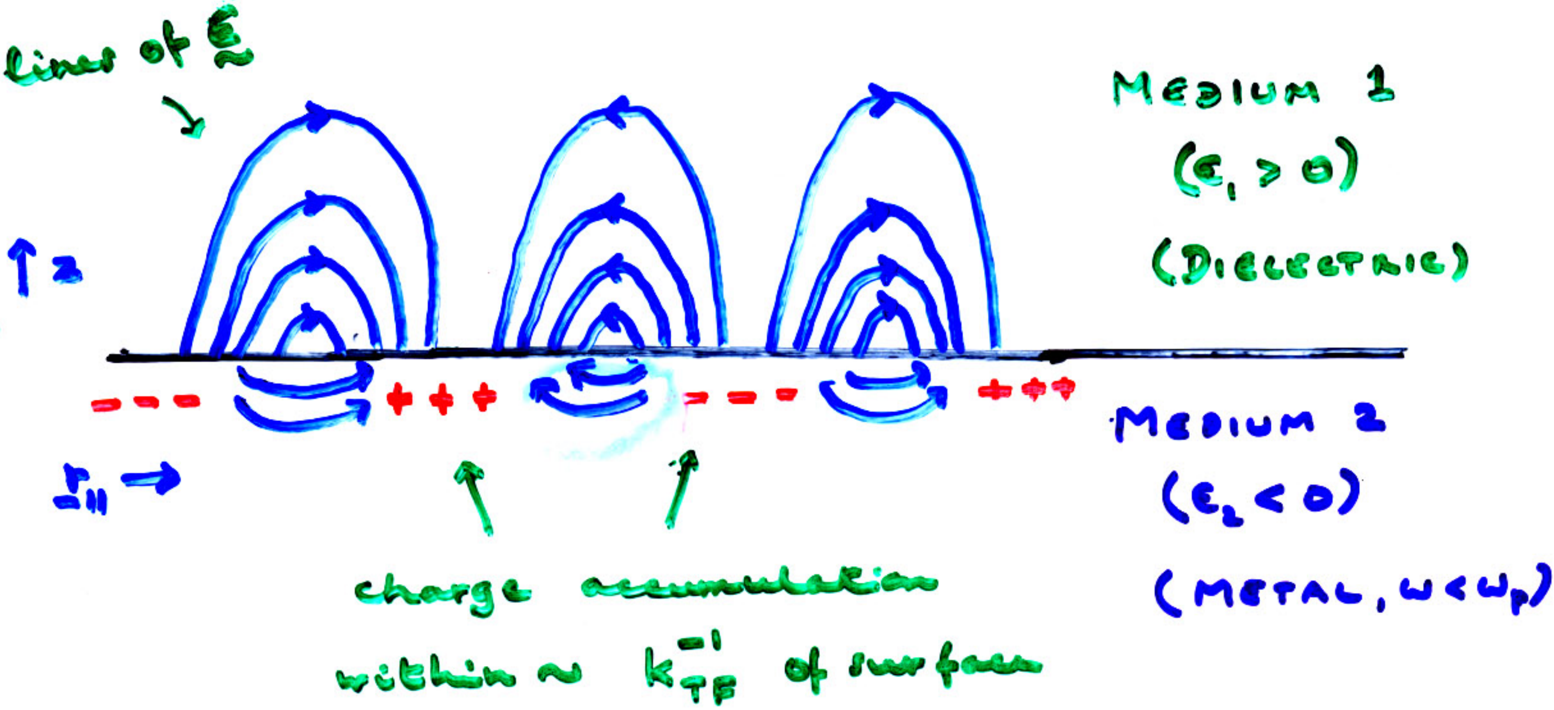
$$\frac{\epsilon_0 A_0^2 c}{\hbar} = \left(\frac{k^2}{\kappa^2(\omega)} \cdot \frac{\lambda}{L}\right) \overset{\omega \ll \omega_p}{\cong} \left(\frac{\lambda_L}{\lambda}\right)\left(\frac{\lambda_L}{L}\right)$$

CONCLUSION: UP TO NUMERICAL FACTORS, ELECTRONS IN WALLS ARE ENTANGLED ONLY TO DEGREE $(\lambda_L/L)(\lambda_L/\lambda) \ll 1$.

END OF DIGRESSION

Back to Surface Plasmons....

What is a Surface Plasmon?



Explicit formula for vector potential $\underline{A}(\underline{r}t)$:

$$A(\underset{\sim}{r}t) = R_e(A(z)\exp - i\omega t),$$

bound to surface

$$A_i(rt) = A_0 \exp i\underline{k}_{\parallel}\cdot\underline{r}_{\parallel} \exp - \kappa_i|z|$$

$$\times\left(\hat{\underset{\sim}{n}}_{\parallel} \pm (i\kappa_i/k_{\parallel})\,\hat{\underline{z}}\right)$$

$$\kappa_i = \frac{\omega}{c}\cdot\left(\frac{-\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}\right)^{1/2}, \quad k_{\parallel} = (\kappa_1\kappa_2)^{1/2}$$

Hence for $|\epsilon_2| \gg \epsilon_1$, field in dielectric is mostly $\perp$ surface, field in metal mostly $\parallel$ surface.

For simplicity take $1 \equiv$ vacuum ($\epsilon_1 = 1$),

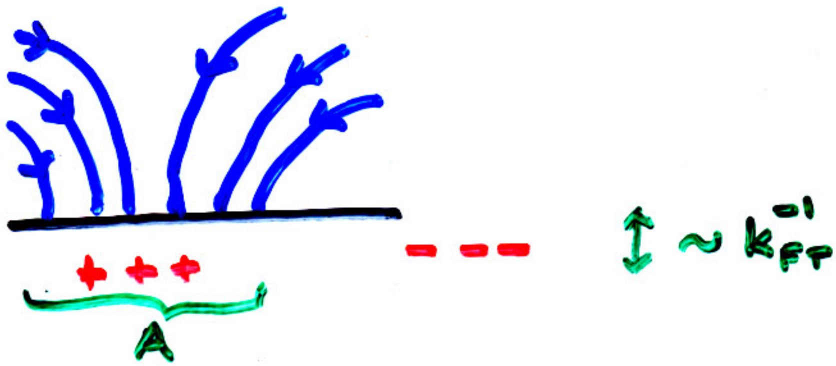
$2 =$ perfect metal ($\epsilon_2 = 1 - \omega_p^2/\omega^2$), and $\omega \ll \omega_p$.

Then:

$$K_1 = \frac{\omega}{\omega_1}\frac{\omega}{c}, \quad K_2 = \frac{\omega_p}{c} \quad (\equiv \lambda_L^{-1})$$

Can go through same argt. as for box, but for charge fluctuations easier to short-circuit:

$(\epsilon_{z1} \gg \epsilon_{z2})$



What is $\langle Q^2 \rangle / A$ when a single plasmon is present?

Since $\sigma \sim \epsilon_0 E_z'$, and normaliz^n: $\longrightarrow$

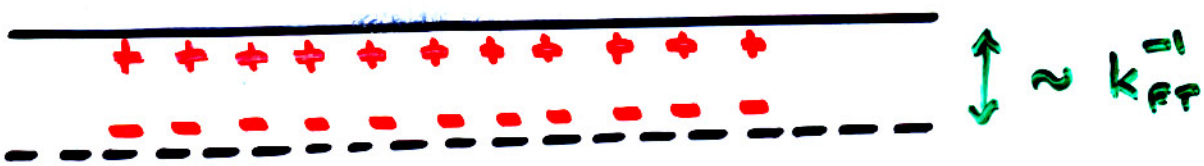
$\epsilon_0 (E_z')^2 K_1^{-1} A = \hbar\omega$, result is $(Q \sim \sigma A)$

$$\left[\langle Q^2 \rangle / A\right]_{1\ \text{plasmon}} \sim \epsilon_0 \hbar\omega \cdot K_1$$

Recap:

$\langle Q^2\rangle/A$ for single surface plasmon $\sim \epsilon_0 \cdot \hbar\omega \cdot \kappa_1$

Now, consider single "bulk" longitudinal plasmon of metal by itself: (or zero-point mode)



$\langle Q^2\rangle_0$ is df. by

$$\langle Q^2\rangle_0/C \sim \hbar\omega_p$$

and $C = \epsilon_0 A/d$.

Hence,

$$\langle Q^2\rangle_0/A = \epsilon_0\cdot\hbar\omega_p/d$$

Thus, finally,

$$\langle Q^2\rangle_{1sp}/\langle Q^2\rangle_0 \sim \frac{\omega}{\omega_p}\cdot\kappa_1 d \sim \left(\frac{\omega}{\omega_p}\right)^2 k_0 d$$

With realistic nos. this is $\lesssim 10^{-5}$!!

(Similar calc. for transverse current fluctns. gives $\sim (\omega/\omega_p)^3$).